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Testing Lorentz Invariance in QED (Part 1)

Brett Altschul

University of South Carolina

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Precis

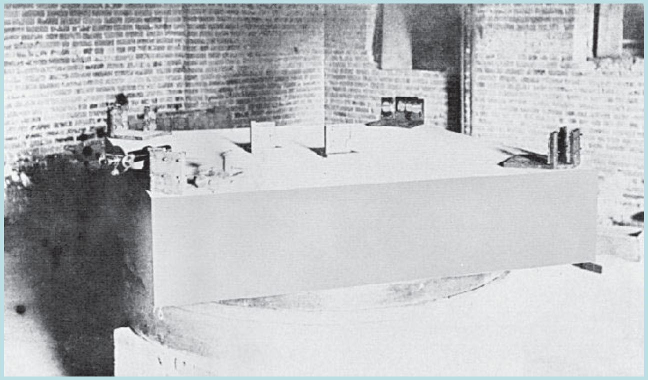
All of physics as we know it exhibits Lorentz symmetry—invariance under rotations and boosts—and CPT symmetry.

These invariances have been tested in matter-antimatter comparisons, meson oscillations, atomic clocks, and astrophysical polarimetery.

There are numerous candidate quantum gravity theories with LV, but nobody knows whether these are the exception or the rule.

We have been testing relativity experimentally for a long time.

The first good test was done in 1887, before special relativity was even understood.

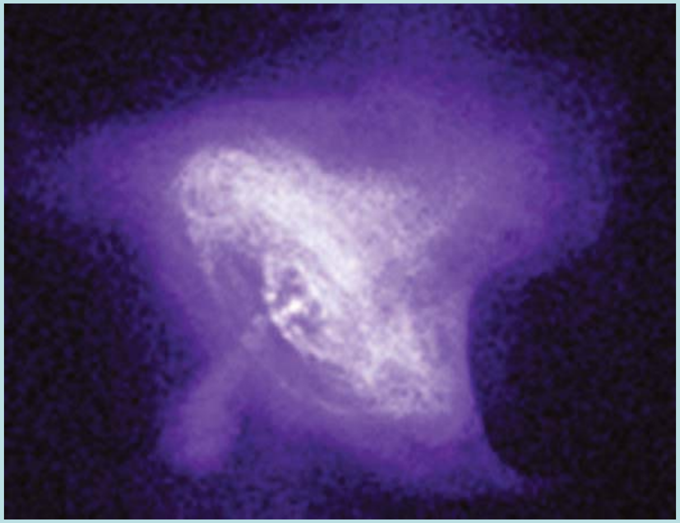


Michelson & Morley, 1887

And even 120 years later, Lorentz tests are still an active area of experimentation.

Outline

- Intro: Why Lorentz and CPT Violation?
- The Standard Model Extension (SME)
- Astrophysical Bounds
- Conclusion



Synchrotron radiation from the Crab.

Introduction

In the last ten years, there has been growing interest in the possibility that Lorentz and CPT symmetries may not be exact.

There are two broad reasons for this interest:

Reason One: Many theories that have been put forward as candidates to explain quantum gravity involve LV in some regime.

(For example, string theory, non-commutative geometry, loop quantum gravity...)

Reason Two: Lorentz symmetry is a basic building block of both quantum field theory and the General Theory of Relativity, which together describe all observed phenomena.

Anything this fundamental should be tested. Much of the story of modern theoretical physics is how important symmetries do not hold exactly.

There is no excellent beauty that hath not some strangeness in the proportion. — Francis Bacon

Although many quantum gravity theories involve LV and CPTV, it is not clear how ubiquitous the violations really are.

For example, the discovery that in string theory the tachyon potential often contains a minimum where Lorentz symmetry would be spontaneously broken spurred a great deal of interest in this subject.

[Kostelecký and Samuel, PRD **39**, 683 (1989)]

However, it now seems that this minimum is probably NOT the true vacuum.

One can try to answer this question using renormalization group techniques. This means looking for forms of Lorentz violation that are enhanced at low energies by quantum corrections.

There have been some interesting early developments in this area, but it's still probably too early to say anything definitive.

[BA and Kostelecký, PLB **628**, 106 (2005);
BA, Bailey, and Kostelecký, PRD **81**, 065028 (2010)]

Ultimately, we don't know where Lorentz violation might come from. However, any theory with CPT violation must also be Lorentz-violating.

[Greenberg, PRL 89, 231602 (2002)]

So it would be good to have a systematic framework for studying any possible Lorentz and CPT violations. This framework is the standard model extension (SME), which uses the known tools of effective field theory to describe all possible forms of Lorentz violation involving standard model fields.

Standard Model Extension (SME)

Idea: Look for all operators that can contribute to Lorentz violation.

