Experimental tests of QED in bound and isolated systems

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Experimental tests of QED in bound systems

Already discussed:

- Spectroscopy of hydrogen and helium atoms including experimental methods
- Pure leptonic atomic systems: positronium and muonium
- Other exotic atoms: muonic helium and antiprotonic helium
- Muonic hydrogen (proton radius puzzle)
- He$^+$ ion (electronic and muonic)
- Highly charged ions: H-like, Li-like and He-like

In this third lecture:

- g – 2 measurements: ions, electron, muon
- Determinations of the fine structure constant
The electron magnetic moment

The magnetic moment of a particle with charge \( q \) and mass \( m \) is related to its angular momentum by the \( g \) factor

\[ \vec{\mu} = g \left( \frac{q}{2m} \right) \vec{j} \]

For an electron, one introduces the Bohr magneton

\[ \mu_B = \frac{-e}{2m_e} \]

and then

\[ \vec{\mu} = g \mu_B \vec{j} \]

Dirac’s theory predicted \( g = 2 \) for a free electron (pure spin momentum)

In a magnetic field, the coupling energy of a given magnetic moment with the field is given by \(- \vec{\mu} \cdot \vec{B}\) so that:

- the magnetic moment precesses around the field at the Larmor frequency

\[ \omega_L = -g \frac{qB}{2m} \]

- energy levels in an atomic bound system are shifted (Zeeman effect) by a quantity which is simply related to the \( g \) factor and to the various quantum numbers of the level
Why to measure the anomalous magnetic moment of electron?

In 1947, the first precise measurement of the hyperfine structure in hydrogen and deuterium showed a discrepancy between measured and calculated values derived from Dirac equation.

\[ g_D = \frac{2}{3} \left[ 1 + 2 \sqrt{1 - (Z \alpha)^2} \right] = 2 \left[ 1 - \frac{1}{3} (Z \alpha)^2 - \frac{1}{12} (Z \alpha)^4 + \ldots \right] \]

This result and the 2S hydrogen Lamb shift, also measured in 1947, gave the first experimental evidence of deviations from the Dirac theory.

For a bound electron in the ground state of an H-like atom, the predictions are:

- If radiative corrections and recoil effect are taken into account:
  \[ g_{e^-} = 2 (1 + a_e) = 2 \left(1 + \frac{Z \alpha}{2 \pi} + \ldots \right) \]
  \[ \frac{g_{e^-}^{(H)}}{g_{e^-}} = 1 - \frac{1}{3} (Z \alpha)^2 - \frac{1}{12} (Z \alpha)^4 + \frac{1}{4} (Z \alpha)^2 \left(\frac{\alpha}{\pi}\right) + \frac{1}{2} (Z \alpha)^4 \left(\frac{m_e}{m_N}\right) + \ldots = 1 - 17.7053 \times 10^{-6} \]

Extremely precise measurements of \( g - 2 \) can be performed in ion traps and then provide stringent tests of QED.
**g factor measurements in light H-like ions**

Experiments performed at Mainz

A single ion is stored in the strong magnetic field ($\sim 4$ T) of a Penning trap

The cyclotron frequency of the trapped ion:

$$\omega_c = \frac{Q}{M_{\text{ion}}} B$$

Larmor frequency of the electron:

$$g_J = 2 \frac{\omega_L}{\omega_c} \cdot \frac{m_e}{M_{\text{ion}}} \cdot \frac{Q}{e}$$

The Larmor frequency is measured through the spin-flip rate of the electron as a function of a driving microwave field (see later)

![Graph showing spin-flip probability for $^{12}$C$^{5+}$ with a width of ~30 mHz]
$g$ factor measurements in light H-like ions

Results:

