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Experimental tests of QED in bound and isolated systems

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Experimental tests of QED in bound systems

Already discussed :

- Spectroscopy of hydrogen and helium atoms
  including experimental methods

- Pure leptonic atomic systems : positronium and muonium

- Other exotic atoms : muonic helium and antiprotonic helium
- Muonic hydrogen (proton radius puzzle)
- He^{+} ion (electronic and muonic)

- Highly charged ions : H-like, Li-like and He-like

In this third lecture :

- g – 2 mesurements : ions, electron, muon

- determinations of the fine structure constant

The electron magnetic moment

The magnetic moment of a particle with charge q and mass m is related to its angular momentum by the g factor

$$\vec{\mu} = g\left(\frac{q}{2m}\right)\vec{J}$$

For an electron, one introduces the Bohr magneton

$$\mu_B = \frac{-e}{2m_e}$$

and then

$$\vec{\mu} = g\,\mu_B\,\vec{J}$$

Dirac's theory predicted $g = 2$ for a free electron (pure spin momentum)

In a magnetic field, the coupling energy of a given magnetic moment with the field is given by $-\vec{\mu}\,.\,\vec{B}$ so that :

- the magnetic moment precesses around the field at the Larmor frequency

$$\omega_L = -g\,\frac{qB}{2m}$$

- energy levels in an atomic bound system are shifted (Zeeman effect) by a quantity which is simply related to the g factor and to the various quantum numbers of the level

Why to measure the anomalous magnetic moment of electron ?

In 1947, the first precise measurement of the hyperfine structure in hydrogen and deuterium showed a discrepancy between measured and calculated values derived from Dirac equation

G. Breit, Phys. Rev. 72, 984 (1947)
J. Schwinger, Phys. Rev. 73, 416 (1948) and 76, 790 (1949)

This result and the 2S hydrogen Lamb shift, also measured in 1947, gave the first experimental evidence of deviations from the Dirac theory

For a bound electron in the ground state of an H-like atom, the predictions are :

- $$g_D = \frac{2}{3}\left[1 + 2\sqrt{1-(Z\alpha)^2}\right] = 2\left[1 - \frac{1}{3}(Z\alpha)^2 - \frac{1}{12}(Z\alpha)^4 + \ldots\right]$$ Dirac

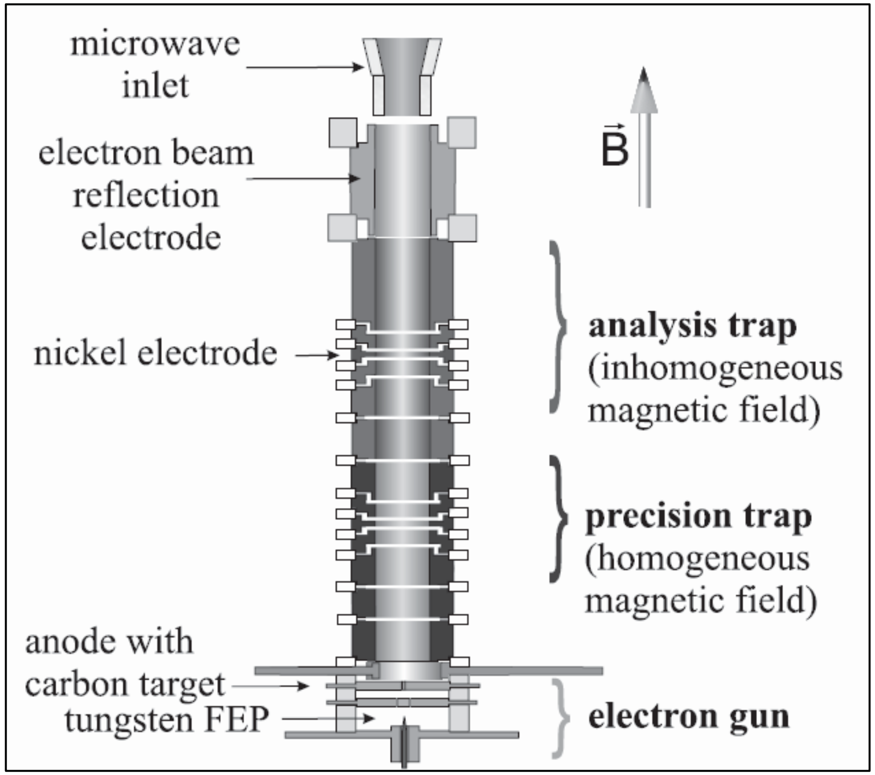
- If radiative corrections and recoil effect are taken into account :

$$g_{e^-} = 2(1+a_e) = 2(1 + \frac{Z\alpha}{2\pi} + \ldots)$$ and :

$$\frac{g_{e^-}(H)}{g_{e^-}} = 1 - \frac{1}{3}(Z\alpha)^2 - \frac{1}{12}(Z\alpha)^4 + \frac{1}{4}(Z\alpha)^2\left(\frac{\alpha}{\pi}\right) + \frac{1}{2}(Z\alpha)^4\left(\frac{m_e}{m_N}\right) + \ldots = 1 - 17.7053\times 10^{-6}$$

Extremely precise measurements of *g - 2* can be performed in ion traps and then provide stringent tests of QED

g factor measurements in light H-like ions



Experiments performed at Mainz

A single ion is stored in the strong magnetic field (~ 4 T) of a Penning trap

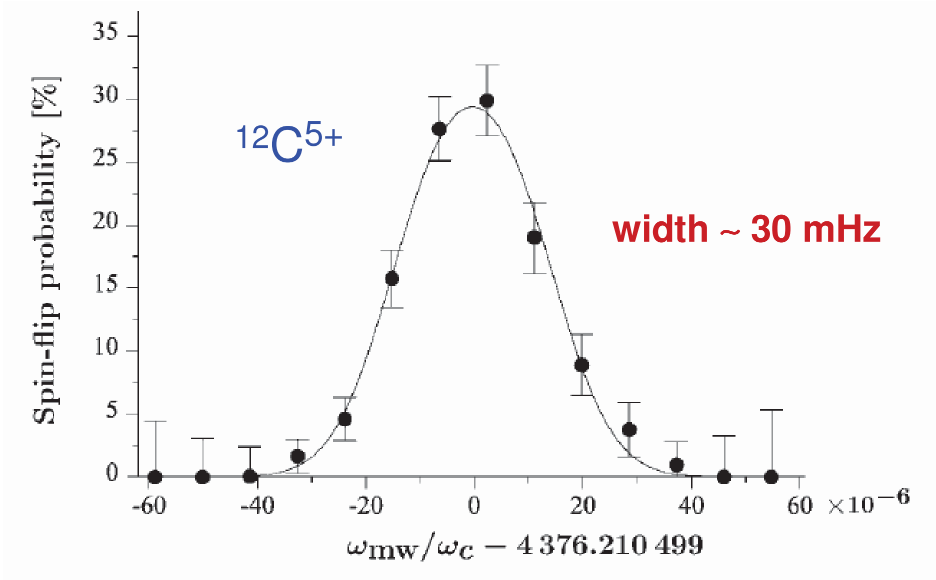
cyclotron frequency of the trapped ion : $\omega_c = \dfrac{Q}{M_{ion}} B$

Larmor frequency of the electron

$$g_J = 2 \frac{\omega_L}{\omega_c} \cdot \frac{m_e}{M_{ion}} \cdot \frac{Q}{e}$$

external input

The Larmor frequency is measured through the spin-flip rate of the electron as a function of a driving microwave field (see later)



g factor measurements in light H-like ions

Results :

- $g_J(^{12}C^{5+}) = 2.001\,041\,596\,4\,(8)\,(6)\,(44)$
  stat syst

H. Häffner *et al.*, Phys. Rev. Lett. 85, 5308 (2000)

the contribution of the electron's atomic mass dominates the error budget

- $g_J(^{16}O^{7+}) = 2.000\,047\,025\,4\,(15)\,(44)$

J. Verdù *et al.*, Phys. Rev. Lett. 92, 093002 (2004)

These results are in very good agreement with theoretical calculations which take into account one-loop all orders in $Z\alpha$ terms for the bound-state QED contributions

They can be used to derive a value of the electron mass

$m_e = 0.000\,548\,579\,909\,6\,(4)$ u more precise than the CODATA value

- $g_J(^{28}Si^{13+}) = 1.995\,348\,958\,7\,(5)\,(3)\,(8)$

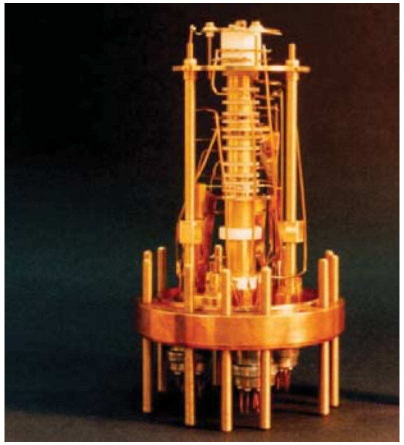
S. Sturm *et al.*, Phys. Rev. Lett. 107, 023002 (2011)

in excellent agreement with the theoretical value

$g_J(^{28}Si^{13+}) = 1.995\,348\,958\,0\,(17)$

including QED contributions up to the two-loop level in $(Z\alpha)^2$ and $(Z\alpha)^4$

g factor measurements in heavy highly charged ions



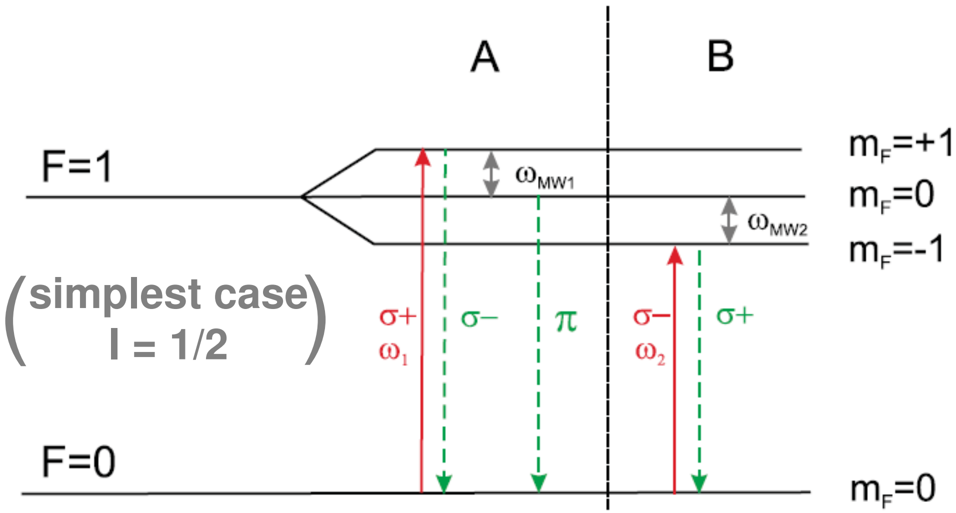
Experiment planned at the HITRAP facility , GSI (Darmstadt)

in a series of H-like elements : Pb^{81+}, Bi^{82+}, U^{91+}

Novel technique of deceleration, trapping and cooling of HCI

Ions injected and confined in a cryogenic Penning trap where a constant homogeneous magnetic field is applied

A double laser-microwave resonance technique will be used to measure the g_F factor of a given hyperfine sublevel on the ppb level



The combined measurement of hfs transition frequency and g_F factor will allow the simultaneous determination of electronic g_J and nuclear g_I *factors*

W. Quint *et al.*, Phys. Rev. A 78, 032517 (2008)

Magnetic moment of free leptonic particles

electron, positron, muon

$$\vec{\mu} = g\left(\frac{q}{2m}\right)\vec{J}$$

The g-value is a dimensionless measure of the magnetic moment

Its experimental determination

- is the most stringent test of QED (*)
- is the most stringent test of CPT invariance with leptons
  (comparaison electron - positron)

and, if one is confident in QED calculations,

- is a test for physics beyond the Standard Model (e.g. electron substructure) (*)
- gives the most precise determination of α if there is no physics beyond
  the Standard Model

(*) if α is known

Measurements of the electron g-factor

For a free electron the g-factor is simply deduced from the ratio between

the Larmor frequency $\omega_L = -g \frac{qB}{2m_e}$ and the cyclotron frequency $\omega_c = \frac{qB}{m_e}$

and its anomaly is defined as $a_e = \frac{g_e - 2}{2}$

In 1947, the first determination of a_e was performed by Kusch and Foley.

