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Quantum Particle Production in Cosmology

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UNIVERSITÉ
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Cargèse, April 2012

Outline

Introduction: The Friedmann-Lemaître universe

The observed Universe is very isotropic and homogeneous on large scales. On large scale its geometry is well approximated by an expanding Friedmann-Lemaître Universe,

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The rate of expansion, $\dot{a}/a$ and the deceleration/acceleration are related to the energy density of the Universe via Einstein's equation (called Friemann equations in this context).

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{K}{a^2} = \frac{8\pi G}{3}\rho + \frac{\Lambda}{3}\,, \qquad \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P) + \frac{\Lambda}{3}$$

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We measure energy densities, cosmological constant, curvature in terms of their contribution to the expansion rate $H \equiv \dot{a}/a$.

$$\Omega_X = \frac{8\pi G\rho_X}{3H^2}\,, \quad \Omega_\Lambda = \frac{\Lambda}{3H^2}\,, \quad \Omega_K = -\frac{K}{a^2H^2}\,,$$

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At present $\Omega_m \simeq 0.3$, $\Omega_b \simeq 0.04$, $\Omega_\gamma \simeq 10^{-4}$, $0.03 > \Omega_\nu > 0.003$, $\Omega_\Lambda \simeq 0.7$, $\Omega_K \simeq 0$.
If $\Lambda = 0$ and $P > -\rho/3$, $\Omega_K = 0$ is an unstable fix point of cosmic expansion.

Introduction: The CMB

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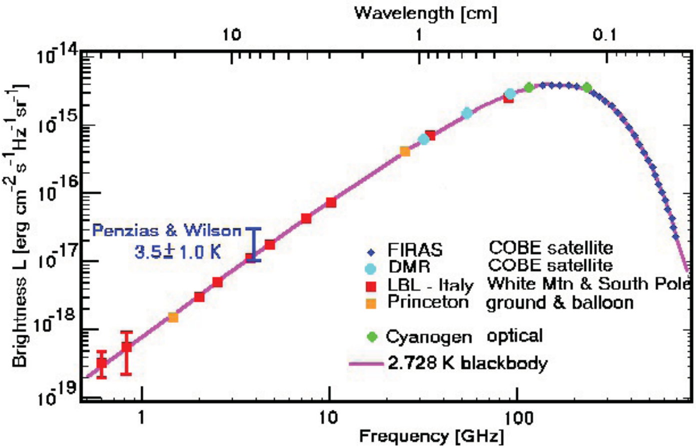
At a temperature of about $T \simeq 3000$K, $z \simeq 1100$, $t \simeq 3 \times 10^5$ years, protons and electrons combined to neutral hydrogen and the Universe became transparent to photons. Today we see these photons in the cosmic microwave background (CMB) which has a perfect Planck spectrum with temperature $T = 2.7255 \pm 0.0006$K.



Introduction: Thermal history

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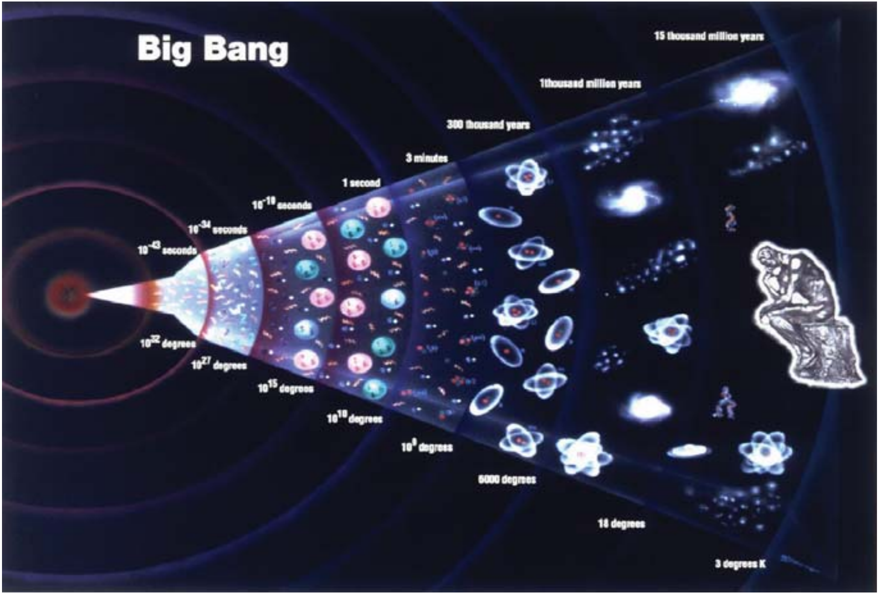

