

Experimental tests of QED in bound and isolated systems

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The fine structure of atoms and the fine structure constant α

As the hyperfine structures, the fine structure frequencies vary as $\alpha^2 R_\infty$

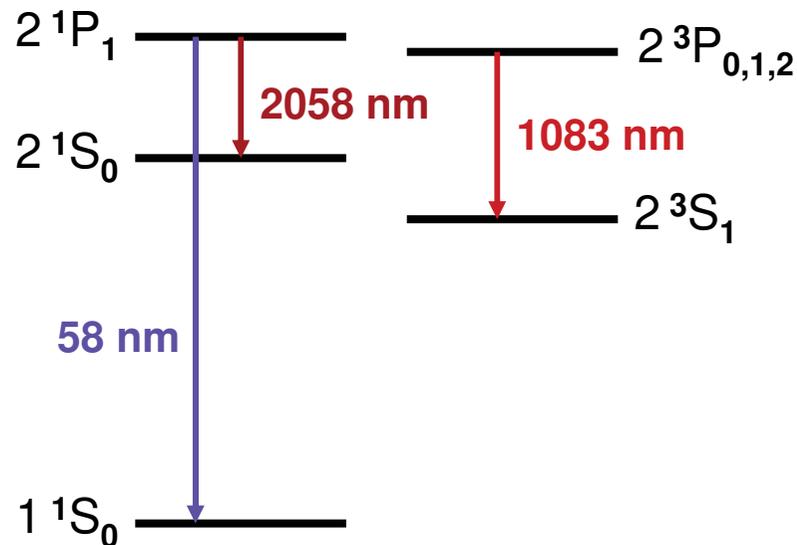
One can determine the $n = 2$ fine structure of hydrogen from the combination of optical measurements involving 2P states

The derived value of $1/\alpha$ is : **137.036003(41)** (accuracy 3×10^{-7})

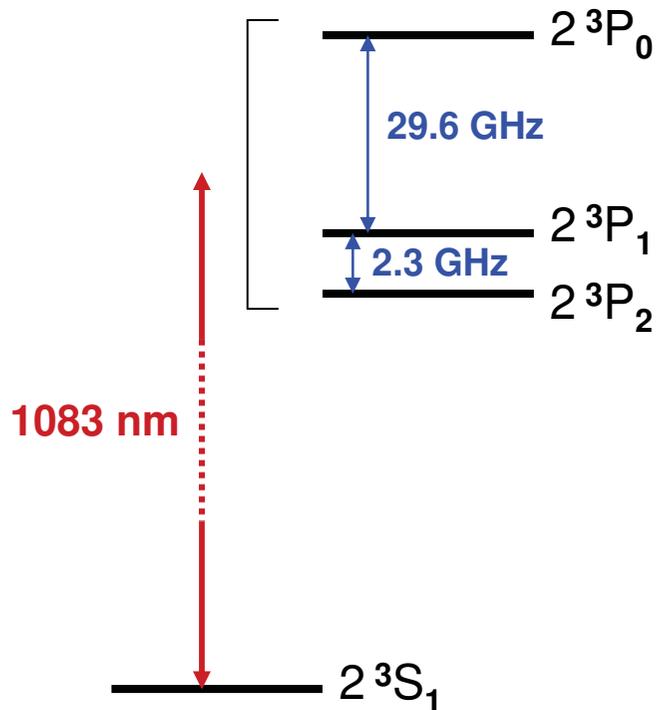
It is not competitive since it is limited by the large natural width of the 2P levels (~ 100 MHz)

On another hand, in helium
the 2^3P states are narrower
(1.6 MHz)
since they cannot easily
decay to the ground state

and the 2^1S_0 and 2^3S_1
states are metastable



The spectroscopy of helium atom : fine structure



A lot of experiments have been performed in ^4He to measure the fine structure intervals of the 2^3P manifold

Metastable 2^3S atoms are populated by discharge either in a cell or in an atomic beam

2^3P_J states are optically excited using a laser diode source at 1083 nm

Frequency intervals have been determined either from optical line structures or by microwave techniques

The spectroscopy of helium atom : fine structure

The various experimental methods

- Laser fluorescence technique with an atomic beam (LENS, Florence) combines sub-Doppler laser spectroscopy and direct microwave measurement

G. Giusfredi et al., Can. J. Phys. 83, 301 (2005)

- Separated oscillatory field microwave measurement of the 2^3P_1 - 2^3P_2 interval uses an optical pulse in a thermal beam followed by two RF pulses (York U.)

J.S. Borbely et al., Phys. Rev. A 79, 060503 (2009)

- Doppler-free saturation spectroscopy in a cell (Harvard U.)

T. Zelevinsky, D. Farkas and G. Gabrielse, Phys. Rev. Lett. 95, 203001 (2005)

- Electro-optic laser technique in an atomic beam (North Texas U.) uses modulated sidebands of a laser diode to measure fine structure and various Zeeman intervals

M. Smiciklas and D. Shyner, Phys. Rev. Lett. 105, 123001 (2010)

All these measurements need very careful study of all systematic effects
(Zeeman, pressure, 2nd order Doppler ...)