Their result was a_e = 0.00119 (5) in agreement with the prediction of Schwinger :

$$\frac{\alpha}{2\pi} = 0.00116$$

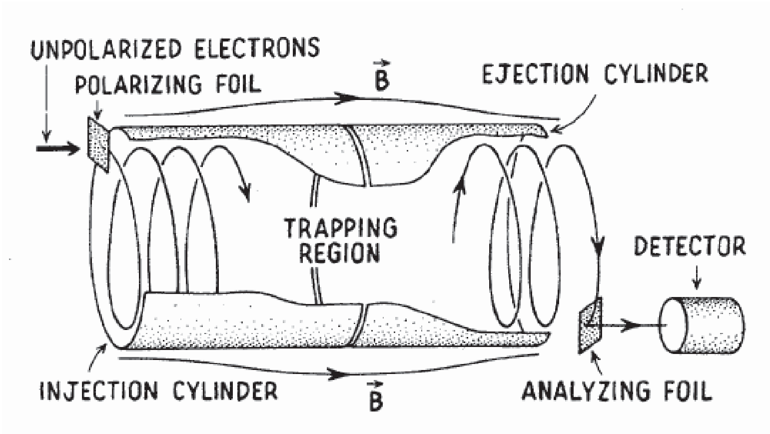
For a review of early measurements, see :

A. Rich and J.C. Wesley, Rev. Mod. Phys. 44, 250 (1972)

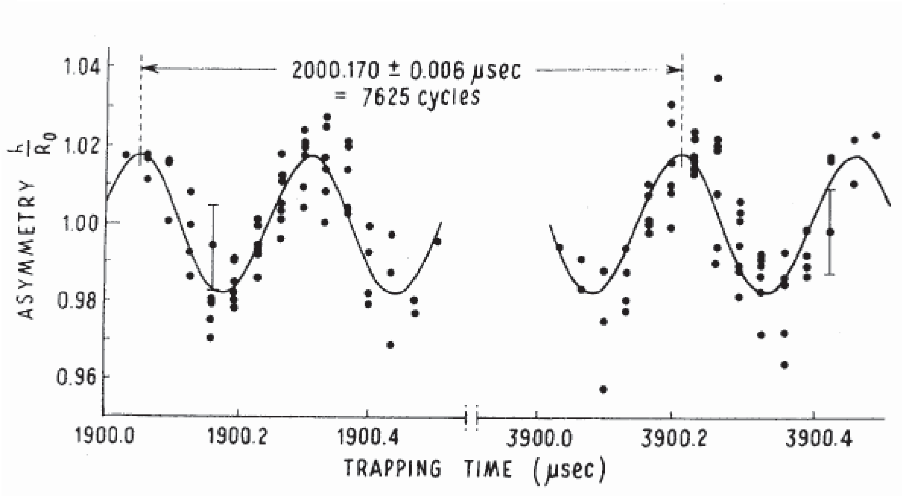
Measurements of the electron g-factor

First experimental method :

- Direct observation of the electron spin precession in a given magnetic field (Michigan)



100 keV electrons (100 ns pulses) are polarized and trapped in a 1 kG magnetic field during a predetermined time T



The spin rotation during this time is recorded versus T

→ 3 ppm accuracy on a_e

J.C. Wesley and A. Rich, Phys. Rev. A 4, 1341 (1971)

The same method is easily extended to the g-factor of positron

J. Gilleland and A. Rich, Phys. Rev. Lett. 23, 1130 (1969)

Measurements of the electron g-factor

Second experimental method :

- Study of transitions induced by a RF field in a Penning trap in a given magnetic field (Washington, Mainz, Stanford, Harvard)

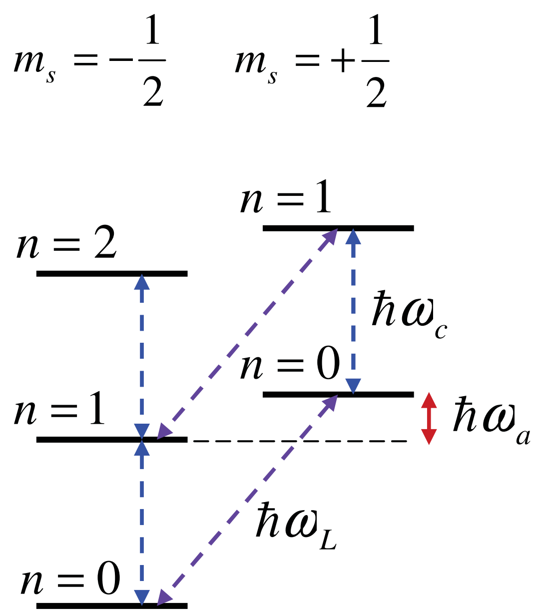
The energy levels of one electron in a magnetic field are given by :

$$E(n, m_s) = \left(n + \frac{1}{2}\right)\hbar\omega_c + m_s\,\hbar\omega_L$$

where $\dfrac{\omega_L}{\omega_c} = \dfrac{g}{2} = 1 + \dfrac{\omega_a}{\omega_c}$

and ω_a is the anomaly frequency $\omega_a = \omega_L - \omega_c$

directly related to a_e $\boxed{a_e = \dfrac{\omega_a}{\omega_c}}$



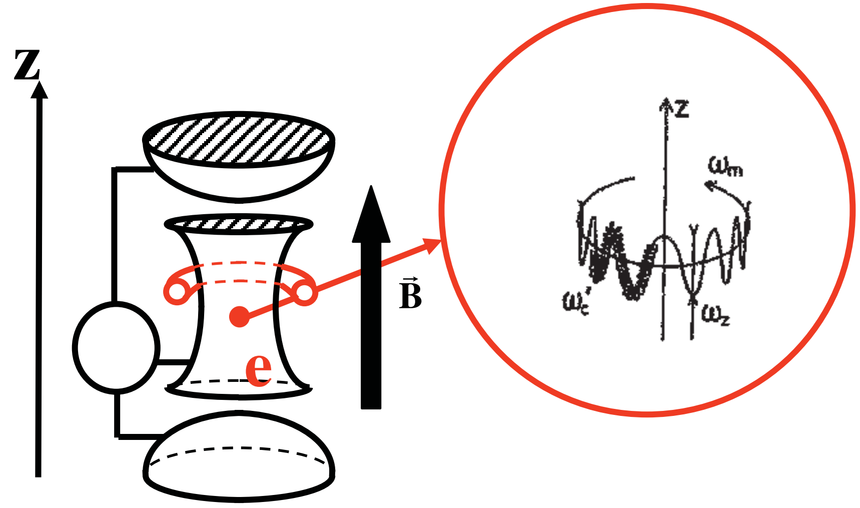
Rabi-Landau levels

In a Penning trap

A quadrupolar electric potential is applied which confine the electron along the z axis

$$V(x,y,z)=\frac{1}{2}m_e\omega_z^2\left[z^2-\frac{x^2+y^2}{2}\right]$$

the transverse confinement is obtained by the application of the magnetic field



hyperbolic Penning trap

The electron movement is the sum of :

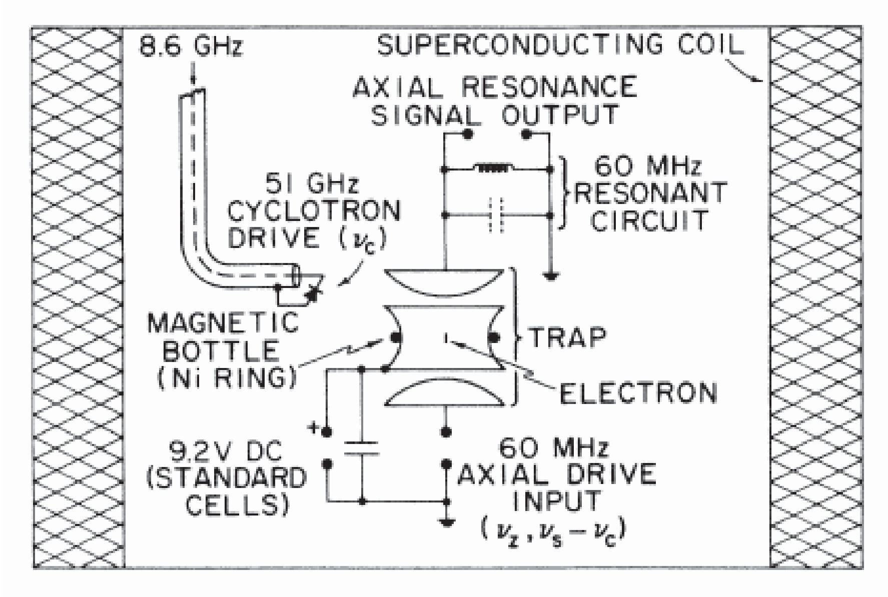
- a cyclotron rotation at a slightly modified frequency
- an oscillation along the z axis
- a slow rotation at the magnetron frequency

An electron suspended in a Penning trap is a homemade atom called geonium

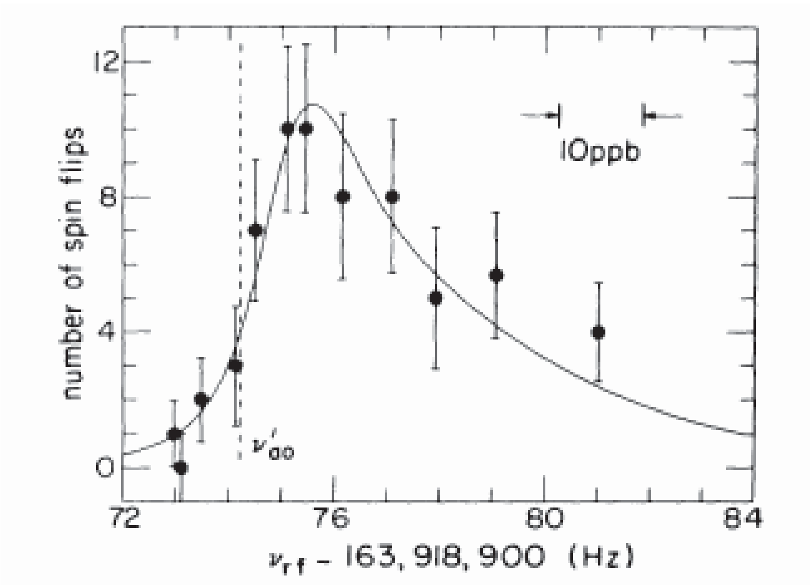
P. Ekstrom and D. Wineland, Scientific American 243, 91 (1980)
L.S. Brown and G. Gabrielse, Rev. Mod. Phys. 58, 233 (1986)