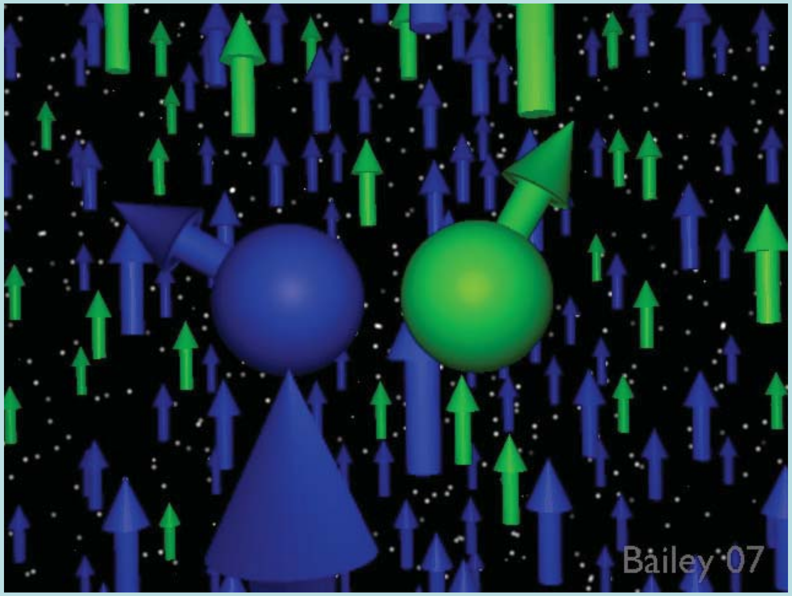
[Kostelecký and Colladay, PRD **58**, 116002 (1998)]

Then one usually adds restrictions:

- locality
- superficial renormalizability
- $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge invariance
- etc...

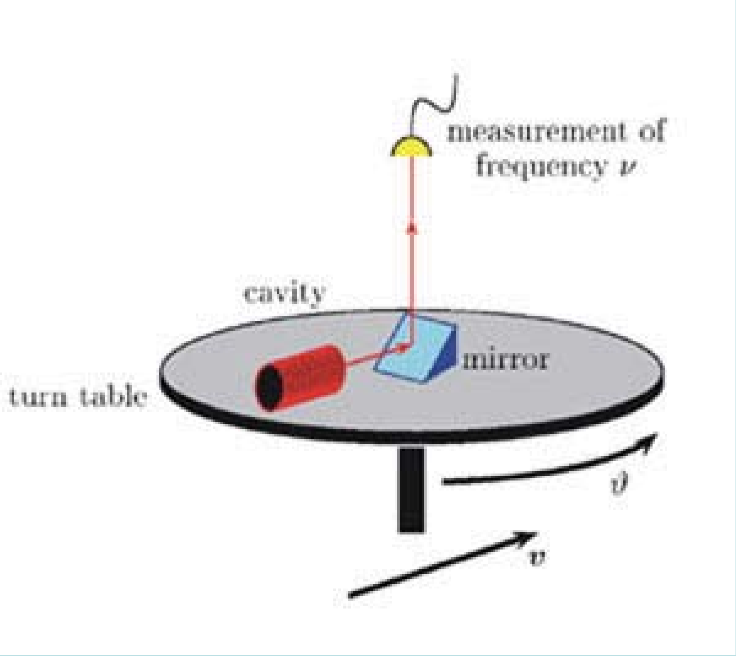
Many other formalisms turn out to be special cases of the SME.

Lorentz violating operators have objects built up from standard model fields, contracted with constant background tensors.



Earth-based laboratories will see slightly different local physics as the planet rotates and revolves.

However, using only the Earth's motion will prevent us from measuring certain Lorentz-violating quantities. (Some newer experiments are using actively rotating apparatuses to get around this.)



This adds sensitivity to a preferred direction parallel to the Earth's rotation axis.

The Lagrange density for a Lorentz-violating free Fermion theory is:

$$\mathscr{L} = \overline{\psi}\left(i\Gamma^{\mu}\partial_{\mu} - M\right)\psi$$

$$M = m + a^{\mu}\gamma_{\mu} + b^{\mu}\gamma_5\gamma_{\mu} + \tfrac{1}{2}H^{\mu\nu}\sigma_{\mu\nu}$$

$$\Gamma^{\mu} = \gamma^{\mu} + c^{\nu\mu}\gamma_{\nu} + d^{\nu\mu}\gamma_5\gamma_{\nu} + e^{\mu} + if^{\mu}\gamma_5 + \tfrac{1}{2}g^{\lambda\nu\mu}\sigma_{\lambda\nu}$$

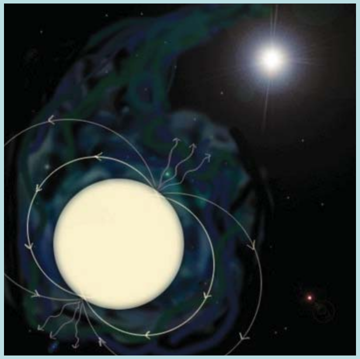
a, b, e, f, and g also violate CPT.

A separate set of coefficients will exist for every elementary particle in the theory.