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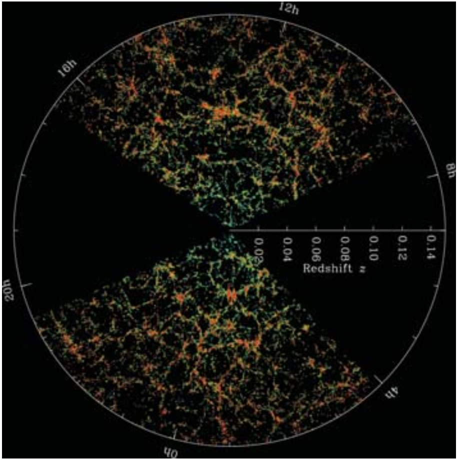

From the Sloan Digital Sky Survey

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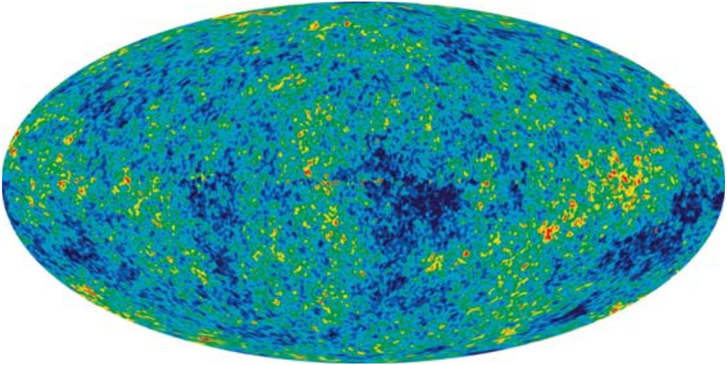
- This corresponds exactly to the fluctuations observed in the cosmic microwave background.



CMB anisotropies (WMAP 7 year data)

Inflation

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- The simplest way to obtain accelerated expansion is a universe dominated by a scalar field, ϕ where $\rho + 3P = \dot\phi^2 - 2V$ becomes negative if the potential dominates over the kinetic term.
- It has been clear already at this time, that an inflationary phase also generates a nearly scale invariant spectrum of scalar V. Mukhanov & G. Chibisov, 1981 and tensor A. Starobinski, 1979 fluctuations.

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- After inflation the scalar field energy is converted into heat (relativistic particles in thermal equilibrium) thereby generating a large amount of entropy.
- The details of this process of pre-heating and reheating are very model dependent, i.e. they depend on the coupling of the inflaton field to other degrees of freedom and to standard model particles.

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- The scalar and tensor fluctuations are expected to be (nearly) Gaussian and hence characterized by their power spectra, $P_S(k)$ and $P_T(k)$ which can be computed in a given inflationary model.

$$k^3\langle\Psi(\mathbf{k})\Psi^*(\mathbf{k}')\rangle = (2\pi)^3\delta(\mathbf{k}-\mathbf{k}')P_S(k) \; ,$$

$$k^3\langle h(\mathbf{k})h^*(\mathbf{k}')\rangle = (2\pi)^3\delta(\mathbf{k}-\mathbf{k}')P_T(k) \; .$$

Anisotropies in the CMB are characterized by their angular power spectrum defined as

$$\left\langle \frac{\delta T(\mathbf{n})}{T}\frac{\delta T(\mathbf{n}')}{T}\right\rangle = \frac{1}{4\pi}\sum_\ell (2\ell+1)C_\ell P_\ell(\mathbf{n}\cdot\mathbf{n}')\,.$$

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After inflation, the evolution of the initial fluctuations can be computed by classical linear cosmological perturbation theory and transfer functions which relate them to the angular power spectrum of the CMB:

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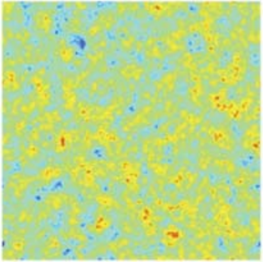
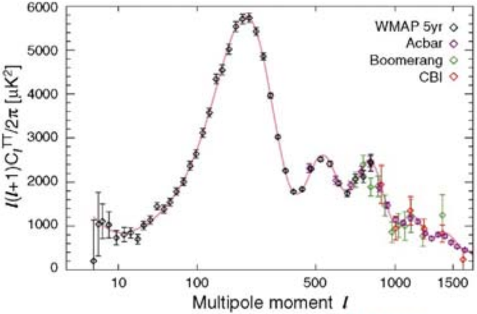
The transfer functions $T_S(\ell,k)$ and $T_T(\ell,k)$ depend on the cosmological parameters. If the inflationary spectra are simple (described by few parameters, e.g.

$$P_S(k) = A_s(k/k_*)^{n_s-1} \qquad P_S(k) = A_t(k/k_*)^{n_t}$$

The observations can be used to determine the cosmological parameters.