Pionneer work in Washington

- Penning trap at 4K
- single electron strored



Measurement of the cyclotron frequency
and of the anomaly frequency



Detection of spin-flip through the induced shift of the axial frequency

Results : $g_{e^-}/2 = 1.001\,159\,652\,200\,(40)$

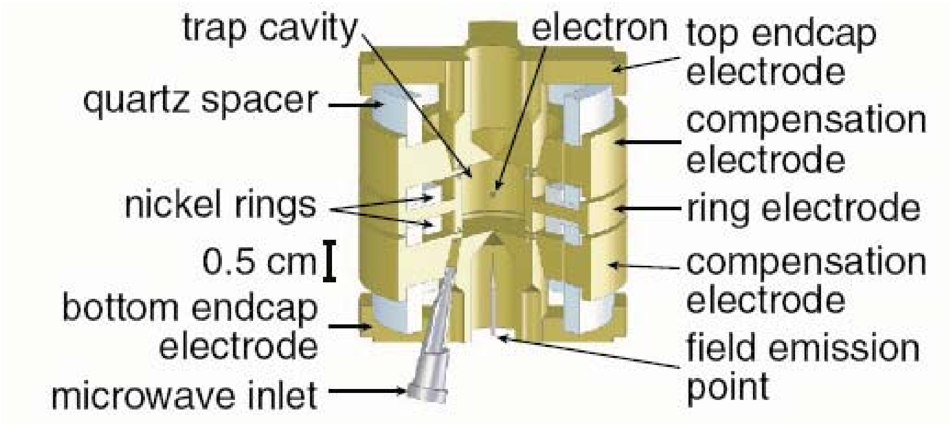
4 x 10^{-11}

and $g_{e^-}/g_{e^+} = 1 + (0.5 \pm 2.1) \times 10^{-12}$

R.S. Van Dyck Jr, P.B. Schwinger and H.G. Dehmelt, Phys. Rev. D 34, 722 (1986)
and Phys. Rev. Lett. 59, 26 (1987)

The Harvard experiment

and the most precise determination of the electron g-factor



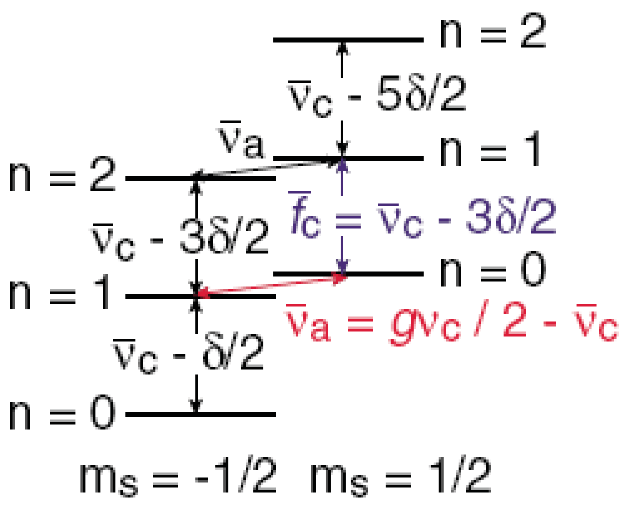
- Cylindrical Penning trap invented to form a microwave cavity that could inhibit spontaneous emission (by a factor of up to 250) → narrowed linewith

- Trap cavity cooled to 100 mK → the electron cyclotron motion is its ground state

- Careful control and probe of radiation field and magnetic field in the trap cavity

The one quantum change in cyclotron motion is resolved

Electron's lowest cyclotron and spin levels



Special relativity makes the transition cyclotron frequency decrease by a very small shift

$$-\delta\left(n+1+m_s\right)$$

where $$\frac{\delta}{\omega_c/2\pi} = \frac{\hbar\omega_c}{mc^2} \approx 10^{-9}$$

not negligible at this level of precision !

- 5.4 T stabilized magnetic field
- very stable trapping potential provided by a charged capacitor

Numerical values : $\omega_c/2\pi \approx 150\ \text{GHz}$

$\omega_z/2\pi \approx 200\ \text{MHz}$

$\omega_m/2\pi \approx 133\ \text{kHz}$

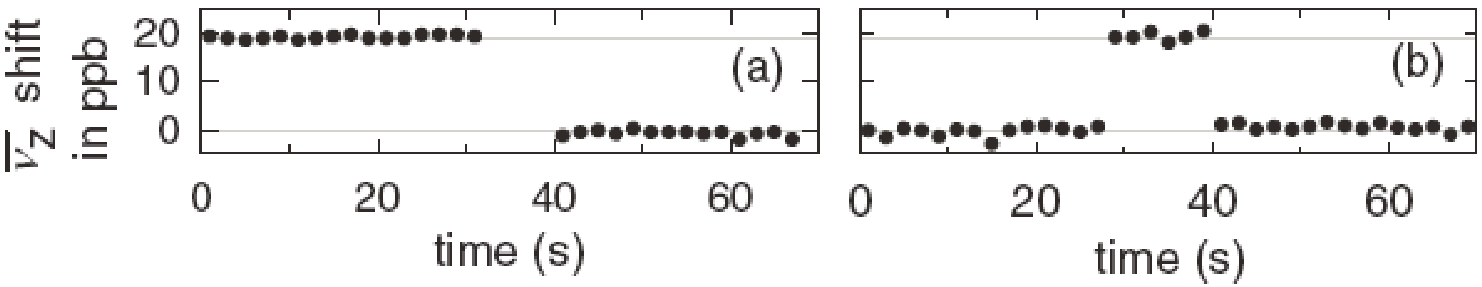
$\omega_a/2\pi \approx 174\ \text{MHz}$
(measured)

Quantum nondemolition (QND) detection

The one-quantum sensitivity is obtained through the coupling of the cyclotron motion to the thermally driven axial motion at 200 MHz frequency where electronic detection is efficient

This coupling is obtained with a magnetic bottle gradient of B (Ni rings)

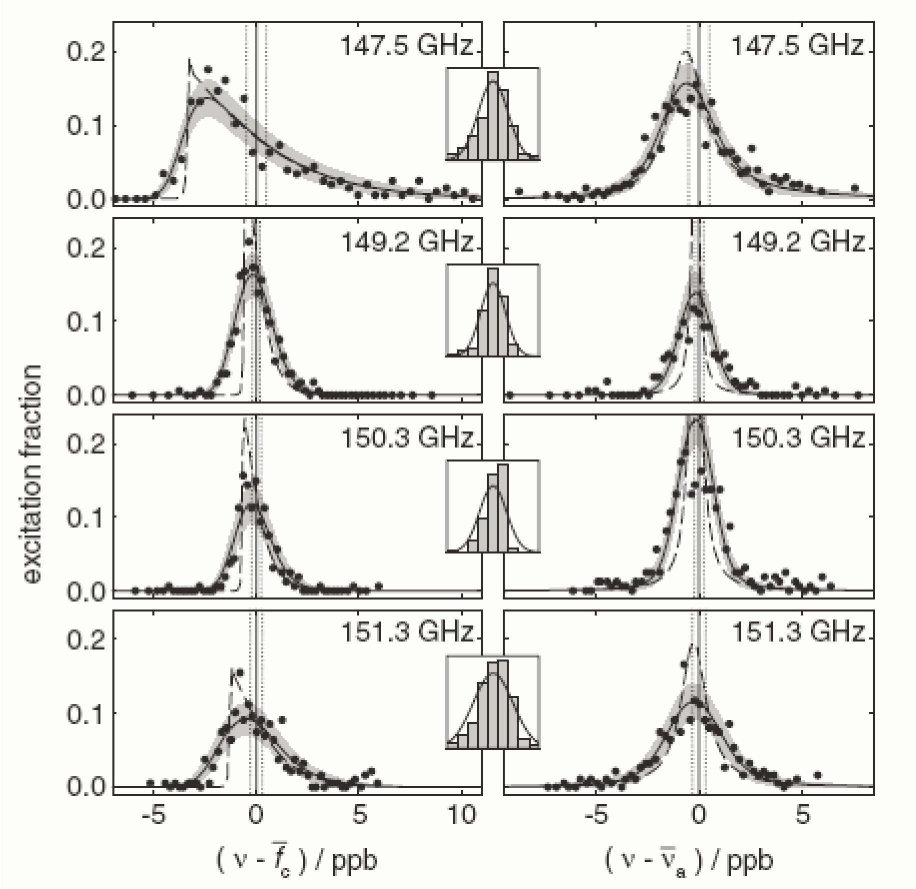
The shift is $\Delta\omega_z = \delta_B \left(n + m_s\right)$ here 4 Hz in 200 MHz



spin flip anomaly transition induced by applying an oscillating potential to the electrodes

one-quantum cyclotron excitation induced by microwaves injected into the cavity

Quantum-jump spectroscopy



cyclotron transitions

anomaly transitions

measuring the quantum jumps per attempt to drive them as a function of drive frequency

Result :

in 2006

$$g/2 = 1.001\ 159\ 652\ 180\ 85\ (76)$$

7.6 x 10^{-13}

in 2008

$$g/2 = 1.001\ 159\ 652\ 180\ 73\ (28)$$

2.8 x 10^{-13}

B. Odom *et al.*, Phys. Rev. Lett. 97, 030801 (2006)
D. Hanneke, S. Fogwell and G. Gabrielse, Phys. Rev. Lett. 100, 120801 (2008) and Phys. Rev. A 83, 052122 (2011)

electron anomaly : discussion

The last result obtained in Harvard is :

$$a_e = 1\ 159\ 652\ 180.73\ (0.28) \times 10^{-12}$$ 2.4 x 10^{-10}

- Taking into account the presence of the muon and tau particles, the QED contribution to the electron g - 2 can be written :

$$a_e = A_1 + A_2\left(m_e/m_\mu\right) + A_2\left(m_e/m_\tau\right) + A_3\left(m_e/m_\mu, m_e/m_\tau\right)$$

where $A_i = A_i^{(2)}\left(\frac{\alpha}{\pi}\right) + A_i^{(4)}\left(\frac{\alpha}{\pi}\right)^2 + A_i^{(6)}\left(\frac{\alpha}{\pi}\right)^3 + .A_i^{(8)}\left(\frac{\alpha}{\pi}\right)^4 ..$ and $A_1^{(2)} = \frac{1}{2}$

- Since the experimental uncertainty is less than 1% of $\left(\frac{\alpha}{\pi}\right)^4 \approx 29 \times 10^{-12}$

the coefficient $A_1^{(8)}$ is needed to match the precision of theory with experiment

- In addition, the total non QED (hadronic) contribution to a_e is 1.72(2) x 10^{-12}

see :T. Kinoshita in *Lepton dipole moments,* Ed. World Scientific (2010)

But the comparison of theory with measured electron anomaly needs also a value of *α* obtained by an independent measurement

electron anomaly and fine structure constant

On another hand, the last measurement of the electron g-factor, combined with recent calculations of $A_1^{(8)}$ cefficient and estimation of $A_1^{(10)}$ one gives the most precise determination of the fine structure constant

$$\alpha^{-1} = 137.035\,999\,084\,(33)\,(39)$$

exp. theor.