When Lorentz symmetry is broken, angular momentum is not conserved.

One can look for this effect directly—by looking for wobbling in pulsars, for example. The effective moment of inertia of a pulsar is:

$$I_{jk} = I_0\left[\delta_{jk} + \frac{1}{2}c_{(jk)}\right]$$



The observed absence of wobbles lets one place bounds of 10^{-8} on the neutron c.

[BA, PRD 75, 023001 (2006)]

Not all the apparently-LV terms appearing the Lagrangian are truly physical. Take a, for instance.

A redefinition of the fermion field

$$\psi' = e^{ia^\mu x_\mu}\psi$$
$$\overline{\psi}' = e^{-ia^\mu x_\mu}\overline{\psi}$$

will eliminate a from the single-fermion Lagrangian. This is like a gauge transformation, but there is no corresponding change to the gauge field.

$$\mathcal{L} = e^{ia^\mu x_\mu} \overline{\psi}' \left[\gamma^\mu \left(i\partial_\mu - a_\mu \right) - m \right] e^{-ia^\mu x_\mu} \psi'$$

$$\mathcal{L} = e^{ia^\mu x_\mu} \overline{\psi}' e^{-ia^\mu x_\mu} \left(i\gamma^\mu \partial_\mu - m \right) \psi'$$

$$- a_\mu e^{ia^\mu x_\mu} \overline{\psi}' e^{-ia^\mu x_\mu} \gamma^\mu \psi' + e^{ia^\mu x_\mu} \overline{\psi}' \gamma^\mu \psi' \left(i\partial_\mu e^{-ia^\mu x_\mu} \right)$$

$$\mathcal{L} = \overline{\psi}' \left(i\gamma^\mu \partial_\mu - m \right) \psi'$$

In terms of the new field variables, there is no LV. This means that a does not affect any observables that do not involve flavor-changing interactions.

A lot can be learned just from how the CPT and Lorentz violation affect the velocity.

Exact solutions of the LV Dirac equation are awkward and difficult to obtain.

A better method is to consider the operator structure of the velocity.

$$\dot{x}_k = i\left[H, x_k\right]$$

$$\dot{\alpha}_k = i\left[H, \dot{\alpha}_k\right]$$

$$H = \beta\Gamma^j p_j + \beta m$$

(Although we have to be a little careful about the definitions of our Dirac matrices.)

In the presence of (spin-independent) c only,

$$H = \alpha_j p_j - c_{lj}\alpha_l p_j - c_{0j} p_j + \beta m$$

This makes the velocity

$$\dot{x}_k = \alpha_k - c_{lk}\alpha_l - c_{0k}$$

Solving the equation of motion for α,

$$-i\dot{\alpha}_k = -2\alpha_k\left(H + c_{0j} p_j\right) + 2p_k - 2c_{kj} p_j$$

$$\alpha_k(t) = \left(p_k - c_{kj} p_j\right)\left(H + c_{0j} p_j\right)^{-1}$$
$$+\left[\alpha_k(0) - \left(p_k - c_{kj} p_j\right)\left(H + c_{0j} p_j\right)^{-1}\right] e^{-2i\left(H + c_{0j} p_j\right)t}$$

We drop the *Zitterbewegung* (although it is important to the Darwin Hamiltonian term).

$$v_k = \left(p_k - c_{kj} p_j - c_{jk} p_j + c_{jk} c_{jl} p_l\right)\left(H + c_{0j} p_j\right)^{-1} - c_{0k}$$

From this expression, we can see when the effective field theory breaks down. The velocity may become superluminal when $E \approx m / \sqrt{c}$. If $c \approx m / M_P$, this is $E \approx \sqrt{m M_P}$.

More generally, momentum eigenstates may not be eigenstates of velocity.

The same velocity can be derived as a group velocity, but it's generally hard to get the dispersion relation. In the presence of generic LV, there can be four inequivalent solutions to the energy eigenvalue equation.

However, with just c, it's not bad:

$$E=\sqrt{m^2+\left(p-c_{kj}p_j\right)\left(p-c_{kl}p_l\right)}-c_{0j}p_j$$

We get the previous velocity just from $\partial E/\partial p_k$.

The photon sector contains more superficially renormalizable couplings.

$$\mathscr{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{4}\left(k_F\right)^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma} + \frac{1}{2}\left(k_{AF}\right)^{\mu}\varepsilon_{\mu\nu\rho\sigma}A^{\nu}F^{\rho\sigma}$$

Most of these couplings are easy to constrain with astrophysical polarimetry.

However, some will require more difficult laboratory measurements (e.g. with resonant cavities).

Both the CPT-odd k_{AF} and the CPT-even k_F cause vacuum birefringence (change in polarization during free propagation), but the effects are rather different.