- $g_J(^{12}\text{C}^{5+}) = 2.001\, 041\, 596\, 4 (8)\, (6)\, (44)_{\text{stat}}\, (syst}$


  the contribution of the electron’s atomic mass dominates the error budget

- $g_J(^{16}\text{O}^{7+}) = 2.000\, 047\, 025\, 4 (15)\, (44)$


  These results are in very good agreement with theoretical calculations which take into account one-loop all orders in $Z\alpha$ terms for the bound-state QED contributions

  They can be used to derive a value of the electron mass

  $m_e = 0.000\, 548\, 579\, 909\, 6 (4)\, u$  more precise than the CODATA value

- $g_J(^{28}\text{Si}^{13+}) = 1.995\, 348\, 958\, 7 (5)\, (3)\, (8)$


  in excellent agreement with the theoretical value

  $g_J(^{28}\text{Si}^{13+}) = 1.995\, 348\, 958\, 0 (17)$

  including QED contributions up to the two-loop level in $(Z\alpha)^2$ and $(Z\alpha)^4$
**g factor measurements in heavy highly charged ions**

Experiment planned at the HITRAP facility, GSI (Darmstadt) in a series of H-like elements: Pb$^{81+}$, Bi$^{82+}$, U$^{91+}$

Novel technique of deceleration, trapping and cooling of HCI

Ions injected and confined in a cryogenic Penning trap where a constant homogeneous magnetic field is applied

A double laser-microwave resonance technique will be used to measure the $g_F$ factor of a given hyperfine sublevel on the ppb level

The combined measurement of hfs transition frequency and $g_F$ factor will allow the simultaneous determination of electronic $g_J$ and nuclear $g_I$ factors

Magnetic moment of free leptonic particles

electron, positron, muon

\[ \bar{\mu} = g \left( \frac{q}{2m} \right) \bar{J} \]

The g-value is a dimensionless measure of the magnetic moment

Its experimental determination

• is the most stringent test of QED (*)

• is the most stringent test of CPT invariance with leptons (comparaison electron - positron)

and, if one is confident in QED calculations,

• is a test for physics beyond the Standard Model (e.g. electron substructure) (*)

• gives the most precise determination of \( \alpha \) if there is no physics beyond the Standard Model

(*) if \( \alpha \) is known
Measurements of the electron g-factor

For a free electron the g-factor is simply deduced from the ratio between the Larmor frequency \( \omega_L = -g \frac{qB}{2m_e} \) and the cyclotron frequency \( \omega_c = \frac{qB}{m_e} \) and its anomaly is defined as

\[
\frac{a_e}{e} = \frac{g_e - 2}{2}
\]

In 1947, the first determination of \( a_e \) was performed by Kusch and Foley. Their result was \( a_e = 0.00119 (5) \) in agreement with the prediction of Schwinger:

\[
\frac{\alpha}{2\pi} = 0.00116
\]

For a review of early measurements, see:

A. Rich and J.C. Wesley, Rev. Mod. Phys. 44, 250 (1972)
Measurements of the electron g-factor

First experimental method:

- Direct observation of the electron spin precession in a given magnetic field (Michigan)

100 keV electrons (100 ns pulses) are polarized and trapped in a 1 kG magnetic field during a predetermined time $T$

The spin rotation during this time is recorded versus $T$

$\rightarrow$ 3 ppm accuracy on $a_e$


The same method is easily extended to the g-factor of positron

Measurements of the electron g-factor

Second experimental method:
• Study of transitions induced by a RF field in a Penning trap in a given magnetic field (Washington, Mainz, Stanford, Harvard)

The energy levels of one electron in a magnetic field are given by:

\[ E(n, m_s) = \left( n + \frac{1}{2} \right) \hbar \omega_c + m_s \hbar \omega_L \]

where \( \frac{\omega_L}{\omega_c} = \frac{g}{2} = 1 + \frac{\omega_a}{\omega_c} \)

and \( \omega_a \) is the anomaly frequency \( \omega_a = \omega_L - \omega_c \)

directly related to \( a_e \)

\[ a_e = \frac{\omega_a}{\omega_c} \]
In a Penning trap

A quadrupolar electric potential is applied which confine the electron along the z axis

\[ V(x, y, z) = \frac{1}{2} m_e \omega_z^2 \left[ z^2 - \frac{x^2 + y^2}{2} \right] \]

the transverse confinement is obtained by the application of the magnetic field

The electron movement is the sum of:
- a cyclotron rotation at a slightly modified frequency
- an oscillation along the z axis
- a slow rotation at the magnetron frequency