3.7×10^{-10}

B. Odom *et al.*, Phys. Rev. Lett. 97, 030802 (2006) and 99, 039902 (2007)
D. Hanneke, S. Fogwell and G. Gabrielse, Phys. Rev. Lett. 100, 120801 (2008)

The other determinations of the fine structure constant are discussed in the following

Measurements of the muon g-factor

- First experiment at the Columbia-Nevis Cyclotron :

  the direct observation of the muon magnetic moment precession in a magnetic field provided the indication that the muon behave like a heavy electron with a g-factor given by

  $$\frac{g}{2} = 1.00122\,(8)$$ in agreement with predictions of QED

  R.L. Garwin *et al.* , Phys. Rev. 118, 271 (1960)   Columbia

- During the following years, experiments were performed at CERN, either at the synchrocyclotron or at the proton synchrotron :

  G. Charpak *et al.*, Phys. Lett. 1, 16 (1962)
  J. Bayley *et al.*, Nucl. Phys. B150, 1 (1979)

  leading to increasing accuracy

Measurements of the muon g-factor

- Experiment in Brookhaven by V. Hughes and collaborators

Study of the spin precession relative to the momentum in a given magnetic field

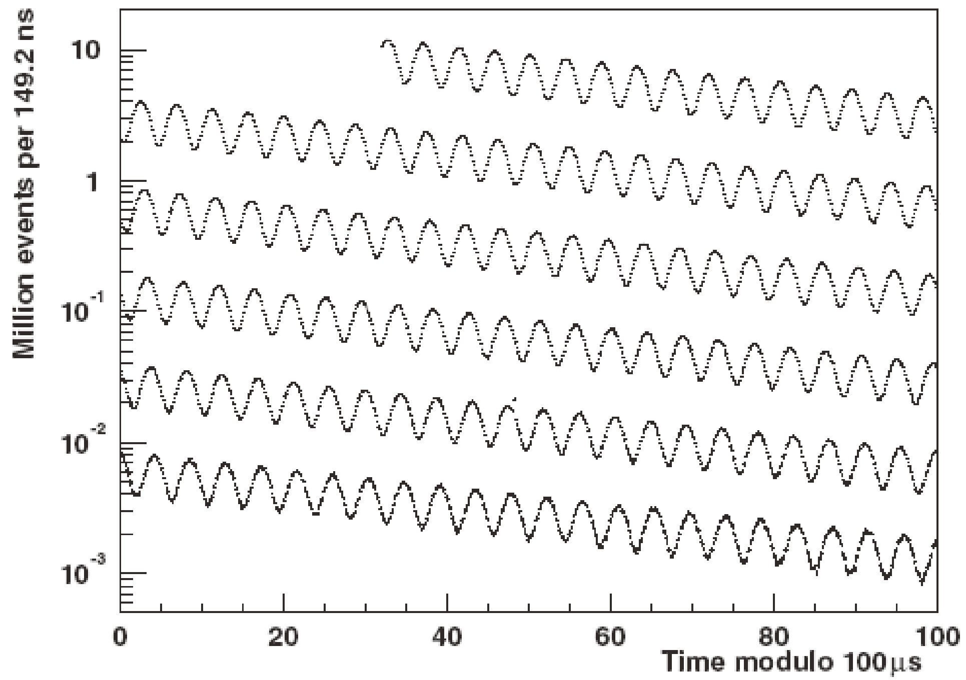
In both CERN and Brookhaven experiments, relativistic muons are stored in circular orbits in a uniform magnetic field and are detected through their decay electrons

The anomaly frequency is observed as an oscillation of the detected electrons

The magnetic field is deduced from proton NMR

Result :

$$a_\mu = 1.165\,920\,91\,(63) \times 10^{-3}$$



G. Bennet *et al.* , Phys. Rev. D 73, 072003 (2006) E821 collaboration

muon anomaly : discussion

The average current experimental result is :

$$a_\mu = 116\,592\,080\,(63) \times 10^{-11}$$

0.5 ppm

- Due to the mass dependence, the muon g - 2 is 4 x 10^4 times more sensitive to hadronic and electroweak effects than the electron g - 2

The hadronic contribution to a_μ is 60 ppm

This result allows to check the contribution of virtual hadrons to the muon anomaly

- Including all contributions, the theoretical value of a_μ in the Standard Model is :

$$a_\mu = 116\,591\,791\,(62) \times 10^{-11}$$

which differs from the experimental value by 289 (86) x 10^{-11} 3.4 σ

Both theory and experiment must be improved to know if it is or not an indicator of physics beyond the standard model

see :T. Kinoshita in *Lepton dipole moments,* Ed. World Scientific (2010)

The various ways to determine α

We have seen that a determination of α can be deduced :
- from the hyperfine structure of muonium with an accuracy of 5.8×10^{-8}
- from the fine structure of helium with an uncertainty of 3.1×10^{-8}
  limited by uncalculated high-order QED terms
- from the electron g - 2 magnetic moment anomaly with an uncertainty of 3.7×10^{-10}
  partly due to both experiment and calculations

Other independent measurements are strongly needed to test QED

Other methods which have been used :

- Gyromagnetic ratio of a shielded proton (H_2O) in high field 3.7×10^{-8}
- Josephson effect 3.1×10^{-7}
- Quantum Hall effect 1.8×10^{-8}
- Diffraction of neutrons 2.4×10^{-8}
- Recoil effect (measurement of the h/M ratio) 6.6×10^{-10}

The two most precise methods only are discussed in the following

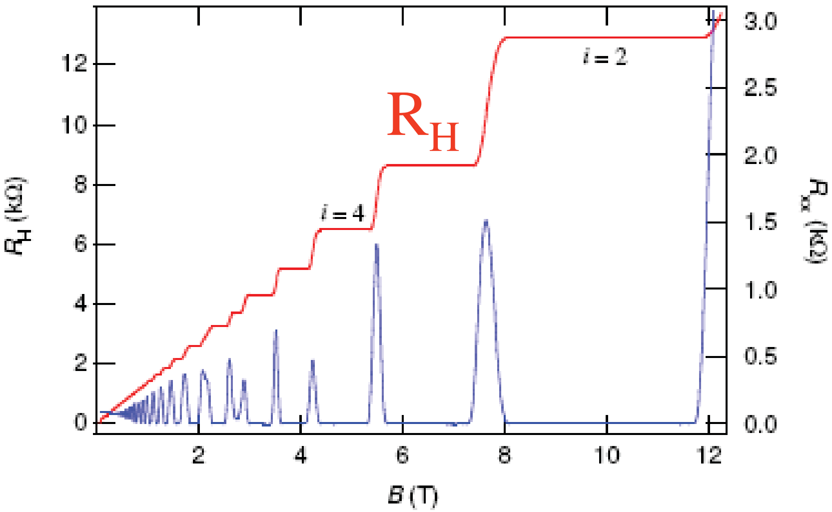
The Quantum Hall effect

Fixed current I in a quantum Hall effect device at low temperature

Plateaus are observed when the resistance is recorded versus the magnetic field

$$R_H(i) = \frac{R_K}{i} = \frac{h}{i e^2}$$

where i is integer

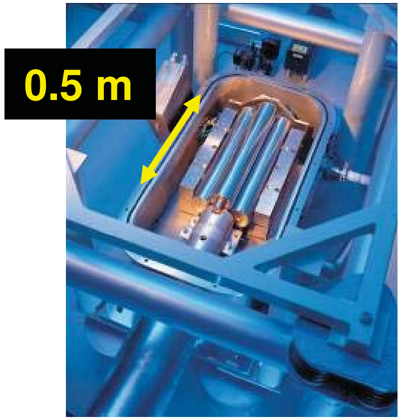


The Von Klitzing constant R_K is simply related to α

($\mu_0 c$ - impedance of vacuum - is exact in SI)

$$R_K = \frac{h}{e^2} = \frac{\mu_0 c}{2\alpha}$$

Its measurement by comparison to a known resistance is done by means of a calculable capacitor

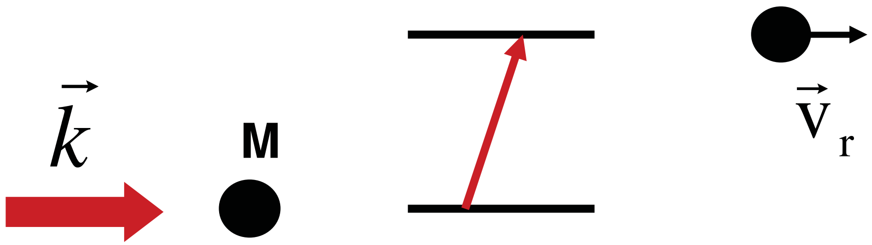


and is limited to 1.8×10^{-8} as the deduced value of α

(NIST, NML, NPL, BNM)

The constant α can also be deduced from recoil

The recoil velocity gives the ratio h/M



$$\vec{v}_r = \frac{\hbar \vec{k}}{M}$$

The recoil velocity can be measured very precisely as a frequency shift

Measurement of h/M allows to determine α

Rydberg constant in terms of energy :

$$h\,c\,R_\infty = \frac{1}{2} m_e c^2 \alpha^2$$

$$\alpha^2 = \frac{2R_\infty}{c} \times \frac{M}{m_e} \times \frac{h}{M}$$

Relative uncertainty on each term :
- Rydberg constant : 5×10^{-12}
- atom-to-electron mass ratio : 4×10^{-10}

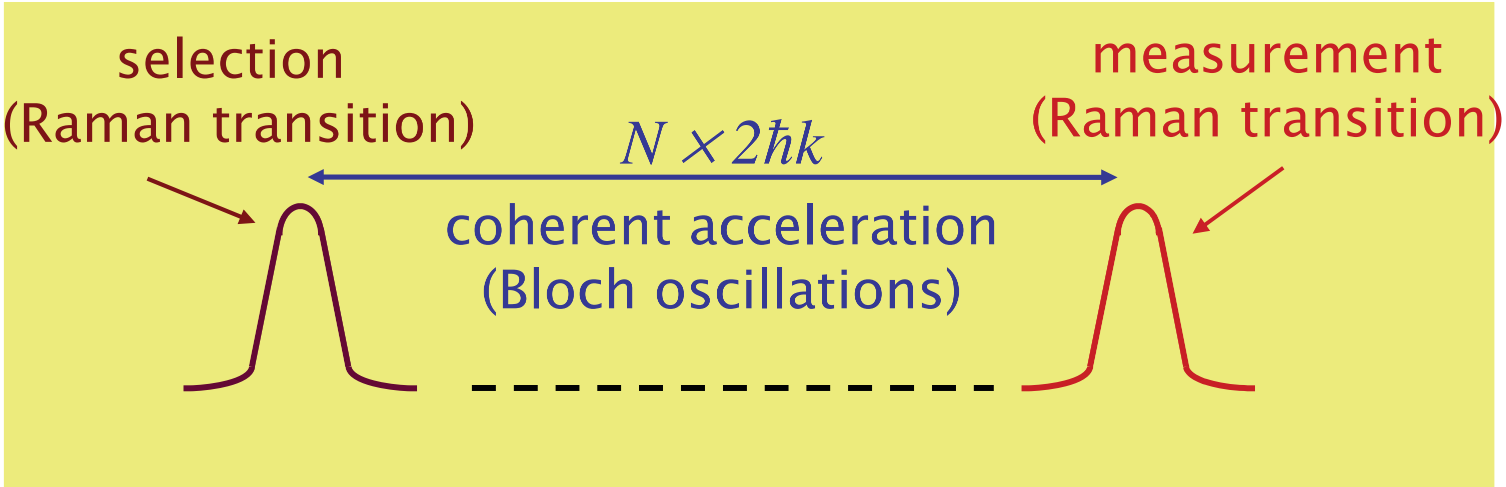
Experiments have been performed : - in Stanford (Cs atoms)
- in Paris (Rb atoms)

The Paris experiment

Ultracold Rb atoms at 4 μK

Principle :

MOT
+
optical molasses

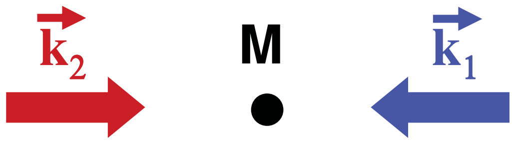


- Selection of an initial subrecoil velocity class
- Coherent acceleration : N Bloch oscillations
  momentum transfer of $2N\,\hbar k$
- Measurement of the final velocity distribution

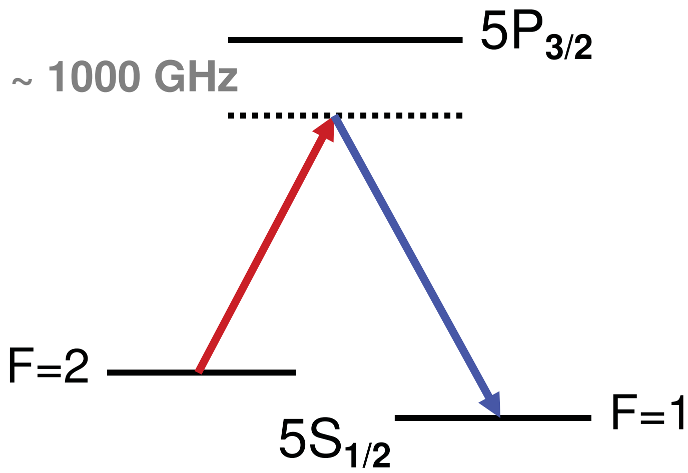
Final uncertainty : $\sigma\,(v_r) = \sigma\,(v)\ /\ 2N$