For a plane wave solution of the modified Maxwell's equations, the modified Ampere's and Faraday's Laws with a k_F give

$$M_{jk}E_k = \left[\delta_{jk}p^2 + p_j p_k + 2(k_F)^{j\alpha\beta k} p_\alpha p_\beta\right]E_k = 0$$

The frequency eigenvalues from this are

$$p_{\pm}^{0} = \left(1 + \rho \pm \sigma\right)\left|\vec{p}\right|$$

$$\rho = -\tfrac{1}{2}\tilde{k}_{\alpha}^{\alpha}$$

$$\sigma = \sqrt{\tfrac{1}{2}\left(\tilde{k}^{\alpha\beta}\right)^{2} - \rho^{2}}$$

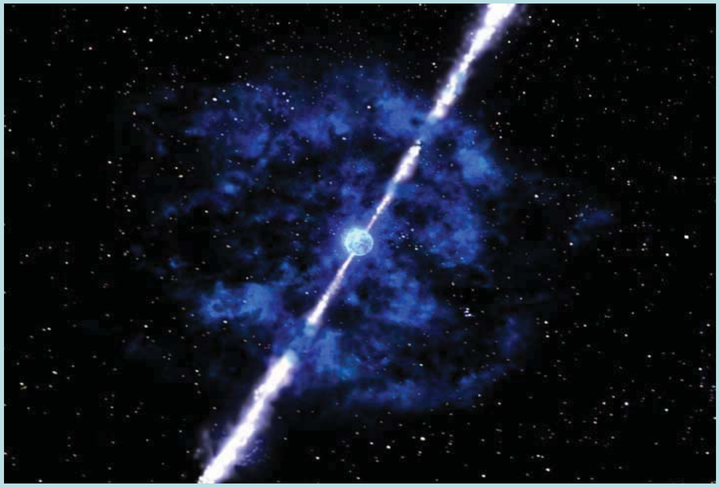
$$\tilde{k}^{\alpha\beta} = \left(k_{F}\right)^{\alpha\mu\beta\nu}\hat{p}_{\mu}\hat{p}_{\nu}$$

$$\hat{p}^{\mu} = p^{\mu} \Big/ \left|\vec{p}\right|$$

Plane waves are not dispersive, but their speed depends intricately on direction.

The existence of two frequencies for one momentum (if $\sigma \neq 0$) means the polarization can oscillate between different linear states.

With k_{AF}, the polarization eigenstates are circularly polarized. In this case, the birefringence causes a linearly polarized wave to rotate as it propagates.



Since we can see polarized sources out to cosmological distances (quasars and GRBs), the birefringent SME terms are constrained extremely tightly ($10^{-33} - 10^{-43}$ levels).

The most sensitive accelerator tests of Lorentz symmetry involve CPT tests with neutral mesons.

CPT-violating quantities, such as the $K^0 - \overline{K}^0$ mass difference are controlled by the phase

$$\delta_K \propto \gamma \frac{v_\mu \left(a_s^\mu - a_d^\mu\right)}{m_{K_L} - m_{K_S}}$$

[Kostelecký, PRL **80**, 1818 (1998)]

The dependence on the meson velocity has important consequences.

Experiments at higher energies are more sensitive, even when they apparently have the same sensitivity to the $K^0 - \overline{K}^0$ mass difference.

The rate of CPT violation also generally depends on the meson direction, and so will change as the Earth-based laboratory rotates.

CPT violation has been searched for in neutral K, D, and B meson systems, using both time-averaged and day-night asymmetry measurements.

Measurement Type	System	Coefficients	log Sensitivity	Source
oscillations	K (averaged)	a (d, s)	–20	E773 Kostelecký
	K (sidereal)	a (d, s)	–21	KTeV
	D (averaged)	a (u, c)	–16	FOCUS
	D (sidereal)	a (u, c)	–16	FOCUS
	B (averaged)	a (d, b)	–16	BaBar, BELLE, DELPHI, OPAL
	neutrinos	a, b, c, d	–19 to –26	SuperK Kostelecký, Mewes
birefringence	photon	k_{AF} (CPT odd)	–43	Carroll, Field, Jackiw
		k_F (CPT even)	–32 to –37	Kostelecký, Mewes
resonant cavity	photon	k_F (CPT even)	–17	Muller et al.
anomaly frequency	e-/e+	b (e)	–23	Dehmelt et al.
	e- (sidereal)	b, c, d (e)	–23	Mittleman et al.
	mu/anti-mu	b (mu)	–22	Bluhm, Kostelecký, Lane
cyclotron frequency	H-/anti-p	c (e, p)	–26	Gabrielse et al.
hyperfine structure	H (sidereal)	b, d (e, p)	–27	Walsworth et al.
	muonium (sid.)	b, d (mu)	–23	Hughes et al.
clock comparison	various	b, c, d (e, p, n)	–22 to –30	Kostelecký, Lane
	He-Xe	b, d (n)	–32	Bear et al. Cane et al.
torsion pend.	spin-polarized solid	b, d (e)	–29	Heckel et al. Hou et al.
gamma-ray astronomy	e- /photons	c, d (e)	–15 to –20	Altschul