Helium fine structure : discussion

All these accurate measurements of the fine structure of the 2^3P level of helium are in good agreement each with others

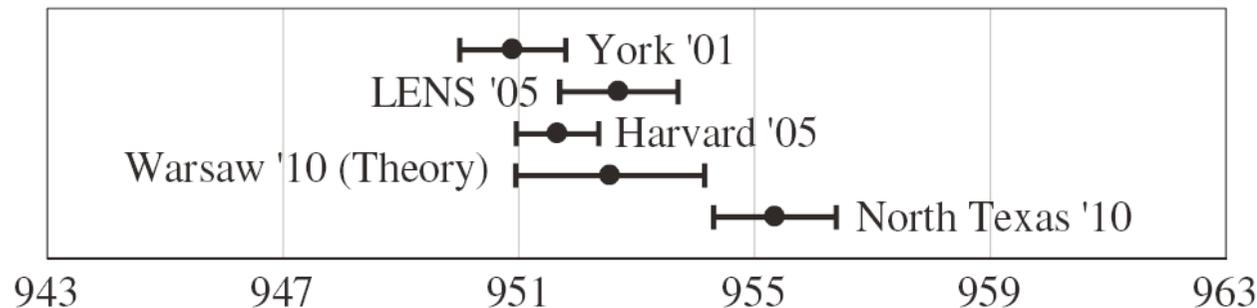
They give a test of QED theory of the electron - electron interaction in bound systems

G.W.F. Drake and Z.-C. Yan, Can. J. Phys. 86, 45 (2008)

Until recently there were two inconsistent calculations which disagreed with experimental results by ~ 15 kHz and more

Recent calculations up to $\alpha^5 R_\infty$ terms finally resolved this discrepancy

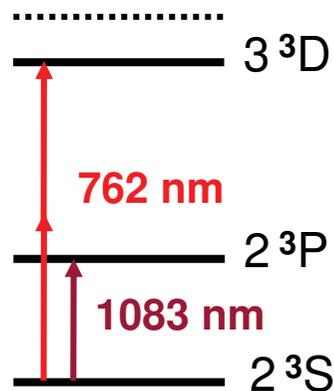
K. Pachucki and V.A. Yerokhin, Phys. Rev. Lett. 104, 070403 (2010)



$2^3P_0 - 2^3P_1$ interval
– 29 646 000 (kHz)
from M. Smiciklas
and D. Shyner (2010)

If the validity of the theory is assumed, experimental fine structure measurements in helium give an independent determination of α with an uncertainty of 2×10^{-8} mainly due to uncalculated high-order QED terms

Other spectroscopic measurements in helium



Precise frequency measurements of optical energy intervals provide a test of two-electron Lamb shift calculations

Optical transitions from the metastable 2^3S_1 state allow the determination of both the 2^3S and 2^3P Lamb shifts and the ionization energy

- Absolute frequency measurements of the 2^3S_1 - $2^3P_{0,1,2}$ transitions at 1083 nm, with a frequency comb, give access to the 2^3S - 2^3P and 2^3P Lamb shifts (Florence)

$$f_0 = 276\,764\,094\,746.9 \text{ (1.3) kHz}$$

$$f_1 = 276\,734\,477\,805.0 \text{ (0.9) kHz}$$

$$f_2 = 276\,732\,186\,818.4 \text{ (1.5) kHz}$$

P. Cancio Pastor et al.,

Phys. Rev. Lett. 92, 023001 (2004)

and 97, 1399 03 (2006)

and 108, 143001 (2012) !

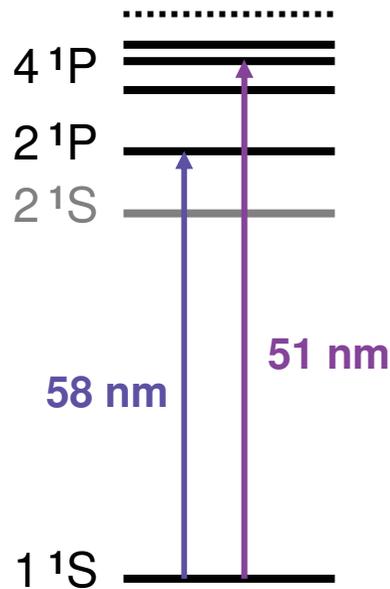
- Measurement of the two-photon 2^3S_1 - 3^3D_1 transition at 762 nm (Paris) provides the ionization energy of the 2^3S state and a precise determination of the 2^3S Lamb shift through theoretical calculations on the 3^3D_1 level

$$f = 786\,823\,850\,002 \text{ (56) kHz}$$

C. Dorrer et al., Phys. Rev. Lett. 78, 3658 (1997)

Theoretical uncertainty of these Lamb shifts : 3000 kHz

Other spectroscopic measurements in helium



The recent development of UV laser sources and the extension of frequency combs in the UV range has opened the way to the study of the resonance line of helium

- Measurement of the 1 ¹S-2 ¹P resonance line at 58 nm (Amsterdam)

K.S.E. Eikema, W. Ubachs, W. Vassen and W. Hogervorst, Phys. Rev. A 55, 1866 (1997)

This experiment yields the 1 ¹S ground state Lamb shift
 $L_1 = 41\,224(45)$ MHz
to be compared to the calculated value 44 233 (35) MHz

- Frequency comb metrology of the 1 ¹S- 4 ¹P and 1 ¹S- 5 ¹P lines (Amsterdam)

D.Z. Kandula et al., Phys. Rev. Lett. 105, 063001 (2010)

This experiment yields the ionization energy of ⁴He

- Metrology of the forbidden 2 ³S₁ - 2 ¹S₀ transition in **cold atoms ...**

Atomic spectroscopy with cold atoms

We have already seen that the atomic frequency standard has been substantially improved by cooling and trapping the Cs atoms

More generally, the development of such techniques is of great interest for high resolution spectroscopy because of :

- the reduced Doppler effect
- the longer interaction time with light
- the advent of new derived physics

The basic idea is to use the **recoil effect** to reduce the atomic velocity



Due to the absorption of a photon the atomic velocity is changed
The recoil velocity is 3 cm/s for a Na atom, since its thermal velocity is 400 m/s