Introduction: CMB anisotropies





WMAP team

$n_s = 0.96$, $A_s = 2.4 \times 10^{-9}$, $r = A_t/A_s$,
$\Omega_m h^2 = 0.12$, $\Omega_b h^2 = 0.023$, $\tau = 0.088$, $H_0 = 71.4$km/sec/Mpc $= h100$km/sec/Mpc,
$K = 0$, $\Omega_\Lambda = 1 - \Omega_m$.

Quantum fluctuations of the inflaton I

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Coupling a quantum field to a time dependent external classical field can lead to particle creation. The energy of the modes which can be generated is determined by the kinetic energy in the time dependence of the external field. In an expanding universe, this is determined by the Hubble parameter $H(t)$ which is nearly constant during inflation,

$$H^2 = \frac{8\pi G}{3}\left(\dot{\phi}^2 + V(\phi)\right) \simeq \frac{8\pi G}{3}V, \qquad \dot{\phi}^2 = \frac{2}{3}\epsilon V \ll V(\phi) \quad \text{(slow roll)}$$

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The small fluctuation in the inflaton also leads to small fluctuations in the geometry,

$$ds^2 = -(1 + 2\Psi)dt^2 + (1 - 2\Psi)a^2(t)d\mathbf{x}^2$$

Einstein's constraint equations relate Ψ and $\delta\phi$ so that this system has only one degree of freedom.

Quantum fluctuations of the inflaton II

Expanding the Lagrangian to 2nd order in the fluctuation one can derive

$$S = -\int d^4x\sqrt{|g_b|}\left(\frac{R_b}{16\pi G} + L(\varphi)\right) + \delta S$$

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$d^4x = d\tau d^3x$, where τ is conformal time, $d\tau = dt/a$.

Expanding the Lagrangian to 2nd order in the fluctuation one can derive

$$S = -\int d^4x \sqrt{|g_b|}\left(\frac{R_b}{16\pi G} + L(\varphi)\right) + \delta S$$

$$\delta S = -\frac{1}{2}\int d^4x \left(\partial_\mu v \partial^\mu v + m^2(t)v^2\right) ,$$

$$v = a(\delta\phi + (\dot\varphi/H)\Psi) , \; m^2 = \ddot z/z , \; z = \dot\varphi/H = \sqrt{\frac{\epsilon m_P^2}{4\pi}}$$

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During inflation H is nearly constant, $a \propto \exp(Ht)$,

$$\tau = \int_{t_i}^{t} dt' \exp(-Ht') \simeq -H^{-1}\exp(-Ht)$$

is negative and $\tau \to 0$ when $t \to \infty$.

Quantum fluctuations of the inflaton III

We now quantize the field v and invoke the canonical commutation relations. We obtain the mode equation in Fourier space

$$\frac{d^2}{d\tau^2} v_k + \left(k^2 - \frac{2 + 9\epsilon - 3\eta}{\tau^2}\right) v_k = 0$$

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Here ϵ and η are the slow roll parameters which we assume to be small and slowly varying,

$$\epsilon = \frac{1}{16\pi G}\left(\frac{V_{,\varphi}}{V}\right)^2 \ll 1\ , \ \eta = \frac{1}{24\pi G}\left(\frac{V_{,\varphi\varphi}}{V}\right) = \frac{m_P^2 m_\varphi^2}{3V} \ll 1.$$

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At early time, $|\tau| \gg 1/k$ the mass term can be neglected and the field is oscillating like a free plane wave. At $1/|\tau| \simeq k$ fluctuations are generated.

Quantum fluctuations of the inflaton IV

The general solution of the mode equation, neglecting the time dependence of the slow role parameters, is

$$v = (k\tau)^{1/2}\left(C_1 H_\mu^{(1)}(k\tau) + C_2 H_\mu^{(2)}(k\tau)\right) \;, \quad \mu = 3/2 + 3\epsilon - \eta$$

Here $H_\mu^{(1,2)}$ is the Hankel function of the first respectively second kind.

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The initial condition for v_k is determined by the requirement, that at very early times v be in the vacuum state, $v_k(\tau) = \frac{1}{\sqrt{2k}}\exp(-ik\tau)$. This requires that $C_1 = 0$ and determines $C_2 \propto 1/\sqrt{k}$.

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What happens here is called squeezing: of the 2 solutions to the mode equation for small $x = k|\tau|$, which are $\propto x^{-\mu+1/2}$ and $x^{\mu+1/2}$, only the first survives. For cosmologically relevant modes, the suppression of the second wrt. the first mode is $x^{2\mu} \sim 10^{-150}$ at the end of inflation.