Synchrotron Emission Bounds

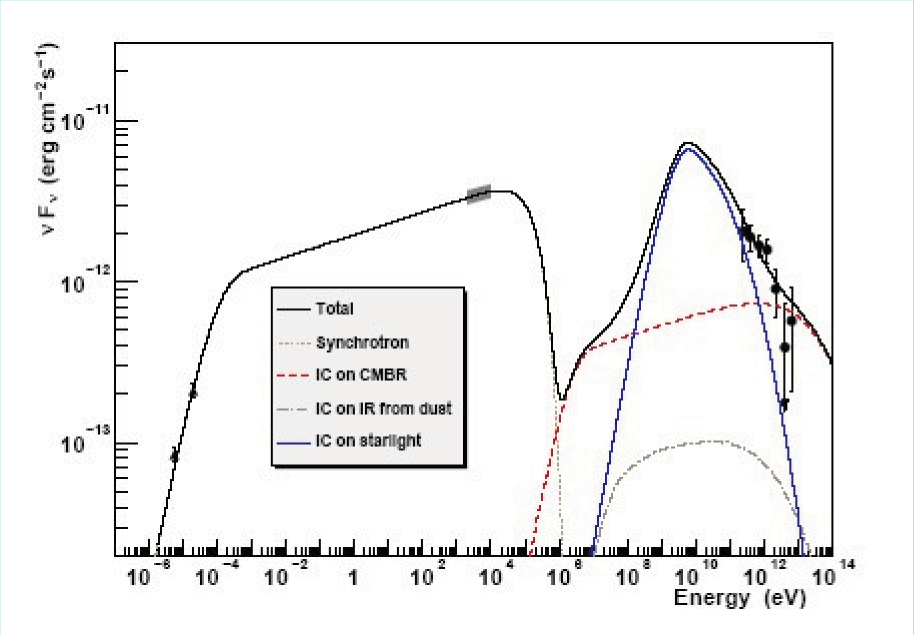
[BA, PRL **96**, 201101 (2006); PRD **72**, 085003 (2005); **74**, 083003 (2006)]

Some of the effects of Lorentz violation should become more important at high energies, so it is natural to look for their effects on astrophysics, where the very highest energies are available.

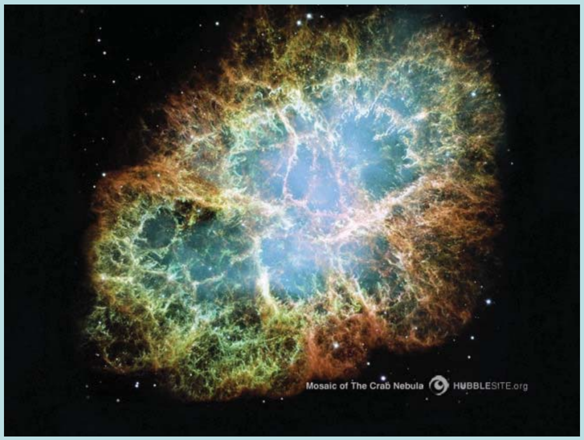
(Other astrophysical bounds may make use of the extremely large distances available, to magnify small light propagation effects.)

There's a lot we can learn from the radiation we see from very energetic sources.

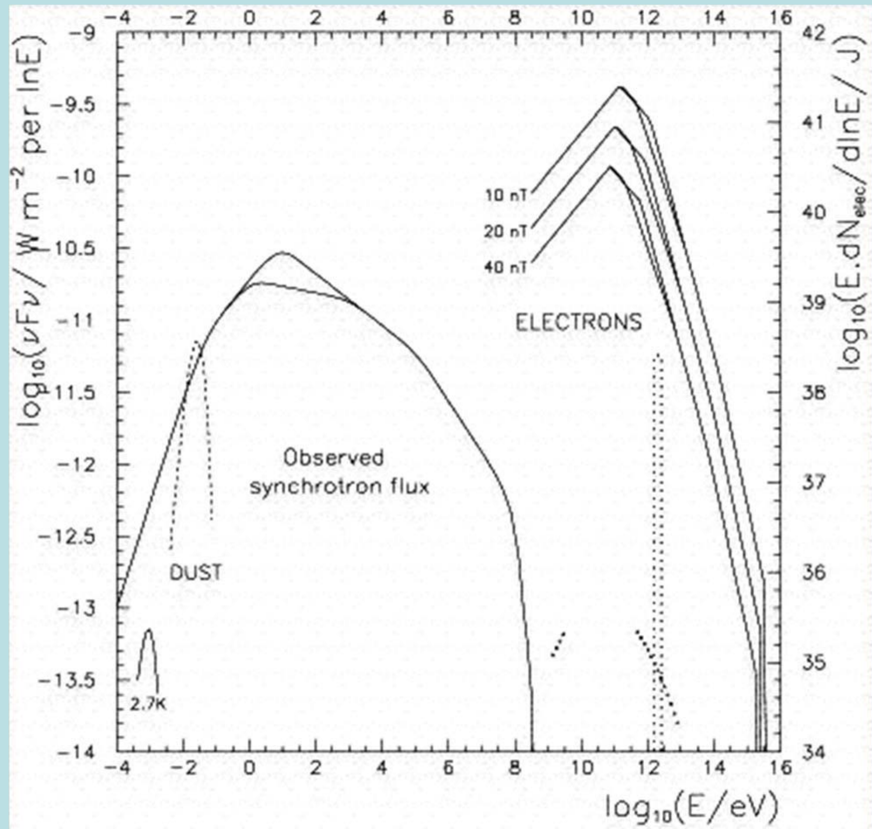
Different parts of the spectrum can tell us about different kinds of Lorentz violation.