Subrecoil velocity selection

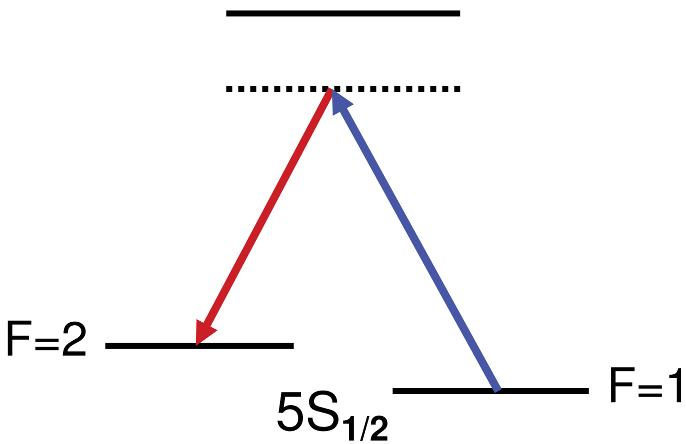
Stimulated Raman transition
in the ground state (π pulse)
between the two hyperfine levels



Contra-propagating laser beams
Doppler shift → velocity selection



(non selected atoms left in F=2 level are blown out)



velocity distribution measurement

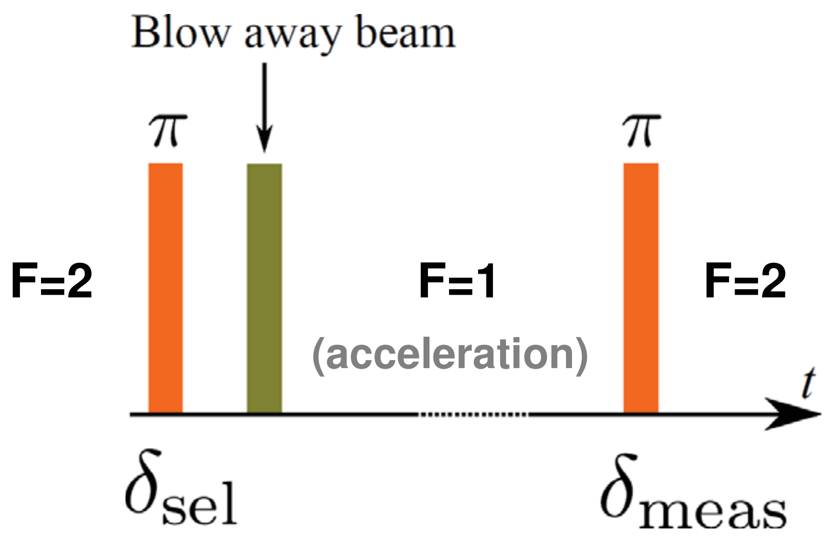
after the coherent acceleration
(momentum transfer)

**Velocity selective stimulated Raman transition
from F = 1 to F = 2**

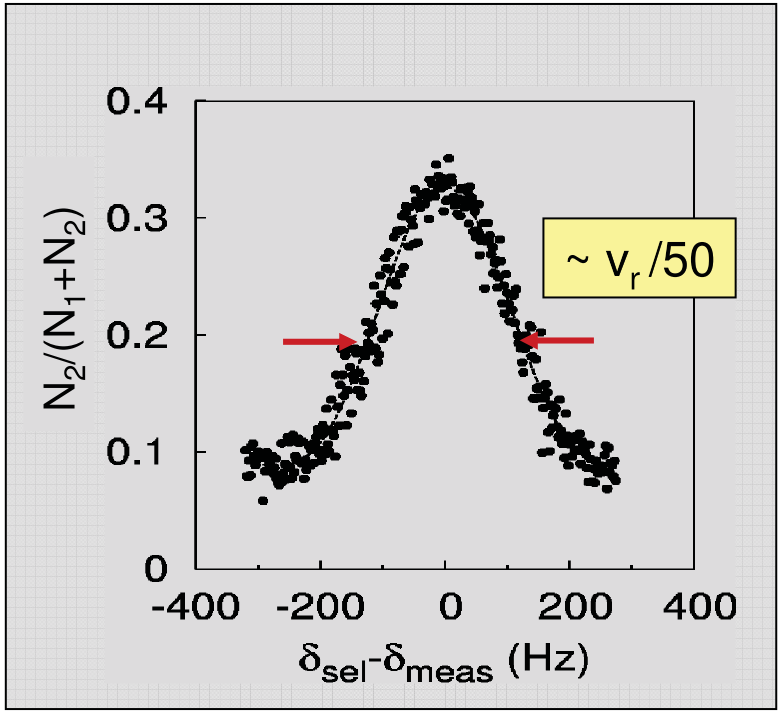
Frequency scan of one beam through the profile

Experimental sequence

for the Raman beams



Signal

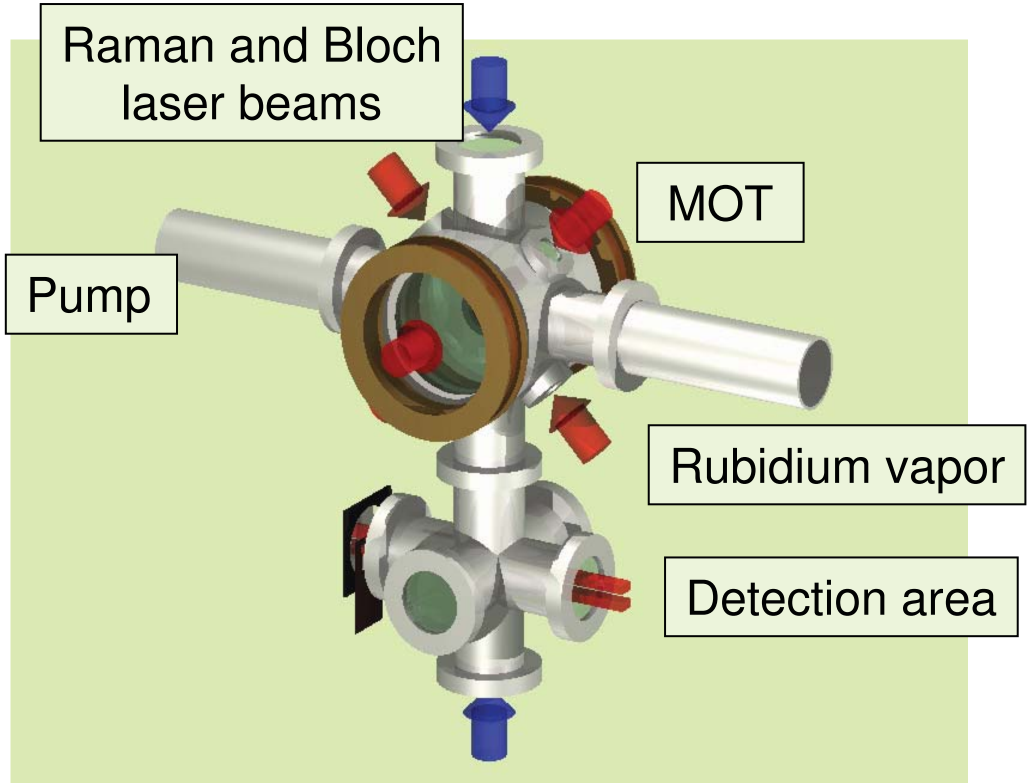


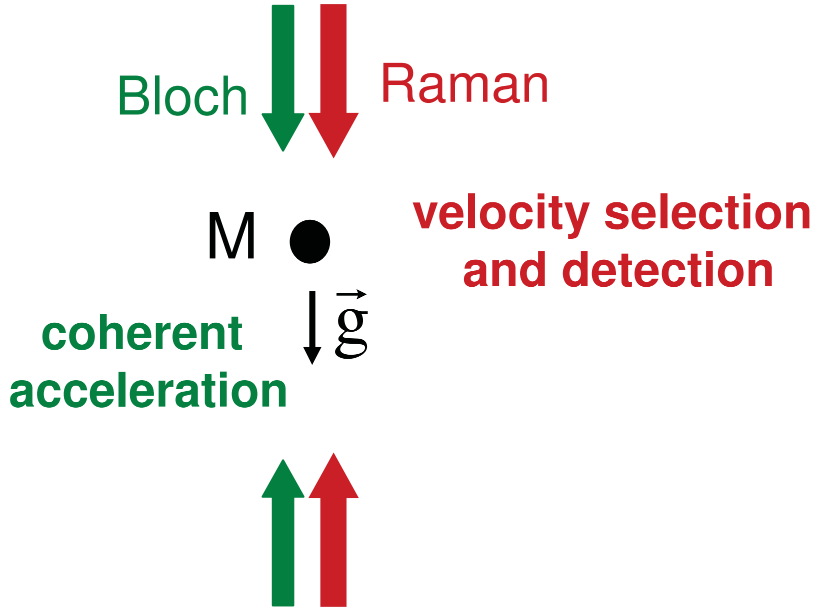
The F = 2 relative population is recorded versus the difference between the selection and the detection laser frequencies

$$\sigma_v \approx 1\,\text{Hz} \leftrightarrow \frac{v_r}{15000}$$

Experimental arrangement

Vertical geommetry





To cancel the effect of gravity,
a differential measurement is used
(up and down accelerations)

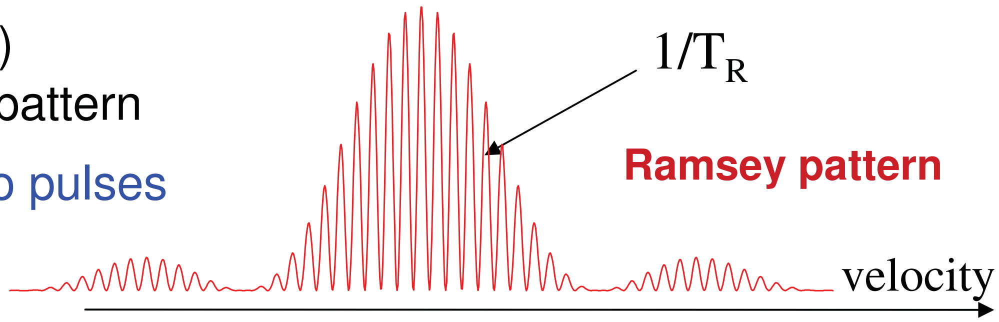
The populations in F=2 and F=1 levels after the velocity detection are successively measured by fluorescence using a time of flight technique

Improvement of the velocity selection

- In our earlier experiment ($\pi-\pi$ configuration), two π Raman pulses were used
  - to select a subrecoil velocity distribution
  - and to measure the final velocity distribution

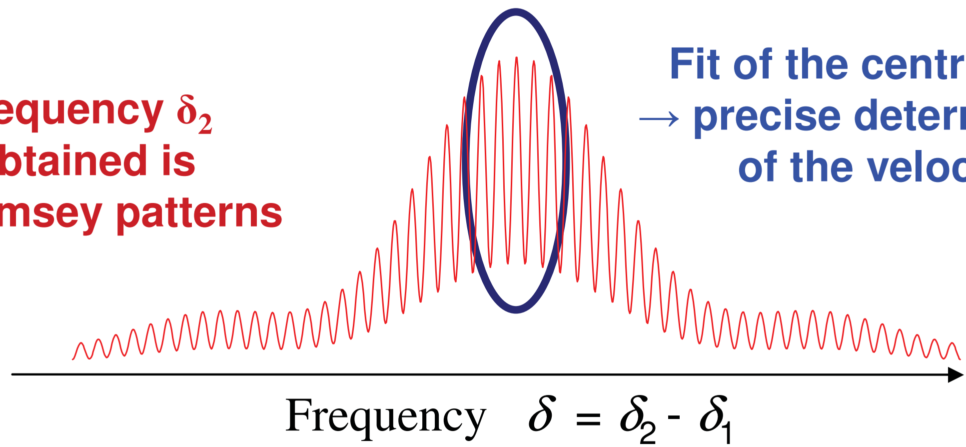
- In our present experiment ($\{\pi/2, \pi/2\}-\{\pi/2, \pi/2\}$ configuration),
  - the first pair of $\pi/2$ pulses (frequency δ_1) selects a velocity pattern