With $P_\zeta(k,t) = k^3\langle 0||v_k(t)|^2|0\rangle/(az)^2$, we obtain the power spectrum of the zeta-variable,

$$P_\zeta(k,t) = \frac{2\pi H^2}{5\epsilon m_P^2}\left(\frac{k}{aH}\right)^{-6\epsilon+2\eta} = A_s(k/k_*)^{n_s-1}\ , \quad k/(aH) \ll 1.$$

The amplitude A_s and the spectral index n_s of the scalar spectrum are determined by the scale of inflation, H and the slow roll parameters, $n_s - 1 = -6\epsilon + 2\eta$.

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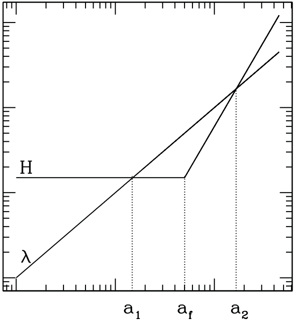
The amplitude A_s and the spectral index n_s of the scalar spectrum are determined by the scale of inflation, H and the slow roll parameters, $n_s - 1 = -6\epsilon + 2\eta$.

After inflation, the curvature fluctuation ζ is simply related to the metric fluctuation by
$P_\Psi = \frac{4}{9}P_\zeta$ in a radiation dominated Universe ($k\tau \ll 1$) and
$P_\Psi = \frac{9}{25}P_\zeta$ in a matter dominated Universe.

Quantum fluctuations of the inflaton: resume



The physical wavelength λ (inverse of frequency) of a mode is growing faster than the Hubble scale H^{-1} during inflation. After inflation, in the radiation phase, $\lambda \propto a \propto \sqrt{t}$ while $H^{-1} \propto t$, the Hubble scale catches up.

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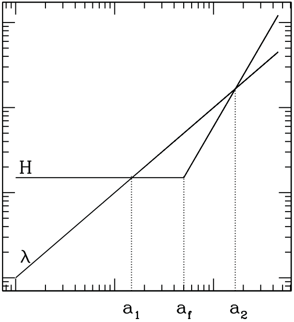

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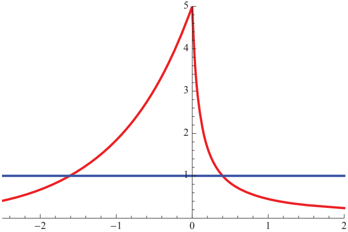

Even simpler, we can regard it as a quantum mechanical scattering process with incoming plane wave and potential $\ddot{z}/(a^2 z)$.

Note: the 'quantum aspect' of the treatment is that we know the initial conditions, the quantum vacuum. As usual, the evolution equation for the amplitudes of the quantum modes is the classical equation of motion.

Quantum fluctuations of the graviton I

The freely propagating mode of gravitation itself is what we call the graviton. It is given by the transverse traceless perturbations of the metric,

$$ds^2 = a^2\left(-d\tau^2 + (\delta_{ij} + 2H_{ij})dx^i dx^j\right) , \quad H^i_j = 0,\; H^{ij}{}_{,j} = 0$$

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To second order, the perturbed action is

$$S + \delta S = -\frac{m_p^2}{16\pi}\int d^4x\sqrt{-(g+\delta g)}(R+\delta R)$$

$$\delta S = \delta S_+ + \delta S_- \ , \ \ \delta S_\pm = \frac{1}{2}\int d^4x\left[(\partial_\tau h_\pm)^2 - (\nabla h_\pm)^2 + \frac{\ddot{a}}{a}h_\pm^2\right]$$

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The fields $h_\pm$ satisfy the same mode equations as v with z replaced by the scale factor a.

$$\frac{d^2}{d\tau^2}h_k + \left(k^2 - \frac{\ddot{a}}{a}\right)h_k = 0$$

Quantum fluctuations of the graviton II

Correspondingly they have the same solutions, just with the Hankel function index μ replaced by $\nu = 3/2 + \epsilon$,

$$h = B_2|\tau|^{1/2}H_\nu^{(2)}(k\tau) \ .$$

With this we obtain the tensor spectrum on super Hubble scales, $k/a \ll H$,

$$k^3 P_T = 32\pi \frac{H^2}{m_P^2}\left(\frac{k}{aH}\right)^{-2\epsilon} = A_t(k/k_*)^{-2\epsilon}$$