The highest energy particles we see are cosmic rays above the GZK limit, but we do not understand them all that well.



So the best thing to concentrate on is high-energy sources that we *do* understand.



At high energies, the c-type Lorentz violation mentioned earlier is the most important.

(Its effects grow as γ^2.)

Neglecting higher order corrections, the maximum electron velocity in a direction $\hat{e}$ is:

$$v < 1 - c_{jk}\hat{e}_j\hat{e}_k - c_{0j}\hat{e}_j$$

This turns out to have readily measurable consequences.

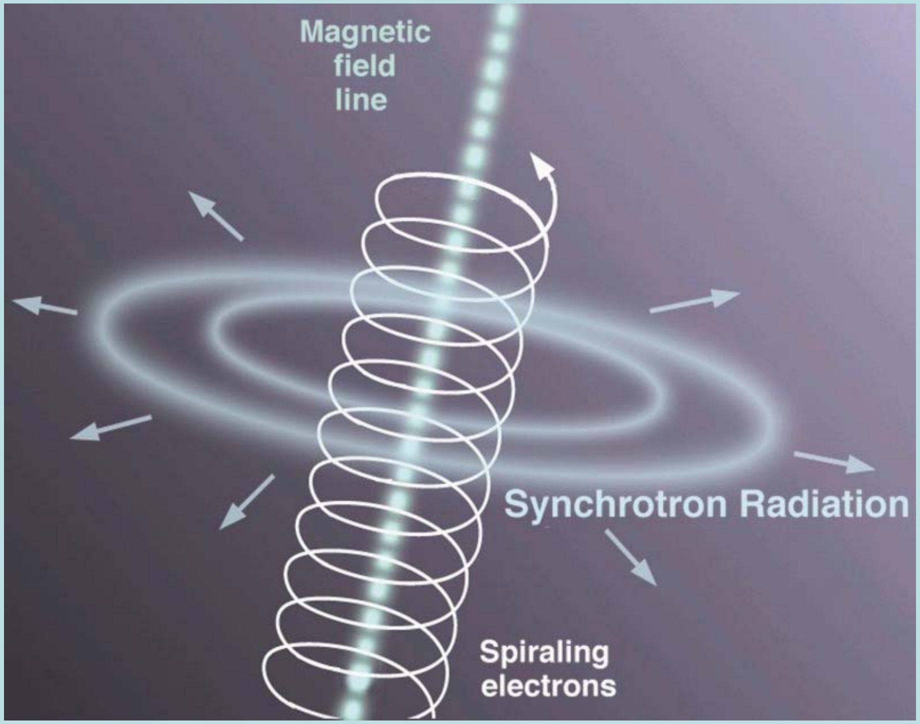
Electromagnetic coupling still enters through the replacement $p^\mu \rightarrow p^\mu - eA^\mu$. This makes the quantity coupled to A in $\mathscr{L}$ precisely the same one as appeared in calculating the velocity.

So we have usual the interaction Lagrangian

$$L_{\text{int}} = -ev^\mu A_\mu$$

Then if a charge's trajectory is known, the radiation it emits is exactly the same as in conventional QED.

In models of PWN, the cutoff of the synchrotron spectrum depends strongly on the maximum velocity (not energy or momentum!) of electrons moving in an Earthward direction:



$$\omega_c^2 \propto \frac{\left|\vec{B}\right|}{1-v_{\max}^2}$$

The magnetic field is “easy” to estimate.

If velocities up to $v_{\max}$ are observed, this limits the Lorentz violation to be smaller than:

$$c_{jk}\hat{e}_j\hat{e}_k + c_{0j}\hat{e}_j < \frac{1}{2\gamma_{\max}^2} = \frac{1-v_{\max}^2}{2}$$

Observations in different directions give bounds on different combinations, but *we can only get bounds this way if the maximum electron speed is less than the speed of light.*

Inverse Compton Bounds

So what happens if the maximum electron speed is greater than the speed of light?

I don't know exactly, but whatever it is, it is definitely "new physics."

One thing we would expect is vacuum Cerenkov radiation.

Can we detect vacuum Cerenkov radiation?

Perhaps. It has some characteristic features, such as a power spectrum linear in the frequency.