An electron suspended in a Penning trap is a homemade atom called geonium

Pionneer work in Washington

- Penning trap at 4K
- single electron stored

Measurement of the cyclotron frequency and of the anomaly frequency

Detection of spin-flip through the induced shift of the axial frequency

Results:
\[ g_{e^-}/2 = 1.001159652200(40) \times 10^{-11} \]

and
\[ g_{e^-}/g_{e^+} = 1 + (0.5 \pm 2.1) \times 10^{-12} \]

The Harvard experiment
and the most precise determination of the electron g-factor

- Cylindrical Penning trap invented to form a microwave cavity that could inhibit spontaneous emission (by a factor of up to 250) → narrowed linewidth

- Trap cavity cooled to 100 mK → the electron cyclotron motion is its ground state

- Careful control and probe of radiation field and magnetic field in the trap cavity

The one quantum change in cyclotron motion is resolved
Electron’s lowest cyclotron and spin levels

Special relativity makes the transition cyclotron frequency decrease by a very small shift

\[- \delta (n + 1 + m_s)\]

where

\[\frac{\delta}{\omega_c / 2\pi} = \frac{\hbar \omega_c}{mc^2} \approx 10^{-9}\]

not negligible at this level of precision!

- 5.4 T stabilized magnetic field
- very stable trapping potential provided by a charged capacitor

Numerical values:

\[\omega_c / 2\pi \approx 150 \text{ GHz}\]
\[\omega_z / 2\pi \approx 200 \text{ MHz}\]
\[\omega_m / 2\pi \approx 133 \text{ kHz}\]
\[\omega_a / 2\pi \approx 174 \text{ MHz} \quad \text{(measured)}\]
Quantum nondemolition (QND) detection

The one-quantum sensitivity is obtained through the coupling of the cyclotron motion to the thermally driven axial motion at 200 MHz frequency where electronic detection is efficient.

This coupling is obtained with a magnetic bottle gradient of $B$ (Ni rings).

The shift is

$$\Delta \omega_z = \delta_B \left( n + m_s \right)$$

here 4 Hz in 200 MHz.

**Figure:**

- **(a)**: Spin flip anomaly transition induced by applying an oscillating potential to the electrodes.
- **(b)**: One-quantum cyclotron excitation induced by microwaves injected into the cavity.
Quantum-jump spectroscopy

measuring the quantum jumps per attempt to drive them as a function of drive frequency

Result:

in 2006

\[ g / 2 = 1.001 \ 159 \ 652 \ 180 \ 85 \ (76) \]

7.6 \times 10^{-13}

in 2008

\[ g / 2 = 1.001 \ 159 \ 652 \ 180 \ 73 \ (28) \]

2.8 \times 10^{-13}

The last result obtained in Harvard is:

\[ a_e = 1\ 159\ 652\ 180.73 \ (0.28) \times 10^{-12} \approx 2.4 \times 10^{-10} \]

• Taking into account the presence of the muon and tau particles, the QED contribution to the electron g - 2 can be written:

\[ a_e = A_1 + A_2 \left( \frac{m_e}{m_\mu} \right) + A_2 \left( \frac{m_e}{m_\tau} \right) + A_3 \left( \frac{m_e}{m_\mu}, \frac{m_e}{m_\tau} \right) \]

where

\[ A_i = A_i^{(2)} \left( \frac{\alpha}{\pi} \right) + A_i^{(4)} \left( \frac{\alpha}{\pi} \right)^2 + A_i^{(6)} \left( \frac{\alpha}{\pi} \right)^3 + A_i^{(8)} \left( \frac{\alpha}{\pi} \right)^4 \]

• Since the experimental uncertainty is less than 1% of \( \frac{\alpha}{\pi} \approx 29 \times 10^{-12} \), the coefficient \( A_i^{(8)} \) is needed to match the precision of theory with experiment.