First spectroscopic observation of recoil effect on a CH_4 line :

J.L. Hall, Ch.J. Bordé, K. Uehara, *Phys. Rev. Lett.* 37, 1339 (1976)

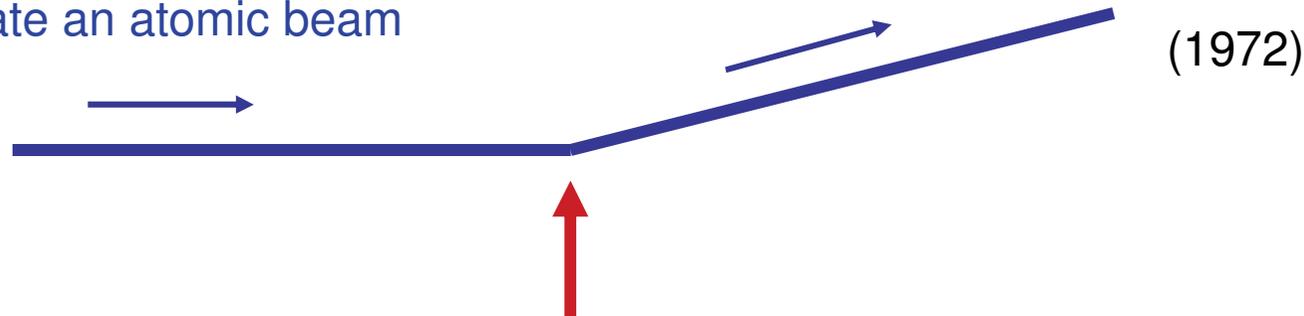
Changing the atomic velocity

The recoil effect is responsible for the radiation pressure

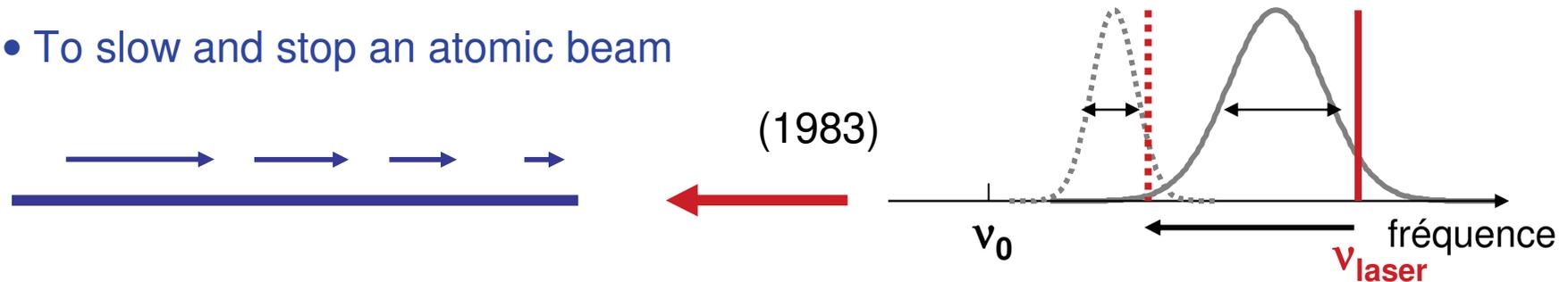
Since spontaneous emission is isotropic, it does not contribute to recoil

Recoil can be used :

- To deviate an atomic beam



- To slow and stop an atomic beam



To keep atoms in resonance with the laser light during the slowing process, one must sweep the laser frequency or the atomic frequency (Zeeman effect)

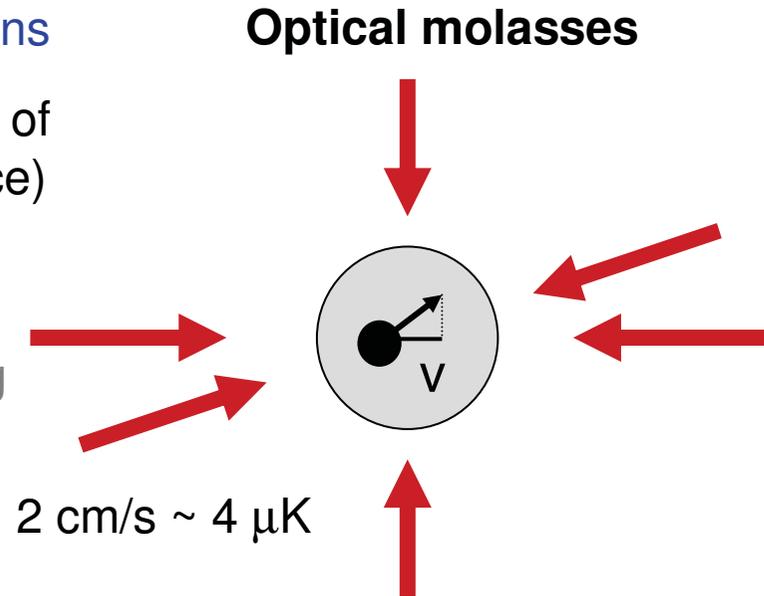
The beam can be stopped on a 1 m distance

Laser cooling and trapping of atoms

- Cooling : reduce the velocity in all directions

Atoms in a gas are enlightened by three pairs of laser beams (one in each direction of space) which frequency is slightly lower than ν_0

Due to Doppler effect, each atom absorbs more the beam which is counterpropagating with its velocity and then is slowed down