However, there are many unanswered questions, and anyway, we expect the radiation to be of short duration. Superluminal electrons could be expected to lose energy at a rate of about 10^{14} GeV/s.

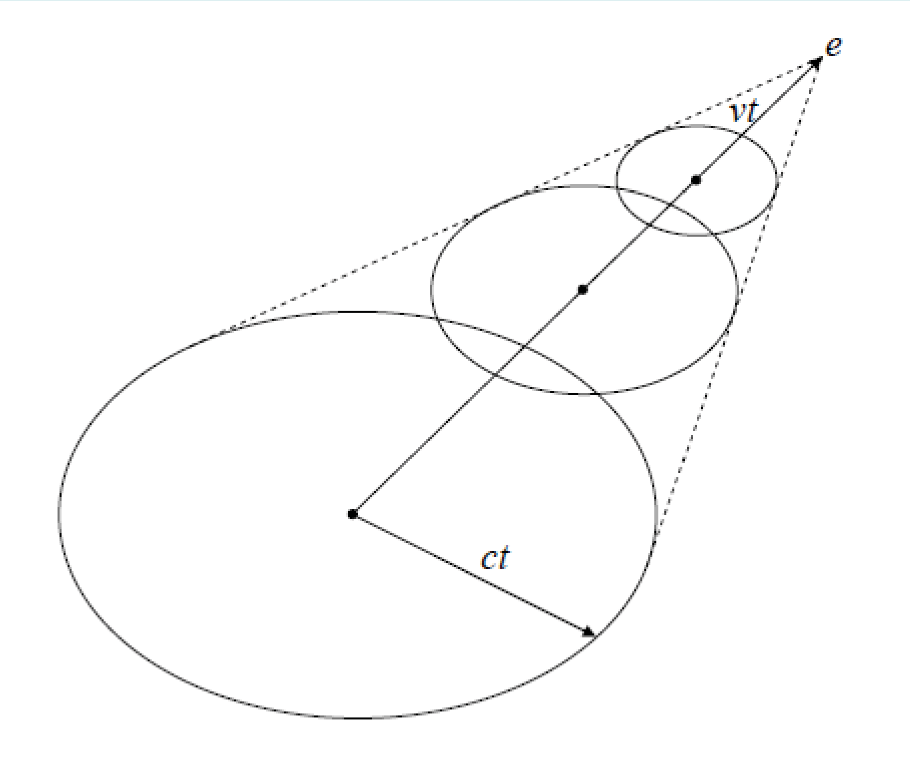
[BA, PRL 98, 041603 (2007)]

Vacuum Cerenkov radiation is much like ordinary Cerenkov emission, which occurs if a charged particle exceeds the phase speed of light in a material—an “optical boom.”

When boost invariance is broken, different species can have different limiting speeds, so the same thing can happen in vacuum.

The usual figure explaining Cerenkov radiation is distorted.

(Of course, this is exaggerated; we expect the cones to be *very* broad.)



Again, we can apply conventional formulas, but with a light speed in the direction of particle motion that may be different from 1.

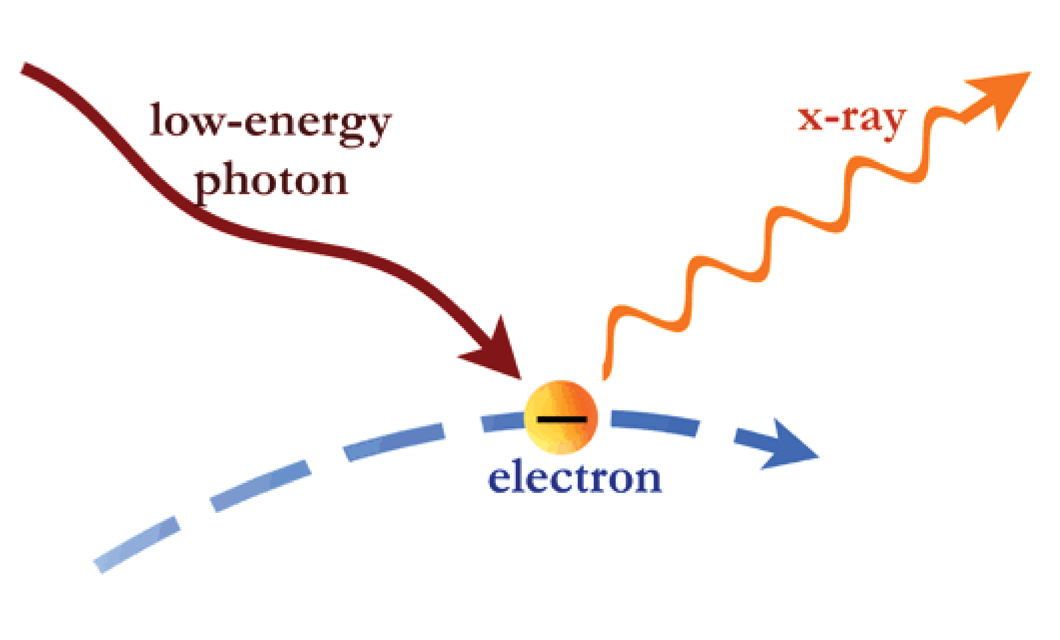
We want to find secure bounds, that don't rely on any inferences about "new physics."