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Comparing it to the amplitude os scalar perturbations one obtains the consistency relation for slow roll inflation

$$r = P_T/P_\zeta = 16\epsilon = -8n_t$$

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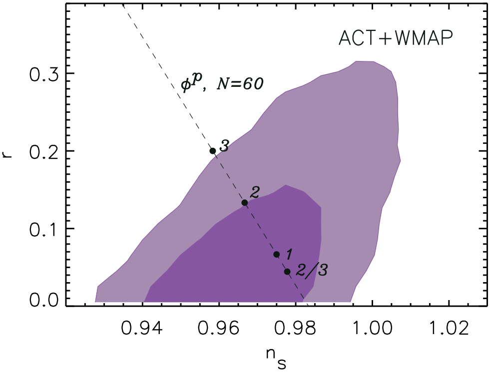
In the CMB temperature anisotropies it is difficult to separate a gravitational wave contribution for the scalar contribution.

Parameter estimation via Markov Chain Monte Carlo (MCMC) methods have so far given $r < 0.3$.



2D marginalized limits (68% and 95% CL) for the tensor-to-scalar ratio r, and the scalar spectral index n_s, for ACT+WMAP data. By measuring the $\ell > 1000$ spectrum, the longer lever arm from ACT data further breaks the n_s - r degeneracy, giving a marginalized limit $r < 0.25$ (95% CL) from the CMB alone. The predicted values for a chaotic inflationary model with inflaton potential $V(\phi) \propto \phi^p$ with 60 e-folds are shown for $p = 3,\ 2,\ 1,\ 2/3$; $p > 3$ is disfavored at $>$ 95% CL.
J. Dunkley et al. 2010.

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- In a decelerating universe, both ζ and H_{ij} are constant on large scales, $k/a(t) \ll H(t)$.

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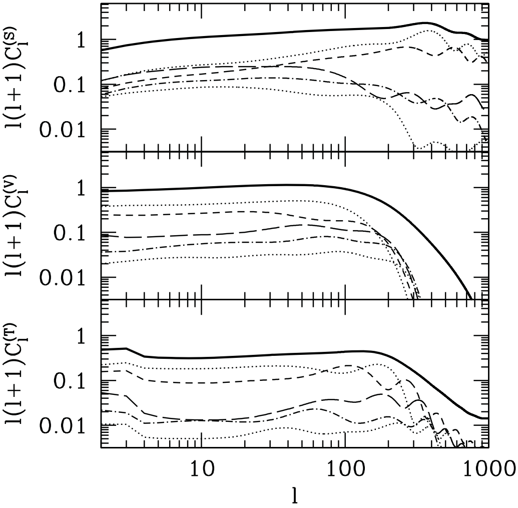
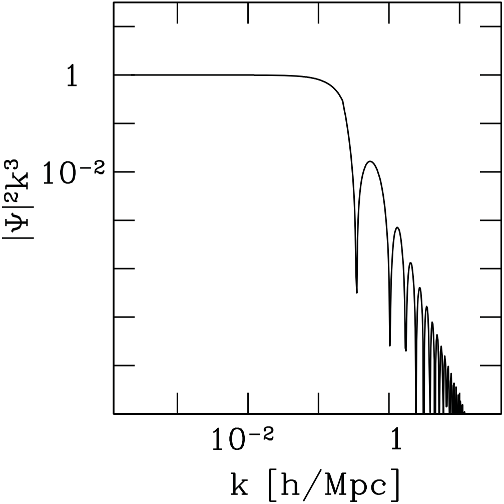
- Non-inflationary perturbations can only be generated once a given scale has entered the horizon. They typically lead to

$$T_{(S,T)}(\ell,k) \propto \min\left\{1, (k\tau_{eq})^{-4}\right\}\cos^2(c_s k\tau_{\rm dec}+\phi)k^3 j_\ell^2(k\tau_{\rm dec})\cdots$$

  with random phase ϕ.



CMB anisotropies from global texture (Bevis et al. 2004).

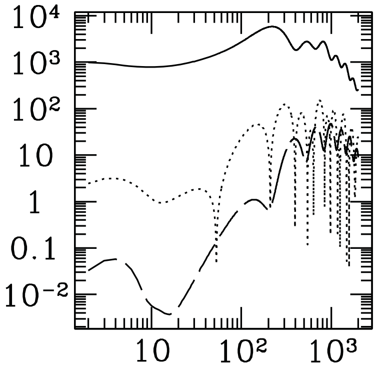


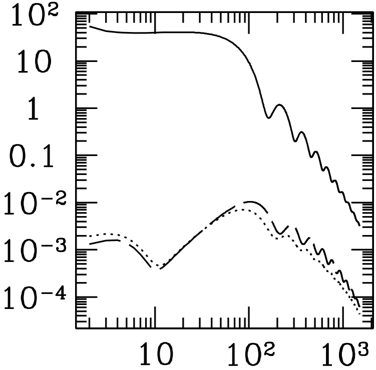
Sketch of the Bardeen potential (proportional to ζ) at late times.
(from RD, '08)