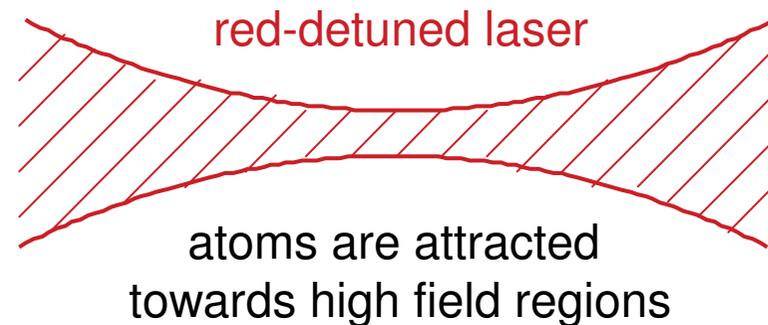
- Trapping : confine the atoms in a given region of space

Magneto-optical trap (MOT)



A spatially varying magnetic field is added to cooling polarized laser beams to induce a minimum Zeeman shift in the center of the trap

Dipole trap



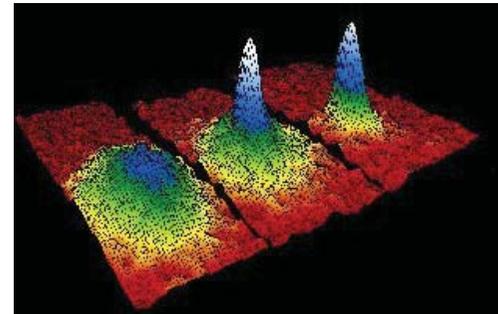
Lower and lower temperatures (300 nK) by evaporating cooling

... and high cold atom density by efficient trapping

Quantum degeneracy is reached when the De Broglie wavelength is of the same order of magnitude as the distance between atoms

$$n\lambda_{DB}^3 \approx 1$$

For bosonic atoms, one can then observe Bose-Einstein condensation



(Rb 1995)

velocity distribution

BEC has been presently obtained in various atoms :

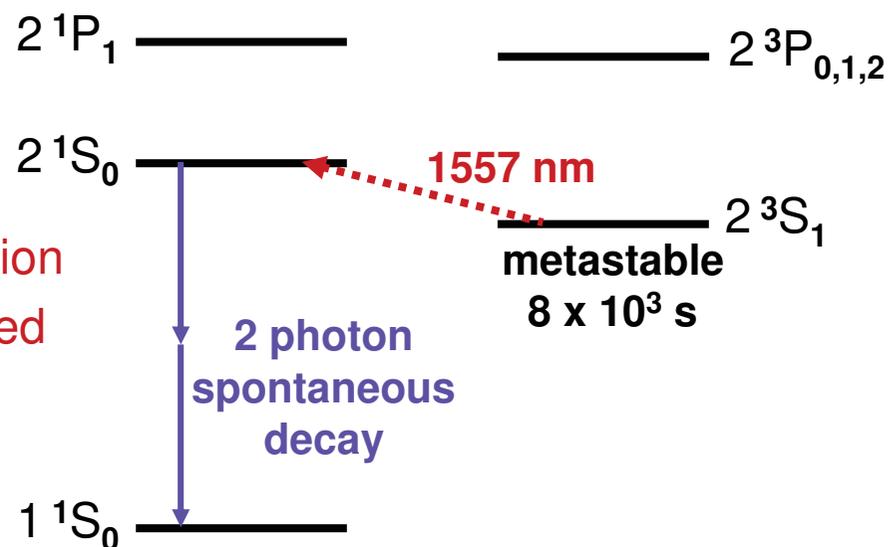
Na, K, Rb, Cs, H, He*, Sr, Yb, Cr ...

Frequency metrology in quantum degenerate helium

Helium in the 2^3S_1 state can be laser-cooled and trapped allowing much longer interaction time with a laser beam and then excitation of a weak transition

The doubly-forbidden $2^3S_1 - 2^1S_0$ transition has been recently observed and measured in both ^4He and ^3He atoms

Its natural width is 8 Hz given by the lifetime of the 2^1S_0 level



This gives a sensitive test of QED calculations and nuclear structure effects

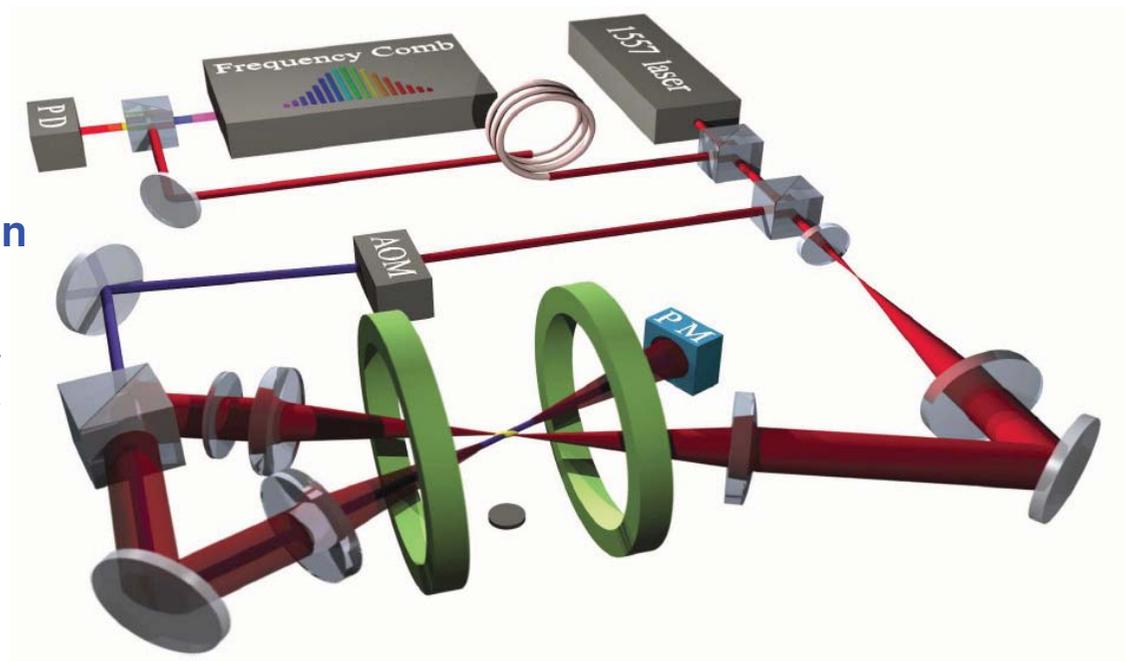
R. van Rooij *et al.*, *Science* 333, 196 (2011)