• In addition, the total non-QED (hadronic) contribution to \( a_e \) is \( 1.72(2) \times 10^{-12} \).


But the comparison of theory with measured electron anomaly needs also a value of \( \alpha \) obtained by an independent measurement.
On another hand, the last measurement of the electron g-factor, combined with recent calculations of $A_1^{(8)}$ coefficient and estimation of $A_1^{(10)}$ one gives the most precise determination of the fine structure constant

$$\alpha^{-1} = 137.035\,999\,084\, (33)\, (39)$$

3.7 x 10^{-10}

exp. theor.


The other determinations of the fine structure constant are discussed in the following
Measurements of the muon g-factor

- First experiment at the Columbia-Nevis Cyclotron:
  the direct observation of the muon magnetic moment precession
  in a magnetic field provided the indication that the muon behaves
  like a heavy electron with a g-factor given by

  \[ \frac{g}{2} = 1.00122 (8) \quad \text{in agreement with predictions of QED} \]


- During the following years, experiments were performed at CERN,
  either at the synchrocyclotron or at the proton synchrotron:

  G. Charpak et al., Phys. Lett. 1, 16 (1962)

  leading to increasing accuracy
Measurements of the muon g-factor

- Experiment in Brookhaven by V. Hughes and collaborators
  Study of the spin precession relative to the momentum in a given magnetic field

In both CERN and Brookhaven experiments, relativistic muons are stored in circular orbits in a uniform magnetic field and are detected through their decay electrons

The anomaly frequency is observed as an oscillation of the detected electrons

The magnetic field is deduced from proton NMR

Result:

\[ a_\mu = 1.165\,920\,91(63) \times 10^{-3} \]

muon anomaly: discussion

The average current experimental result is:

\[ a_\mu = 116\,592\,080 \pm 63 \times 10^{-11} \]

which differs from the experimental value by \( 289 \pm 86 \times 10^{-11} \) \( 3.4 \sigma \)

- Due to the mass dependence, the muon \( g-2 \) is \( 4 \times 10^4 \) times more sensitive to hadronic and electroweak effects than the electron \( g-2 \).

The hadronic contribution to \( a_\mu \) is 60 ppm.

This result allows to check the contribution of virtual hadrons to the muon anomaly.

- Including all contributions, the theoretical value of \( a_\mu \) in the Standard Model is:

\[ a_\mu = 116\,591\,791 \pm 62 \times 10^{-11} \]

Both theory and experiment must be improved to know if it is or not an indicator of physics beyond the standard model.

The various ways to determine $\alpha$

We have seen that a determination of $\alpha$ can be deduced:
- from the hyperfine structure of muonium with an accuracy of $5.8 \times 10^{-8}$
- from the fine structure of helium with an uncertainty of $3.1 \times 10^{-8}$
  limited by uncalculated high-order QED terms
- from the electron $g - 2$ magnetic moment anomaly with an uncertainty of $3.7 \times 10^{-10}$
  partly due to both experiment and calculations

Other independent measurements are strongly needed to test QED

Other methods which have been used:
- Gyromagnetic ratio of a shielded proton (H$_2$O) in high field $3.7 \times 10^{-8}$
- Josephson effect $3.1 \times 10^{-7}$
- Quantum Hall effect $1.8 \times 10^{-8}$
- Diffraction of neutrons $2.4 \times 10^{-8}$
- Recoil effect (measurement of the h/M ratio) $6.6 \times 10^{-10}$

The two most precise methods only are discussed in the following
The Quantum Hall effect

Fixed current $I$ in a quantum Hall effect device at low temperature.