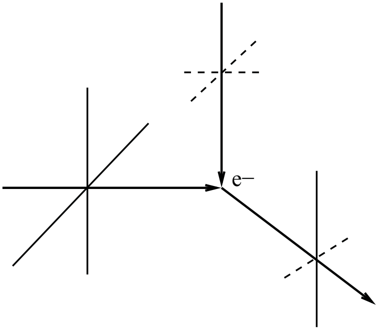
scalar (in units of $(\mu K)^2$)

tensor (for $r = 0.1$)

(from RD, '08)





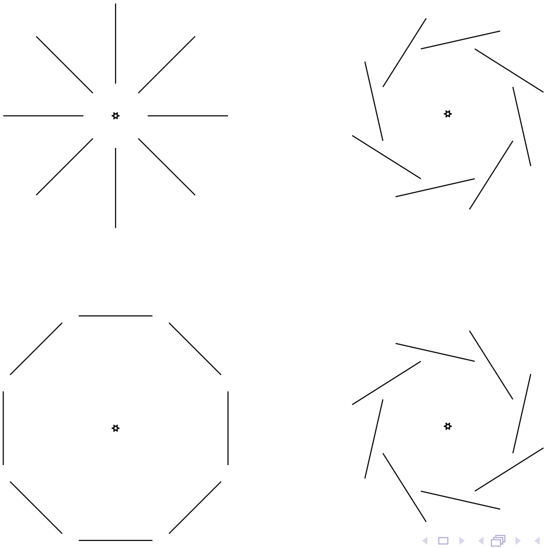


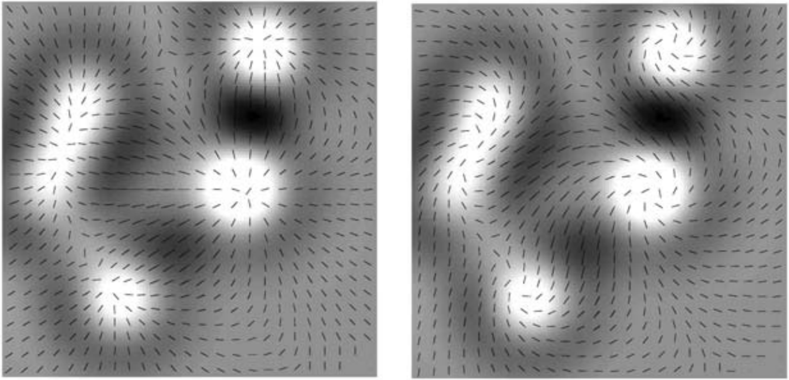
(from RD, '08)

$$E_{\parallel}^{(c)} = \sqrt{\frac{3}{8\pi} n_e \sigma_T} \cos\theta E_{\parallel}$$
$$E_{\perp}^{(c)} = \sqrt{\frac{3}{8\pi} n_e \sigma_T} E_{\perp}$$

The Thomson cross section depends on polarization. A quadrupole anisotropy on the last scattering surface induces polarisation.

E–type polarisation pattern B–type polarisation pattern

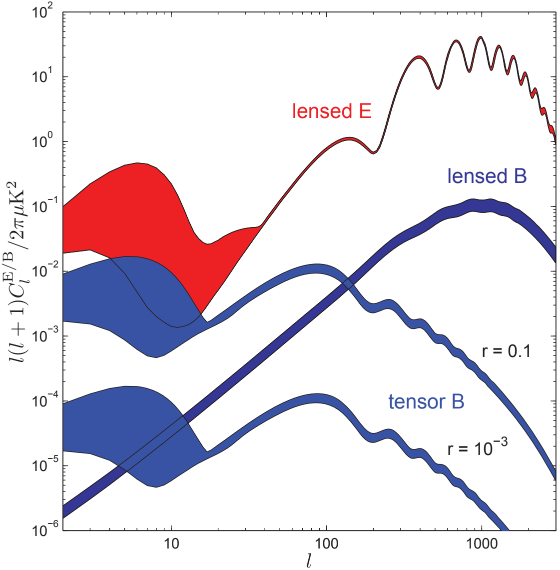




(from RD, '08)

A E-polarization pattern (left) is compared with B-polarization. The function $\tilde{\mathcal{E}} = \tilde{\mathcal{B}}$ is indicated in grey scale, and the polarization directions are drawn. E-polarization is tangential along the dark negative regions while it is radial from the white white positive regions. The B-polarization pattern can be obtained by rotating the polarization directions by 45^o.