Frequency metrology in quantum degenerate helium

Experimental set-up

2 $^3\text{S}_1$ atoms are produced by electron impact in a discharge
They are slowed and trapped by laser techniques at 1083 nm

- $^4\text{He}^*$ are evaporatively cooled toward Bose Einstein condensation
- $^3\text{He}^*$ reach quantum degeneracy by sympathetic cooling with $^4\text{He}^*$



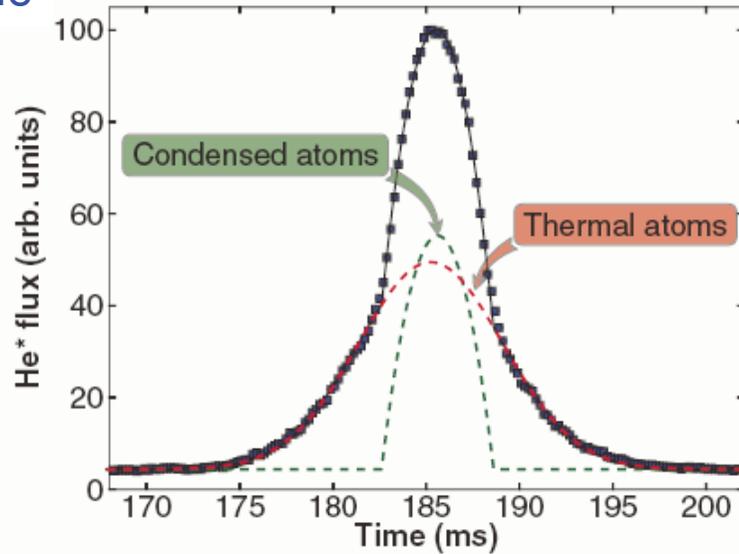
$\sim 10^6$ ultracold atoms are transferred into a crossed beam optical dipole trap where they are illuminated by a resonant light at 1557 nm

After free falling, remaining atoms in the 2 $^3\text{S}_1$ state are detected by a microchannel plate detector

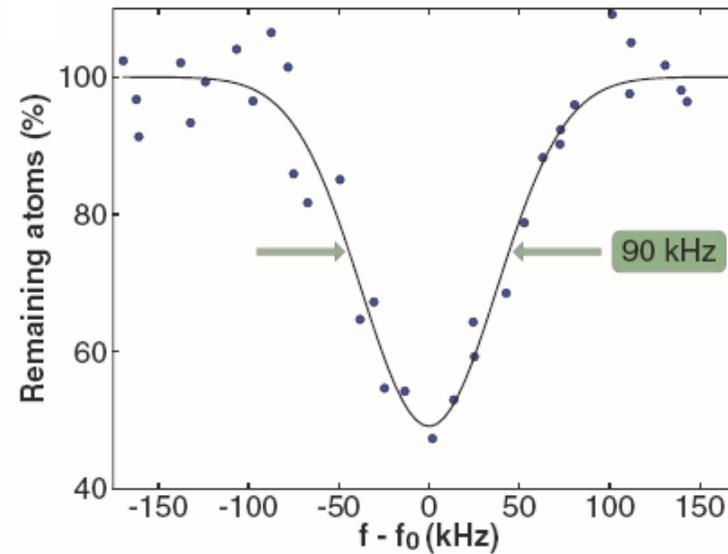
Frequency metrology in quantum degenerate helium

Signals

^4He



bimodal time-of-flight distribution



Gaussian fit of the line profile

Results

Frequencies measured with a frequency comb

- in ^4He : $f_4 = 192\,510\,702\,145.6$ (1.8) kHz
- in ^3He : $f_3 = 192\,504\,914\,426.4$ (1.5) kHz

accuracy 8×10^{-12}

This experiment also allows to deduce the isotope shift

The spectroscopy of helium atom : conclusion

During the last years, various accurate frequency measurements have been performed in helium :

- Fine structure intervals in the 2^3P state
- Optical transitions from the 2^3S metastable state
- UV transitions from the 1^1S ground state
- Inter-combination $2^3S - 2^1S$ line
- Hyperfine structure of ^3He and isotope shift (non discussed here)

and significant advances have been accomplished in theoretical calculations so that the level of precision is approaching the experimental one (except for the $2S$ Lamb shift)

The recent experiments provide :

- the fine and hyperfine structures
- the binding energies
- the Lamb shifts

and then allow to test electron - electron interaction and QED effects in this 3-body atomic system

and also to deduce the fine structure constant at a level of 2×10^{-8}

Experimental tests of QED in bound systems

Already discussed :

Spectroscopy of hydrogen and helium atoms ($Z = 1$ and 2)
including experimental methods :

- microwave measurements
- high resolution laser spectroscopy
- metrology of optical frequencies
- cold atoms techniques

Lamb shift measurements give stringent test of QED calculations
but are limited by the knowledge of nuclear charge radius