Plateaus are observed when the resistance is recorded versus the magnetic field

$$R_H(i) = \frac{R_K}{i} = \frac{h}{i e^2}$$

where $i$ is integer

The Von Klitzing constant $R_K$ is simply related to $\alpha$

$$(\mu_0 c$ - impedance of vacuum - is exact in SI)

$$R_K = \frac{h}{e^2} = \frac{\mu_0 c}{2 \alpha}$$

Its measurement by comparison to a known resistance is done by means of a calculable capacitor

and is limited to $1.8 \times 10^{-8}$ as the deduced value of $\alpha$

(NIST, NML, NPL, BNM)
The constant $\alpha$ can also be deduced from recoil

The recoil velocity gives the ratio $h/M$

The recoil velocity can be measured very precisely as a frequency shift

Measurement of $h/M$ allows to determine $\alpha$

Rydberg constant in terms of energy:

\[
h c R_{\infty} = \frac{1}{2} m_e c^2 \alpha^2
\]

Relative uncertainty on each term:

- Rydberg constant: $5 \times 10^{-12}$
- Atom-to-electron mass ratio: $4 \times 10^{-10}$

Experiments have been performed:
- In Stanford (Cs atoms)
- In Paris (Rb atoms)
The Paris experiment

Ultracold Rb atoms at 4 μK

Principle:

- Selection of an initial subrecoil velocity class
- Coherent acceleration: $N \times 2\hbar k$
  momentum transfer of $2N\hbar k$
- Measurement of the final velocity distribution

Final uncertainty: $\sigma(v_f) = \sigma(v) / 2N$
Subrecoil velocity selection

Stimulated Raman transition in the ground state (π pulse) between the two hyperfine levels.

Contra-propagating laser beams

Doppler shift → velocity selection

( non selected atoms left in F=2 level are blown out )

velocity distribution measurement

after the coherent acceleration (momentum transfer)

Velocity selective stimulated Raman transition from F = 1 to F = 2

Frequency scan of one beam through the profile
Experimental sequence

for the Raman beams

N_2/(N_1+N_2) \approx \frac{v_r}{50}

The F = 2 relative population is recorded versus the difference between the selection and the detection laser frequencies.

\begin{align*}
\sigma_v &\approx 1 \text{ Hz} \iff \frac{v_r}{15000}
\end{align*}
The populations in F=2 and F=1 levels after the velocity detection are successively measured by fluorescence using a time of flight technique. To cancel the effect of gravity, a differential measurement is used (up and down accelerations).
Improvement of the velocity selection

- In our earlier experiment (\(\pi-\pi\) configuration), two \(\pi\) Raman pulses were used
  - to select a subrecoil velocity distribution
  - and to measure the final velocity distribution

- In our present experiment (\(\{\pi/2, \pi/2\} - \{\pi/2, \pi/2\}\) configuration),
  - the first pair of \(\pi/2\) pulses
    (frequency \(\delta_1\))
    selects a velocity pattern
  - the second pair of \(\pi/2\) pulses (frequency \(\delta_2\)) selects another velocity pattern

\(T_R\) : time between the two pulses

When the detection frequency \(\delta_2\)
is swept, the signal obtained is
the convolution of two Ramsey patterns

\[\delta = \delta_2 - \delta_1\]

Interferometric method

Fit of the central part → precise determination of the velocity
**Atom interferometry**

The manipulation of ultracold atoms with laser beams has opened the way to atom interferometry.

With a resonant laser light, a $\pi$ pulse acts as a mirror and a $\pi/2$ pulse as a beam splitter.

With 3 or 4 pulses, one realizes an interferometer.

*Ramsey-Bordé*
The Stanford experiment

Atomic fountain of cold Cs atoms

The two pairs of $\pi/2$ pulses are used to separate and recombine the two paths of the interferometer. The dephasing between the two paths is proportional to the recoil velocity. $N\pi$ pulses are applied to increase the number of transferred recoils.