T_R : time between the two pulses



- the second pair of $\pi/2$ pulses (frequency δ_2) selects another velocity pattern

When the detection frequency δ_2 is swept, the signal obtained is the convolution of two Ramsey patterns

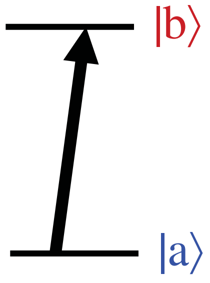


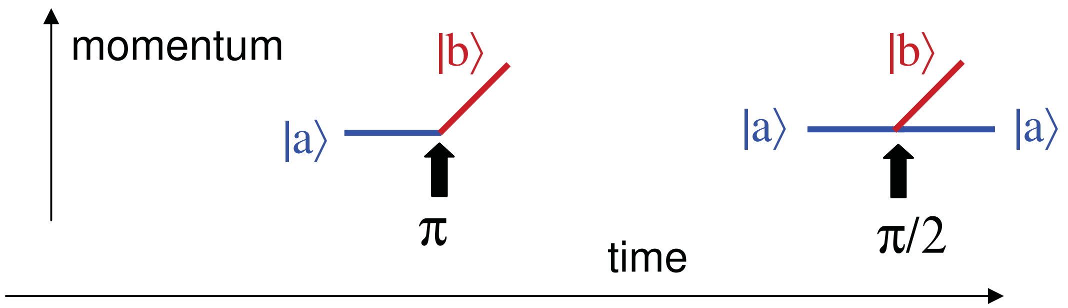
Fit of the central part → precise determination of the velocity

Interferometric method

Atom interferometry

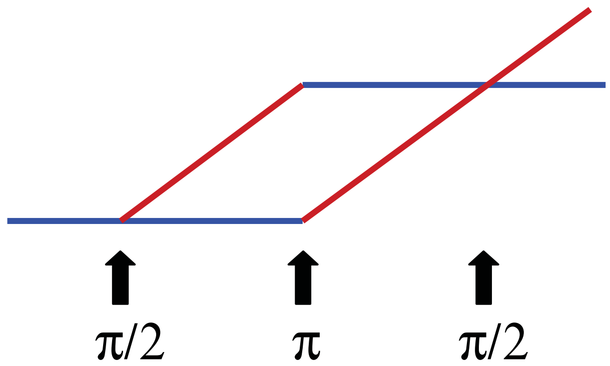
The manipulation of ultracold atoms with laser beams
has opened the way to atom interferometry

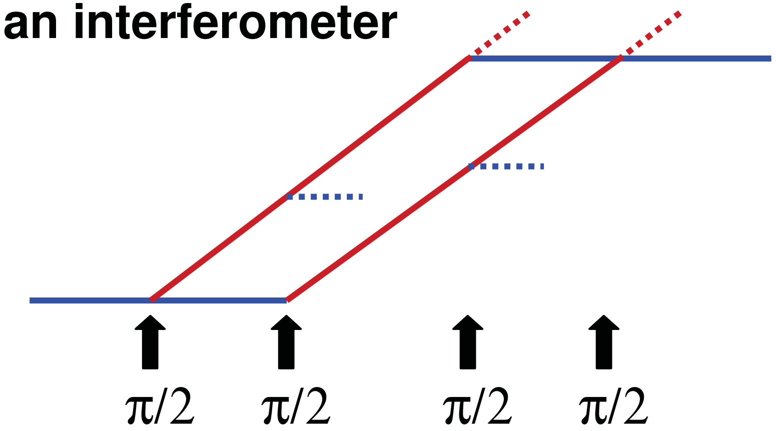




With a resonant laser light, a π pulse acts as a mirror and a π/2 pulse as a beam splitter

With 3 or 4 pulses, one realizes an interferometer





Ramsey-Bordé

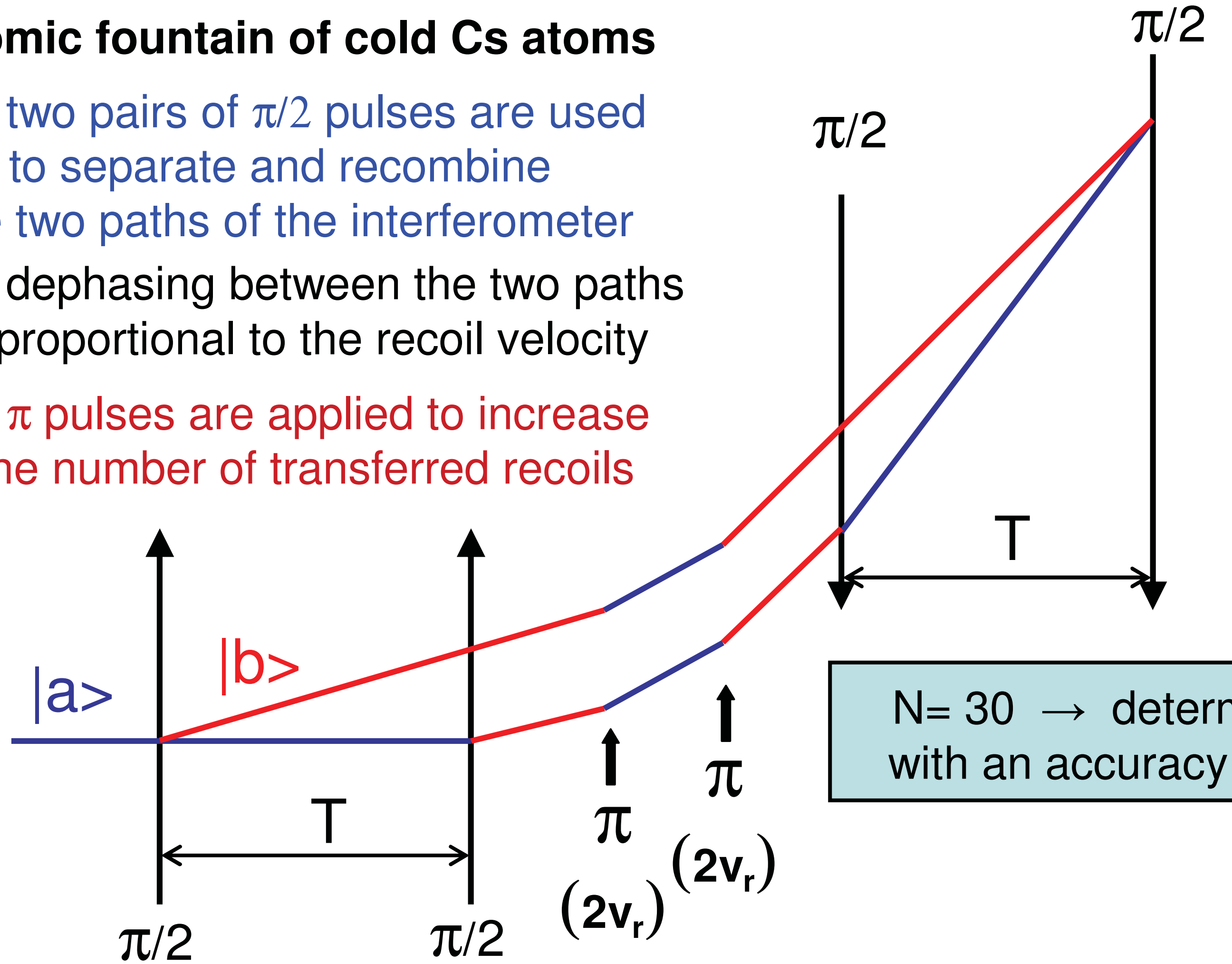
The Stanford experiment

Atomic fountain of cold Cs atoms

The two pairs of $\pi/2$ pulses are used to separate and recombine the two paths of the interferometer

The dephasing between the two paths is proportional to the recoil velocity

N π pulses are applied to increase the number of transferred recoils



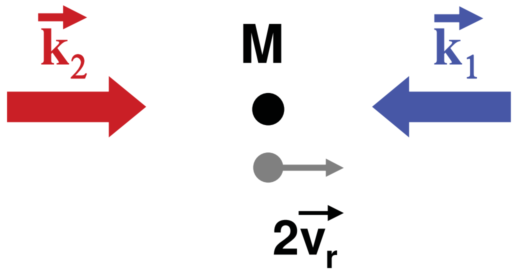
N= 30 → determination of α with an accuracy of 7.7×10^{-9}

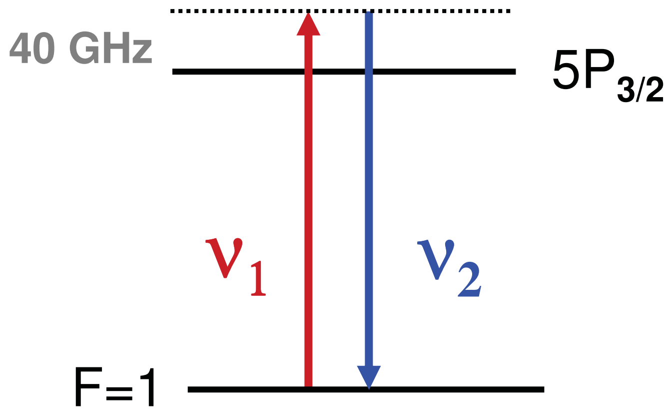
D.S. Weiss et al, Phys. Rev. Lett. 70, 2706 (1993)

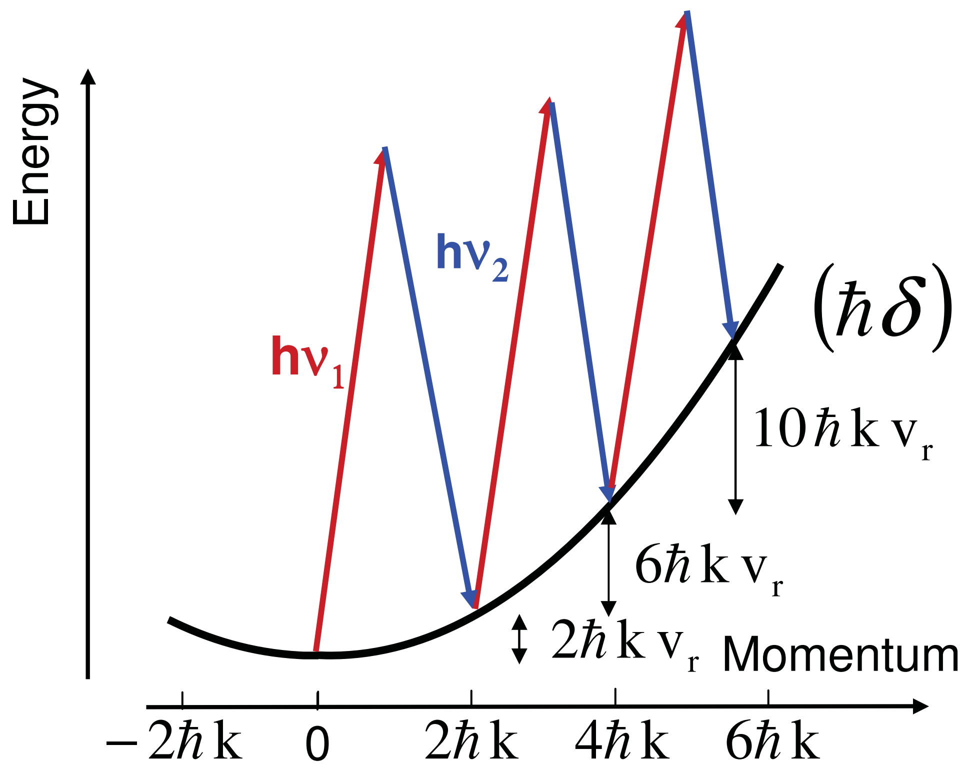
A. Wicht, J.M. Hensley, E. Sarajlic and S.Chu, Phys. Scr. T102, 82 (2002)

Coherent acceleration of the atoms (I)

Stimulated Raman transition
in a given hyperfine level







Linear frequency sweep of the frequency of one beam :

$$\delta = \nu_1 - \nu_2 \propto t$$

Momentum transfer :