In the following of this lecture :

- We will see if it is possible to overcome this limitation
 - by the study of pure leptonic atomic systems
 - or by the determination of the proton radius
- And we will consider highly charged ions to test QED corrections in high Z systems

The positronium $e^+ - e^-$

It is the lightest exotic hydrogen-like atom and a purely leptonic system and allows to test relativistic two-body and QED corrections

Electron and positron have same mass, same spin, but opposite charges

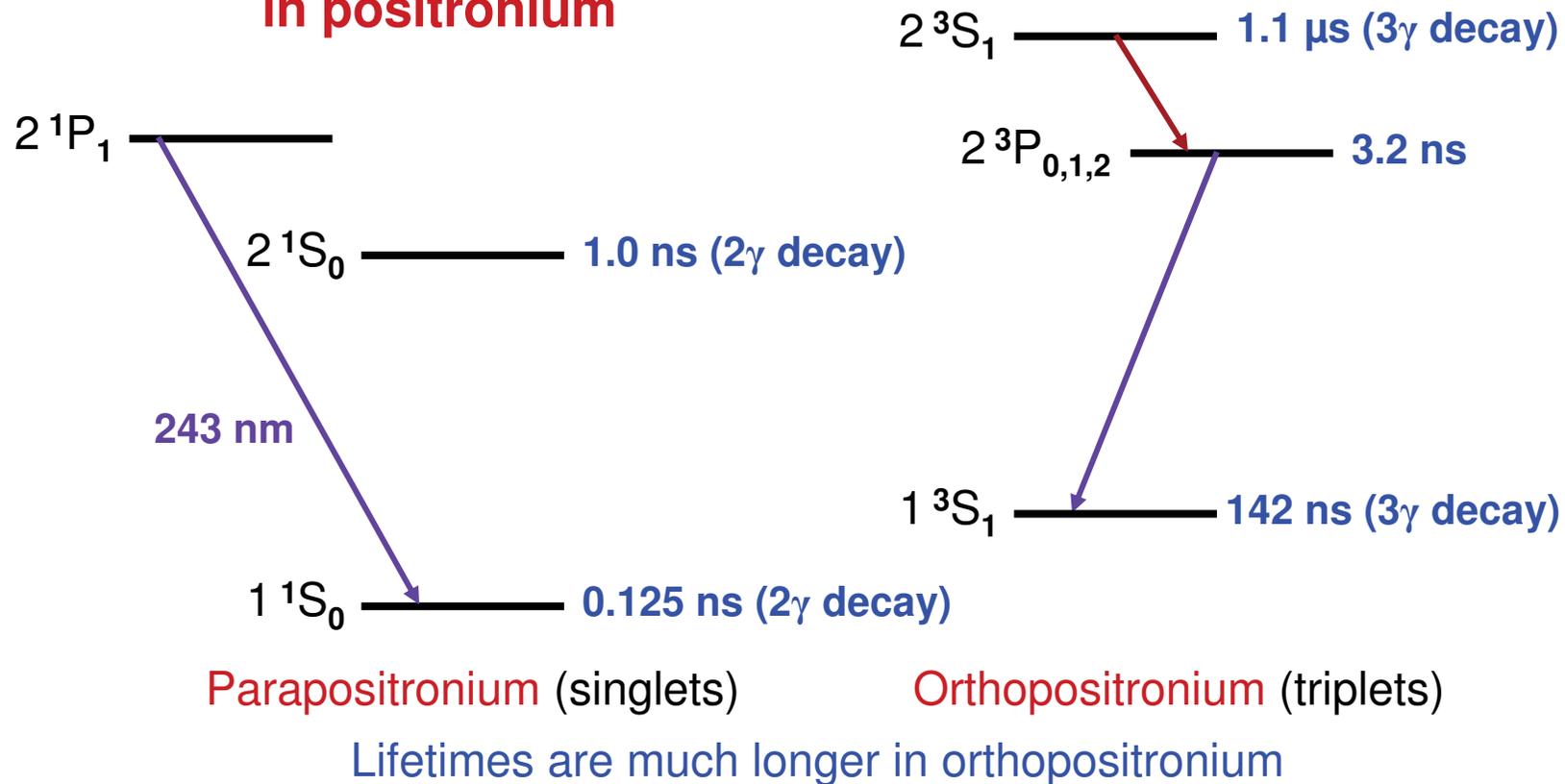
Energetic positrons are produced by radioactive sources

Positronium can be formed :

- by stopping energetic positrons in a gas or in a powder
- or by charge exchange of slow positrons in a thin foil or a gas target
- or by interaction of slow positrons with the surface of a solid in vacuum

The $e^+ - e^-$ annihilation is responsible for specific terms in energy levels and limits the lifetimes

Energy levels and lifetimes in positronium



Because of present precision in theory and experiments, the experimental data which can be used to test QED predictions in positronium are :

- the annihilation rates in triplet and singlet ground states
- the $n = 1$ hyperfine interval
- the $n = 2$ fine structure interval
- the 1S-2S triplet interval

Lifetimes of ortho- and para- positronium in their ground states

They result from $e^+ - e^-$ annihilation

Because of parity conservation, the decay process is different :

2γ decay for the singlet (0.125 ns)

3γ decay for the triplet (142 ns)

A series of measurements have been performed (U. of Michigan) determining the decay rate of a positronium beam either in a gas or in vacuum

D.W. Gidley, A. Rich, E. Sweetman and D. West, *Phys. Rev. Lett.* 49, 525 (1982)

C.I. Westbrook, D.W. Giley, R.S. Conti and A. Rich, *Phys. Rev. Lett.* 58, 1328 (1987)