$|a\rangle$ $|b\rangle$

$\pi/2$ $\pi/2$

$\pi$ $(2v_r)$ $(2v_r)$

$N=30 \rightarrow$ determination of $\alpha$ with an accuracy of $7.7 \times 10^{-9}$

Coherent acceleration of the atoms (I)

Stimulated Raman transition in a given hyperfine level

$\vec{k}_2 \rightarrow \vec{k}_1$

$M$

$2\nu_r$

Linear frequency sweep of the frequency of one beam:

$\delta = \nu_1 - \nu_2 \propto t$

Momentum transfer:

$2\hbar k$ per cycle
Coherent acceleration of the atoms (II)

In the laboratory frame: accelerated optical lattice

In the accelerated frame at $a = \frac{\pi(v_1 - v_2)}{kt}$: standing wave but inertial force

$\vec{F} = -M \vec{a}$

Bloch oscillations in the fundamental energy band

M. Ben Dahan et al.,

up to 1000 recoils
Transfer efficiency of 99.95% per B.O.


Experimental sequence

Raman beams (interferometric method) + Bloch oscillations

Signal

- upwards and downwards accelerations to cancel the effect of gravity
- exchange between Raman beams to cancel systematic shifts

Four spectra recorded in 5 min. → 6 ppb accuracy on h/M
Recent measurements performed in Paris


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\[ \alpha^{-1} = 137.035\,999\,037\,037 \pm 0.001 \]
Comparaison between various measurements of $\alpha$

$\Gamma_{p,h-90}$

He fine structure

hfs muonium

Quantum Hall effect

Solid state physics

$h/m$ (neutron)

$g - 2$ of the electron (UW)

$g - 2$ of the electron (Harvard)

$h/m$ (Cs)

$h/m$ (Rb)

$2006$

$2008$

$2010$

$\mu = h/\lambda_{DB}$

$v_r = \hbar k/m$

The measurement of $\alpha$ in different domains of physics is a test of the consistency of theory
CODATA deduced from all experiments except $g - 2$

Recommended value

Value used to test $g - 2$
The comparison between recent h/M and g-2 determinations of $\tilde{\alpha}$ gives a test of QED at a level better than $10^{-9}$
Electron anomaly and recoil effect: conclusion

\[ a_e = C_1 \left( \frac{\alpha}{\pi} \right) + C_2 \left( \frac{\alpha}{\pi} \right)^2 + C_3 \left( \frac{\alpha}{\pi} \right)^3 + C_4 \left( \frac{\alpha}{\pi} \right)^4 + a(m_e / m_\mu, m_e / m_\tau, \text{weak, hadron}) + \ldots \]

\[ \delta(a_e) = a_e (\text{meas}) - a_e (\text{theo}) = -(40 \pm 89) \times 10^{-14} \]

In red: theoretical uncertainties

- First test of the QED at the $10^{-9}$ level (up to 4-loop terms)
- The accuracy is now at the level of muonic and hadronic corrections
General conclusion

The spectroscopy of atomic systems and free particles at low energy provides very accurate tests of QED.

A lot of experiments give results in good agreement with calculated predictions.

The most accurate test is the comparison between determinations of the fine structure constant derived from electron $g-2$ and $\hbar/M$ measurement.

Previous discrepancies are now solved …
- fine structure of helium
- orthopositronium lifetime

… but others are not solved or recently appeared!
- $g$-2 anomaly of muon $> 3 \sigma$
- Lamb shift of muonic hydrogen (proton size puzzle) $\sim 5 \sigma$

The work must continue for both theoreticians and experimentalists.
Bibliography


• "Introduction to the Physics of Highly Charged Ions" Ed. H.F. Beyer and V.P. Shevelko, Series in Atomic and Molecular Physics, Institute of Physics (2003)

Many thanks to all my colleagues for their help, comments and suggestions ...

... and thank you for your attention!