CMB polarisation from scalar and tensor fluctuations III



(from Challinor & Lewis '06)

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One can show that in usual circumstances this is OK. The quantum fluctuations do decohere and become classical fluctuations with the same spectrum (see eg. Kiefer et al., 1998). It is easy to see that the occupation numbers are large. But the state is strongly squeezed (only the growing mode survives).

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In this situation a squeezed state can be approximated by a classical random field with random amplitude and fixed phase. The quantum to classical transition is induced not by the usual decoherence due to interactions with an environment, but by the decay of the quantum correlation between the growing and the decaying mode ”decoherence without decoherence”.

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The fact that the phase of the fluctuations (of a given wave number) is fixed is actually very important. It is responsible the the acoustic pics in the CMB anisotropy spectrum.

Post-inflationary fluctuations

Since they originate from quantum fluctuations of a nearly free field we expect the fluctuations to be nearly Gaussian. One can show that for single field slow roll inflation, non-Gaussianities are small, of the order of the slow roll parameters, i.e. the bi-spectrum defined by

$$k_1^3 k_2^3 \langle \zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\zeta(\mathbf{k}_2)\rangle \equiv (2\pi)^6 \delta(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) B(k_1, k_2, k_3)$$

is $k^6 B = f_{nl}(k^3 P)^2 \simeq \epsilon(k^3 P)^2$, (an $f_{nl} \sim 1$ is expected from the non-linearity of the fluctuations).

There are many models of inflation, some of them not slow-roll, which can lead to a fluctuation spectrum compatible with observations. Improving limits on non-Gaussianity and on B–polarisation from gravitational waves are crucial to distinguish them.

Planck satellite

The **Planck satellite** of ESA launched in 2009 has observed the microwave sky in more than 20 frequencies with a resolution of a few arc minutes for about 2.5 years (5 full sky coverages). Results are expected soon.
Can detect $r \geq 0.05$, $f_{nl} \gtrsim$ a few.

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In addition better observations and understanding of the more complicated matter distribution is needed. This gives us 3-dimensional information while the CMB is only 2-dimensional.

Conclusions

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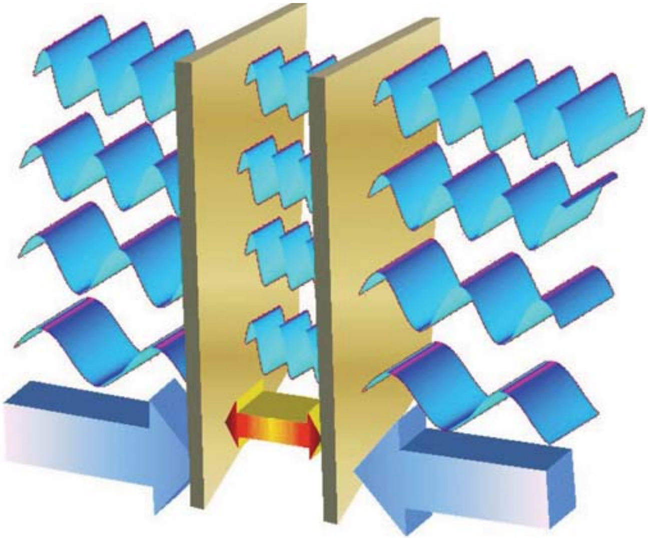
- If we can detect tensor fluctuations in the CMB this would be direct evidence of the quantum nature of spacetime.

RD & M. Ruser, arXiv:0704.0756, PRL 99, 071601 (2007)
M. Ruser & RD, arXiv:0704.0790, PRD 76, 104014 (2007)



Braneworlds

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Braneworlds

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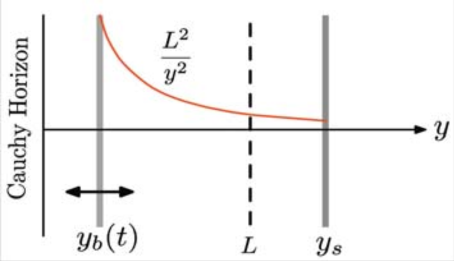
- Here we consider the RS model (Randall & Sundrum, 99) : a 4d brane moving through a 5d AdS spacetime.
- Like a moving metal plate can lead to the generation of photons by the dynamical Casimir effect, a moving brane can lead to the generation of gravitons.

The 5d AdS metric in conformal coordinates is

$$ds^2 = \left(\frac{L}{y}\right)^2 \left[\eta_{\mu\nu} dx^\mu dx^\nu + dy^2\right]$$



The Einstein equations, $G_{ab} = -\Lambda g_{ab}$ give $\Lambda = -6/L^2$.
$M_5^3 = M_4^2/L$. For $L \sim 0.1$mm $M_4 = 10^{19}$GeV yields $M_5 \simeq 10^5$TeV.