This means we can only look at electrons moving subluminally, and if the maximum electron speed is greater than one, there will be a maximum energy for these electrons:

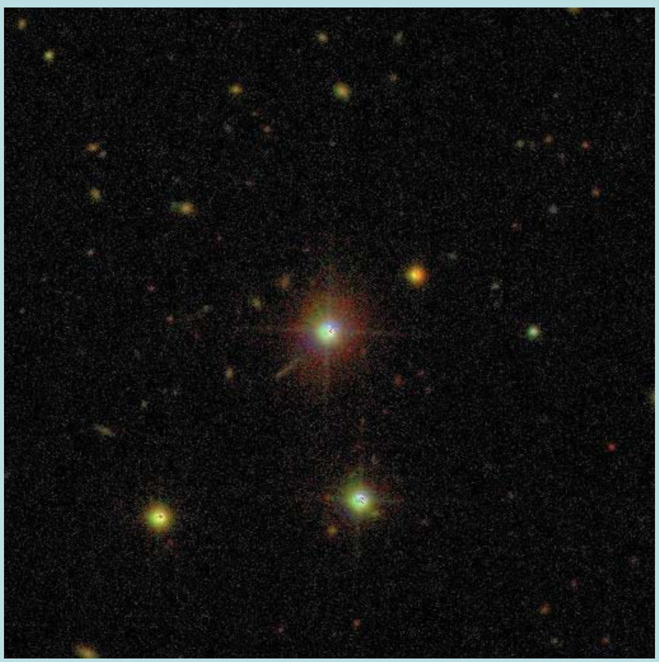
$$E_{\max}=\frac{m}{\sqrt{-2c_{jk}\hat{e}_j\hat{e}_k-2c_{0j}\hat{e}_j}}$$

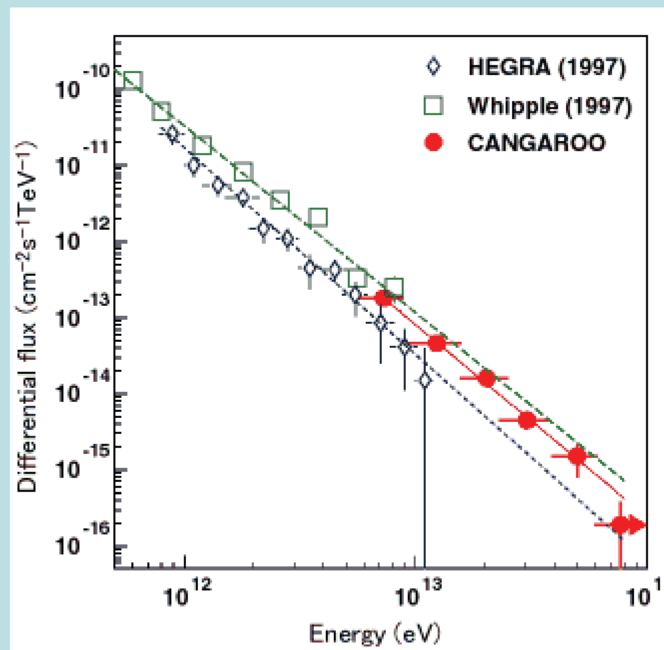
This also turns out to be something we can measure.

In inverse Compton scattering, $e^- + \gamma \rightarrow e^- + \gamma$, an ultrarelativistic electron collides with a lower energy photon and transfers a sizeable fraction of its energy.



Sources with superluminal electrons would exhibit anomalous emission features. The absence of such features indicates there are only subluminal particles, and the energy of observed inverse Compton photons is a lower limit on the highest electron energies.





This gives us the other half of the bound we need:

$$-\frac{1}{2\left(E_{obs}/m\right)^2} < c_{jk}\hat{e}_j\hat{e}_k + c_{0j}\hat{e}_j < \frac{1}{2\gamma_{\max}^2}$$

The best bounds of this sort are at the 10^{-20} level, coming from electrons with energies above 1 PeV.

By looking at different sources spread across the sky, we can get enough bounds to restrict all nine components of c to lie in a bounded region of parameter space.

With linear programming, one can then place independent bounds on the individual components. These are typically at the 10^{-15} level or better.

Currently, these are the best bounds on the electron c coefficients.

Conclusion

Tests of special relativity are still interesting and relevant.

Many Lorentz-violating coefficients are strongly constrained, but LV remains an interesting candidate to appear in a fundamental theory.

Emissions from astrophysical sources provide some of the best bounds for electrons and photons.

Thanks to V. A. Kostelecký, E. Pfister-Altschul.