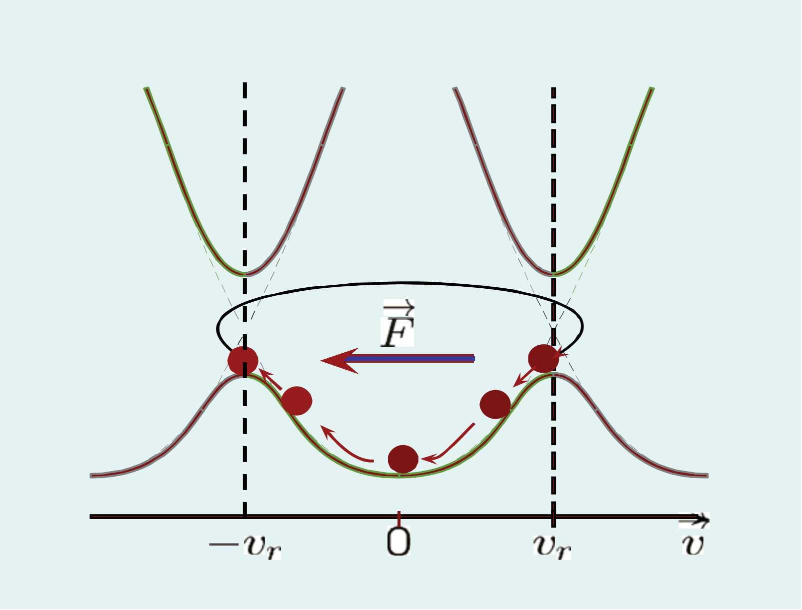
$2\hbar k$ per cycle

Coherent acceleration of the atoms (II)

In the laboratory frame : accelerated optical lattice

In the accelerated frame at $a = \dfrac{\pi(\nu_1 - \nu_2)}{k\,t}$: standing wave but inertial force

$$\vec{F} = -M\vec{a}$$



Bloch oscillations in the fundamental energy band

M. Ben Dahan *et al.*,
Phys. Rev. Lett. 76, 4508 (1996)

up to 1000 recoils

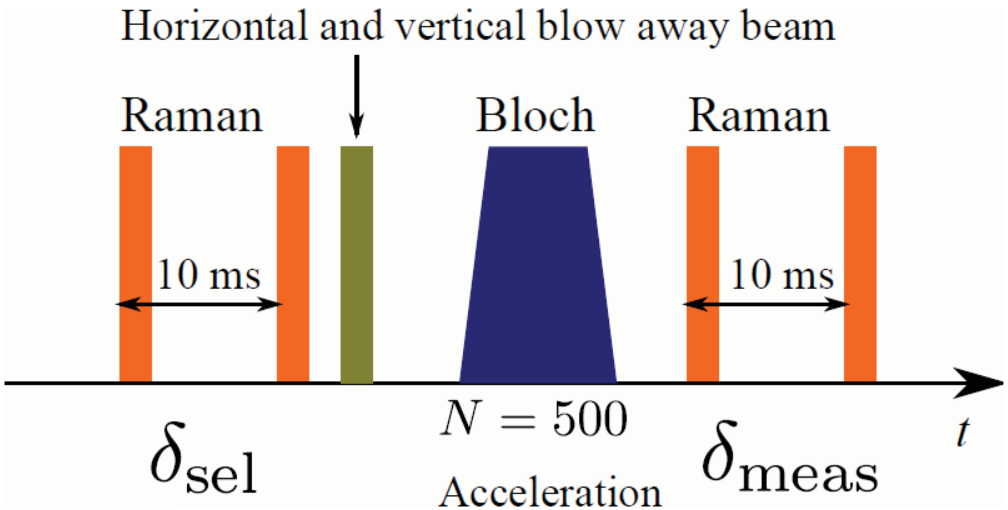
Transfer efficiency of 99.95% per B.O.

Principle : R. Battesti *et al.* , Phys. Rev. Lett. 92, 253001 (2004)

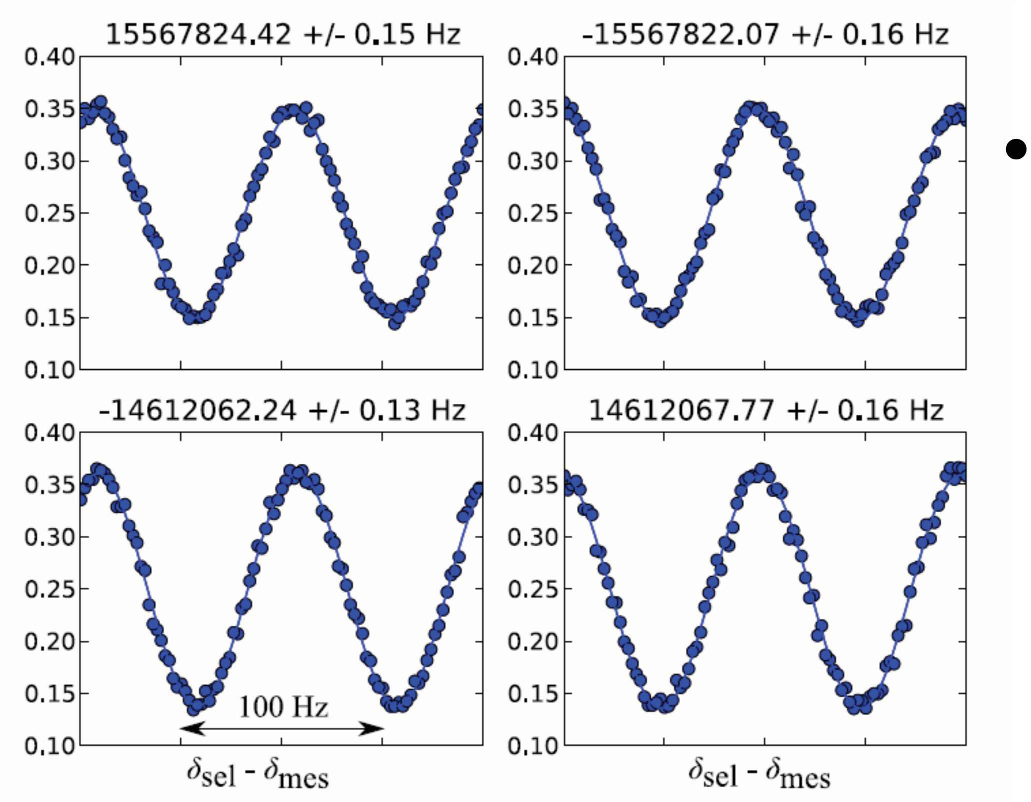
First measurement : P. Cladé *et al.* , Phys. Rev. Lett. 96, 033001 (2006)
and Phys. Rev. A 74, 052109 (2006)

Experimental sequence

Raman beams
(interferometric method)
+
Bloch oscillations



Signal



- upwards and downwards accelerations to cancel the effect of gravity

- exchange between Raman beams to cancel systematic shifts

Four spectra recorded in 5 min.
→ 6 ppb accuracy on h/M

Recent measurements performed in Paris

M. Cadoret *et al.*, Phys. Rev. Lett. 101, 230801 (2008) — 4.6×10^{-9}

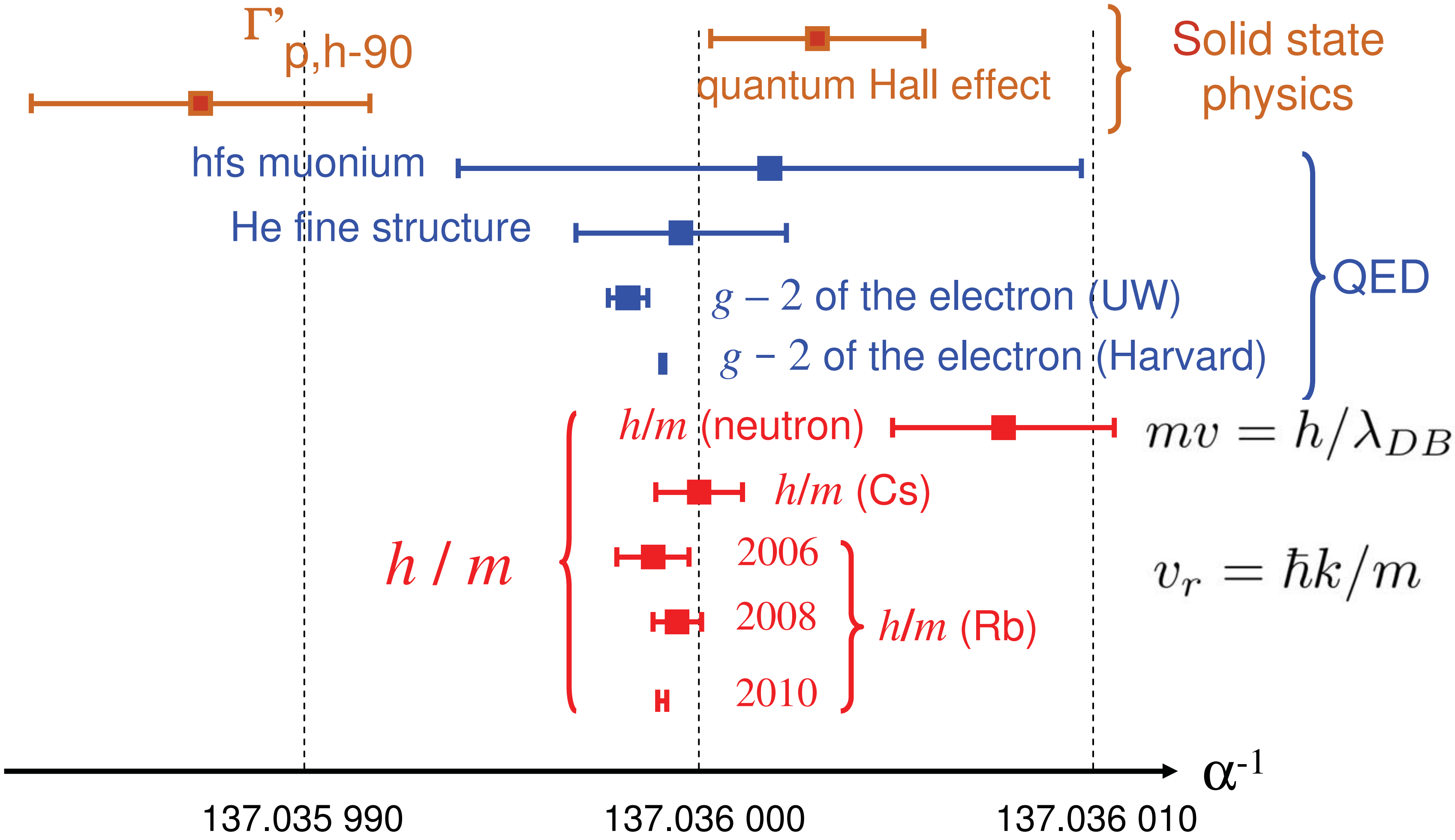
R. Bouchendira *et al.*, Phys. Rev. Lett. 106, 080801 (2011) — 6.6×10^{-10}

TABLE I. Error budget on the determination of $1/\alpha$ (systematic effect and relative uncertainty u in part per 10^{10}).