The Dynamical Casimir effect in Braneworlds

4d metric on the brane: $ds^2 = (L/y_b(t))^2[-d\eta^2 + \delta_{ij}dx^i dx^j]$

$a = L/y_b$, $d\eta = (1 - v^2)^{1/2}dt = \gamma^{-1}dt$, $v = dy_b/dt$

Junction condition (at low energy):

$$18H^2 = \kappa_5^2\lambda\rho = 6\kappa_4\rho \qquad (8\pi G_4 =)\ \kappa_4 = \kappa_5^2\lambda/6$$

RS fine tuning condition:

$$\kappa_5^2\lambda^2 = 36/L^2\ . \qquad \text{With} \quad \kappa_5 = L_s^3 \ \text{and} \ \kappa_4 = L_P^2$$

this yields:

$$\frac{L}{L_s} = \left(\frac{L_s}{L_P}\right)^2$$

The Dynamical Casimir effect in Braneworlds

One can now compute exactly like for the ordinary dynamical Casimir effect, the generation of 4d gravitons due to the motion of the moving brane (our Universe).

$$ds^2 = \left(\frac{L}{y}\right)^2 \left[-d\tau^2 + (\delta_{ij} + 2h_{ij})dx^i dx^j + dy^2\right]$$

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$$h_k(\tau, y) = \sum_\alpha q_\alpha(\tau)\phi_\alpha(k, \tau, y) \quad \alpha \in \mathbb{N}_0, .$$

$\alpha = 0$ corresponds to the usual massless 4d graviton while $\alpha = j \in \mathbb{N}$ are massive Kaluza-Klein modes. Their mass is given by their momentum in y-direction.

The time dependent boundary conditions lead to coupled second order differential eons. for $q_\alpha(t)$ which determines the particle production. The produced massive gravitons, $\omega_\alpha^2 = k^2 + m_\alpha^2$ have a time dependent mass, $m_j \simeq j\pi/(y_s - y_b)$ which is redshifted with the expansion of the Universe.

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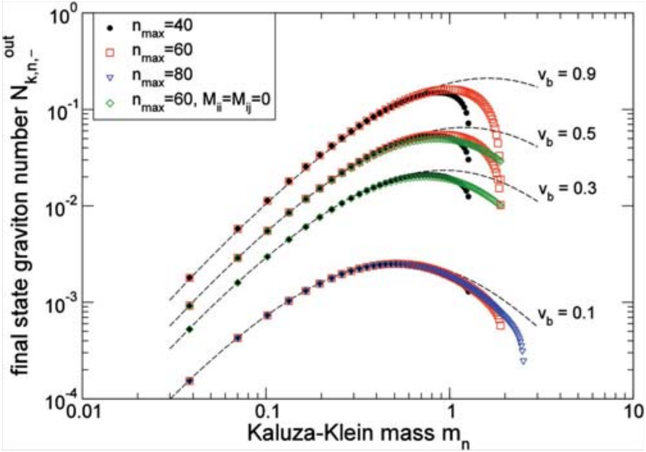
The energy density of the massless graviton goes like

$$\rho_0 \propto |h'|^2/a^2 \propto k^2|\phi_0|^2/a^2 \propto a^{-4} \qquad \text{(radiation)}$$

While the density of the massive modes goes like

$$\rho_n \propto |h'|^2/a^2 \propto \omega^2|\phi_n|^2/a^2 \propto a^{-6} \qquad \text{(stiff matter)}$$



for $y_s = 100$

$$\Omega_{h0} \simeq v_b \Omega_{\rm rad} \qquad \frac{\Omega_{hKK}(\tau_b)}{\Omega_{\rm rad}(\tau_b)} \simeq 100 v_b^3 \frac{L}{y_s}\left(\frac{L}{L_s}\right)^2 \sim v_b^3 \frac{L}{y_s} 10^{42}$$

The Dynamical Casimir effect in Braneworlds

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- Braneworld cosmology where expansion is mimicked by a brane moving through a warped higher dimensional spacetime leads to particle creation via the dynamical Casimir effect for bulk modes.
- KK gravitons scale like stiff matter, $1/a^6$, and can therefore not represent dark matter.
- In an ekpyrotic type scenario with an AdS5 bulk, the nucleosynthesis bound on gravitational waves requires $v_b < 0.2$.
- In ekpyrotic type scenarios back reaction of KK gravitons on the evolution of spacetime is most probably not negligible.