J.S. Nico, D.W. Gildey and A. Rich, *Phys. Rev. Lett.* 65, 1344 (1990)

Extensive efforts have been carried out to more and more improve the precision and check all the systematic effects

In 1990, the results were :

$\lambda_S = 7.994 (0.011) \text{ ns}^{-1}$ in agreement with theoretical prediction

$\lambda_T = 7.0516 (0.0013) \mu\text{s}^{-1}$ (in gas)

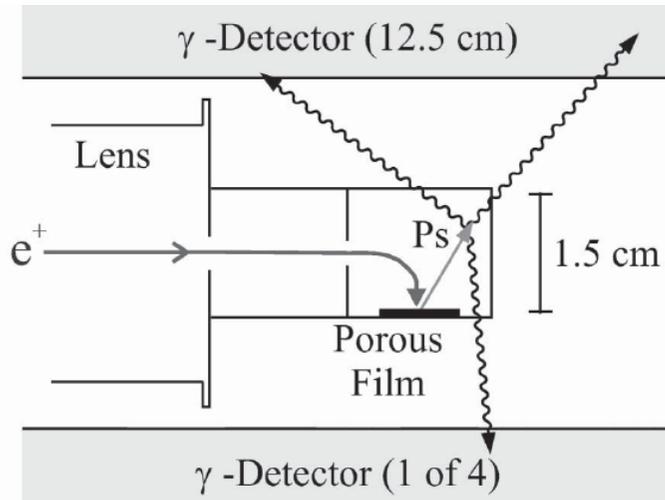
$\lambda_T = 7.0482 (0.0016) \mu\text{s}^{-1}$ (in vacuum)

Long standing discrepancy by more than 6σ with theoretical prediction on λ_T ...

... now resolved !

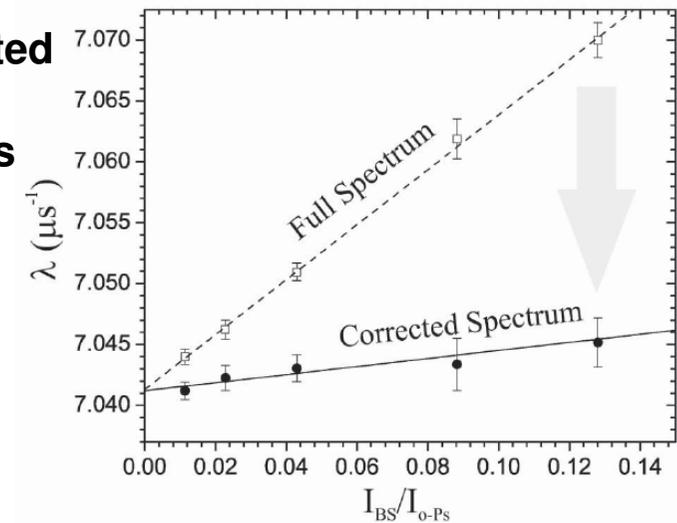
R.S. Vallery, P.W. Zitewitz and D.W. Gidley, *Phys. Rev. Lett.* 90, 203402 (2003)

(Michigan)



Ps is formed in a porous silica film in near thermal equilibrium and confined in a small cavity having negligible wall quenching

The spectrum is corrected from backscattering due to high energy Ps



The result is : $\lambda_T = 7.0404 (10)(8) \mu\text{s}^{-1}$

in agreement with theory : $\lambda_T = 7.039 979 (11) \mu\text{s}^{-1}$

G.S. Adkins, R.N. Fell and J. Sapirstein,
Ann. Phys. (N.Y.) 295, 136 (2002)