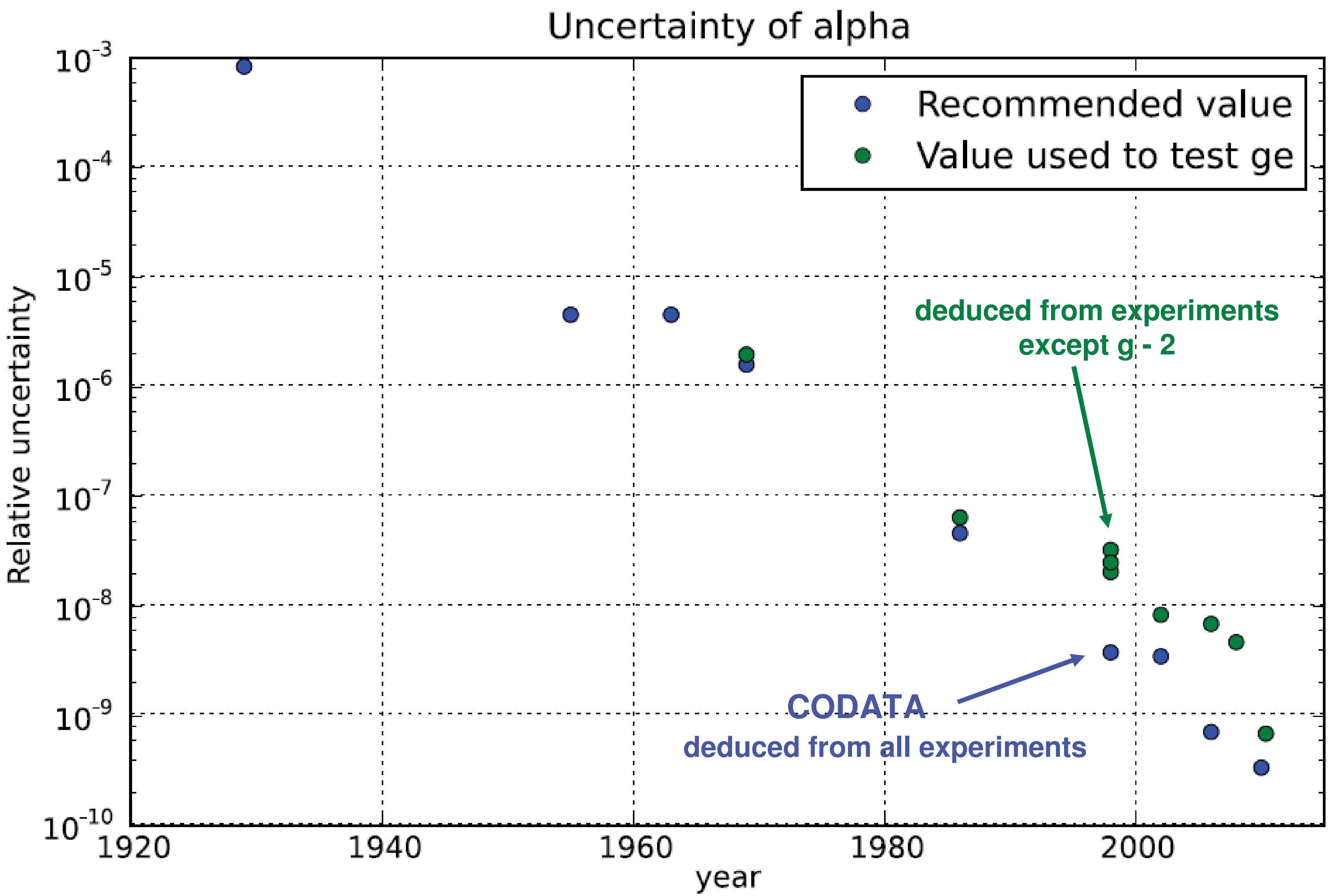
Source	Correction	u
Laser frequencies		1.3
Beams alignment	−3.3	3.3
Wave front curvature and Gouy phase	−25.1	3.0
2nd order Zeeman effect	4.0	3.0
Gravity gradient	−2.0	0.2
Light shift (one photon transition)		0.1
Light shift (two photon transition)		0.01
Light shift (Bloch oscillation)		0.5
Index of refraction atomic cloud and atom interactions		2.0
Global systematic effects	−26.4	5.9
Statistical uncertainty — 1370 spectra have been recorded		2.0
Rydberg constant and mass ratio		2.2
Total uncertainty		6.6

Result : $\alpha^{-1} = 137.035\,999\,037\,(91)$

Comparaison between various measurements of α



The mesurement of α in different domains of physics is a test of the consistency of theory



Uncertainty of alpha
Recommended value
Value used to test ge
deduced from experiments except g - 2
CODATA
deduced from all experiments
Relative uncertainty
year
10^-3
10^-4
10^-5
10^-6
10^-7
10^-8
10^-9
10^-10
1920
1940
1960
1980
2000

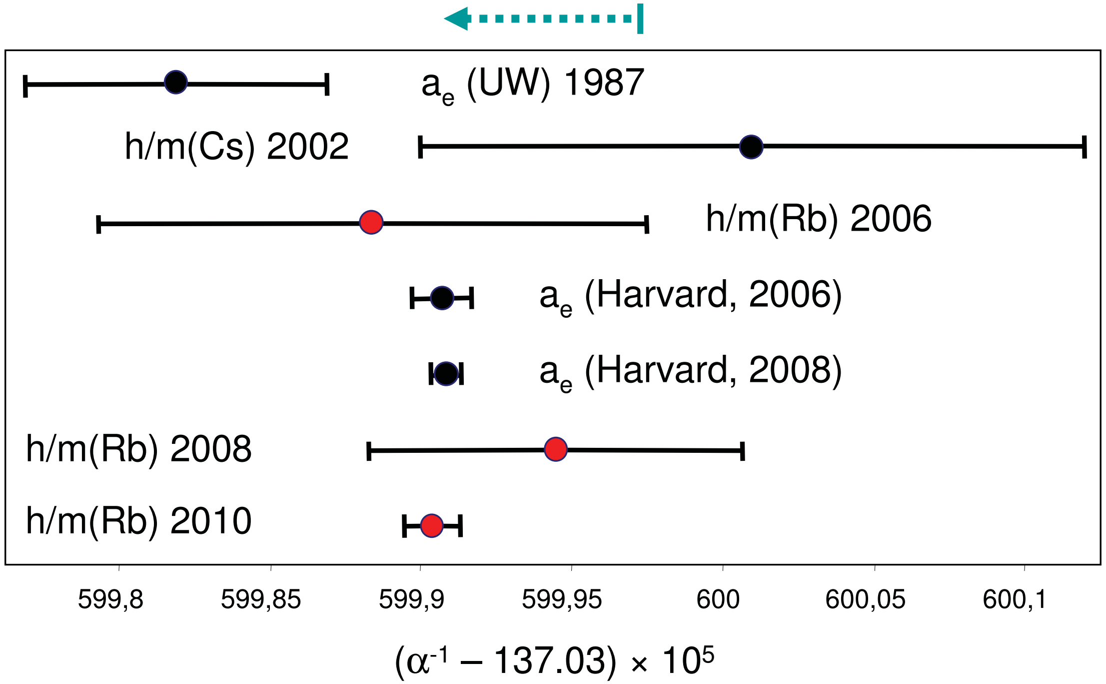
Fine structure constant : discussion

The value of α deduced from a_e is the most accurate one
but is fully dependent on QED calculations

It has been shifted by 4.7 x 10^{-9} in 2007 after a correction in calculations

T. Aoyama *et al.*, Phys. Rev. Lett. 99, 110406 (2007)

G. Gabrielse *et al.*, Phys. Rev. Lett. 99, 039902 (2007)



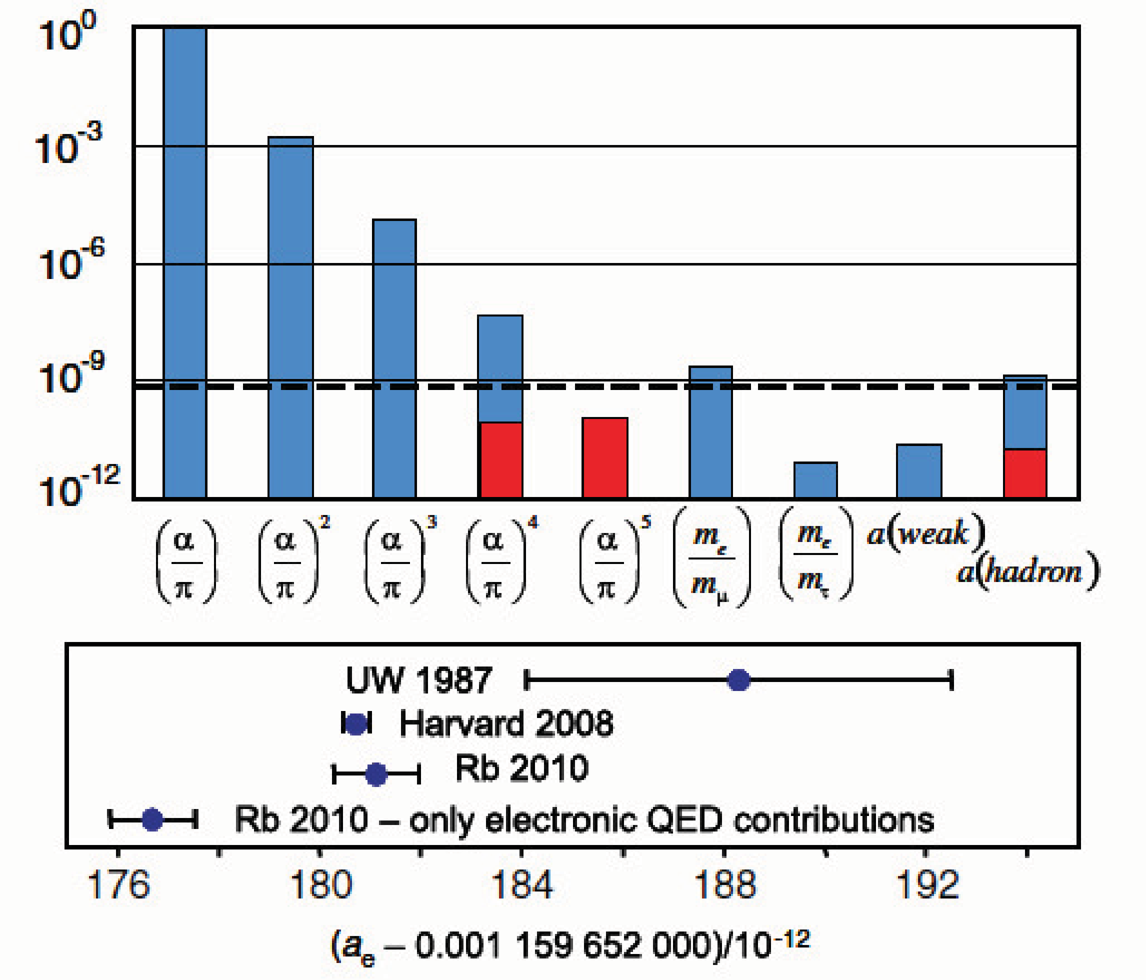
The comparison between recent h/M and g-2 determinations of α gives a test of QED at a level better than 10^{-9}

Electron anomaly and recoil effect : conclusion

$$a_e = C_1\left(\frac{\alpha}{\pi}\right) + C_2\left(\frac{\alpha}{\pi}\right)^2 + C_3\left(\frac{\alpha}{\pi}\right)^3 + C_4\left(\frac{\alpha}{\pi}\right)^4 + a\left(m_e/m_\mu, m_e/m_\tau, weak, hadron\right) + \ldots$$

$$\delta(a_e) = a_e(meas) - a_e(theo) = -(40 \pm 89) \times 10^{-14}$$

g - 2   α from h/M



In red :
theoretical uncertainties

- **First test of the QED at the 10^{-9} level (up to 4-loop terms)**
- **The accuracy is now at the level of muonic and hadronic corrections**

General conclusion

The spectroscopy of atomic systems and free particles at low energy provides very accurate tests of QED

A lot of experiments give results in good agreement with calculated predictions

The most accurate test is the comparison between determinations of the fine structure constant derived from electron g - 2 and h/M measurement

Previous discrepancies are now solved …

- fine structure of helium
- orthopositronium lifetime

… but others are not solved or recently appeared !

- g-2 anomaly of muon $> 3\,\sigma$
- Lamb shift of muonic hydrogen (proton size puzzle) $\sim 5\,\sigma$

The work must continue for both theoreticians and experimentalists

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Many thanks to all my colleagues for their help, comments and suggestions ...

... and thank you for your attention !