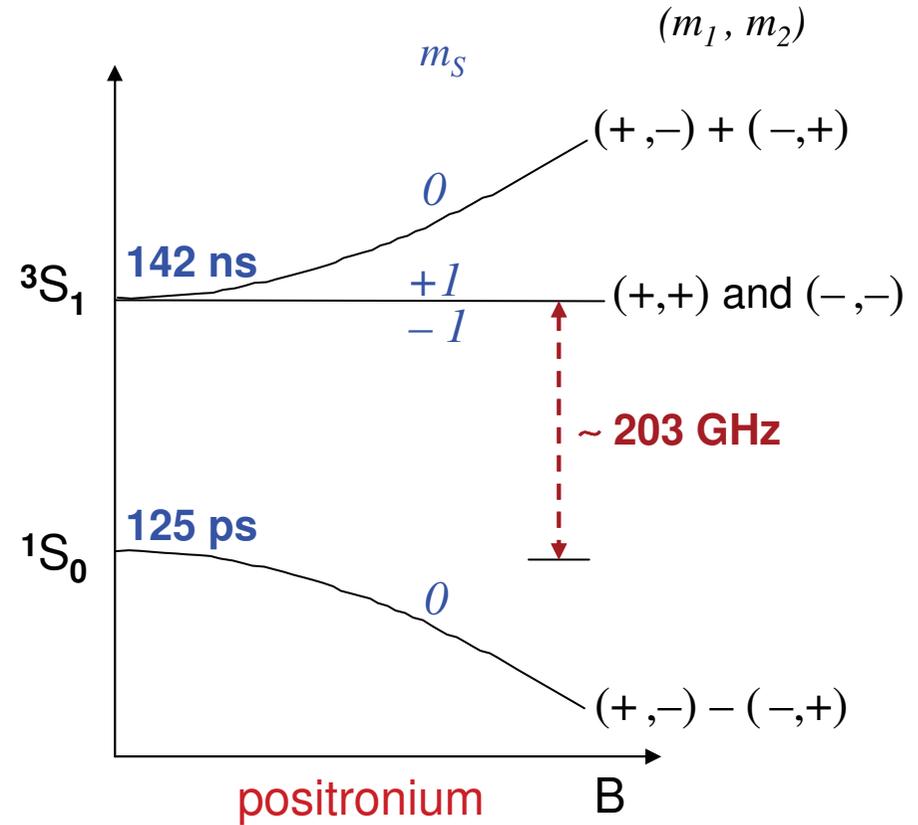
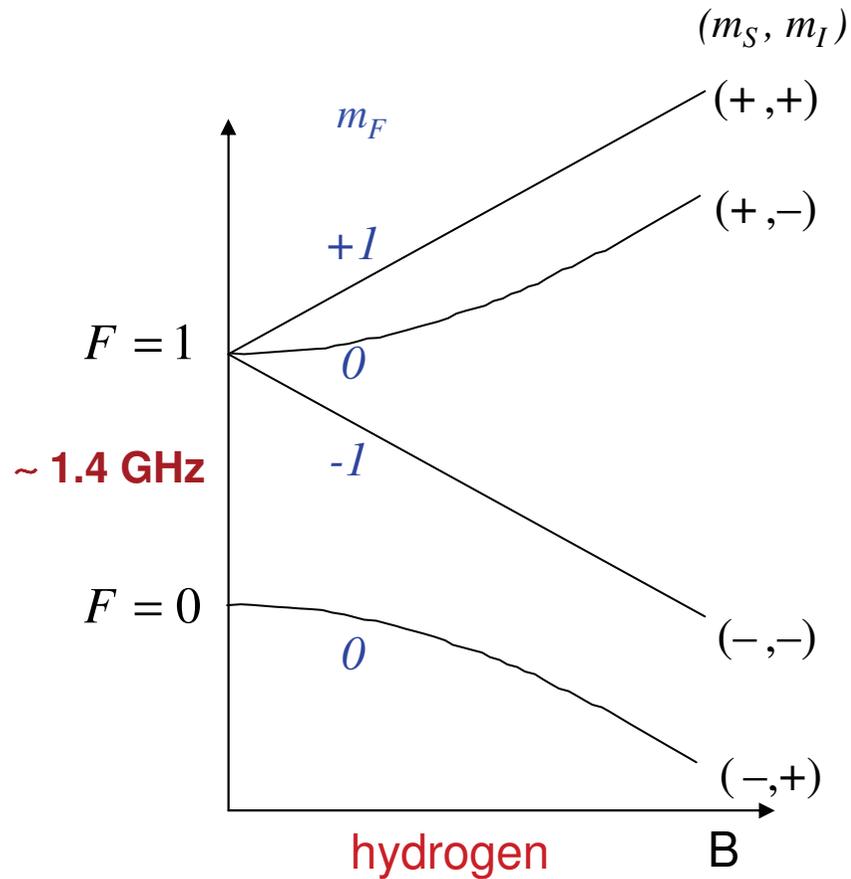
Another recent measurement performed in Tokyo confirms this excellent agreement

Y. Kataoka, S. Asai and T. Kobayashi, *Physics Letters B* 671, 219 (2009)

The combined experimental result is now : $\lambda_T = 7.0401 (7) \mu\text{s}^{-1}$

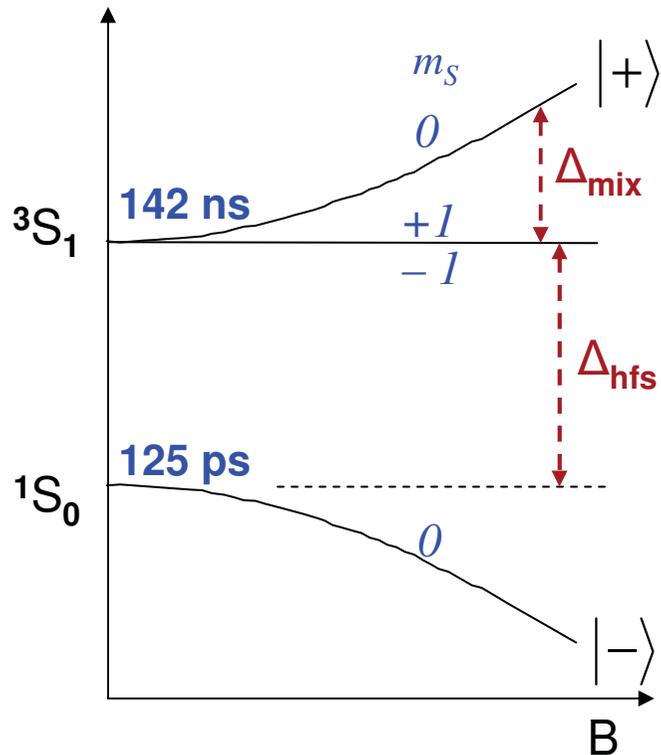
giving a good test of validity for the second order $O(\alpha^2)$ corrections

The hyperfine structure of the ground state of positronium



e^+ and e^- have opposite gyromagnetic ratios so that $(+, +)$ and $(-, -)$ states have null magnetic momentum

Measurement of the hyperfine structure of positronium through the Zeeman splitting



Δ_{mix} is related to Δ_{hfs} by a simple formula

Two approaches :

- Microwave excitation at frequency Δ_{mix} (~ 3 GHz in 0.8 T) followed by an increase of 2γ decay
M.R. Ritter *et al.*, *Phys. Rev. A* **30**, 1331 (1984)

Result : $\Delta_{\text{hfs}} = 203.389\ 10\ (74)$ GHz (Yale)
(3.9 σ from theory)

- Quantum oscillation

Y. Sasaki and al., (Tokyo)

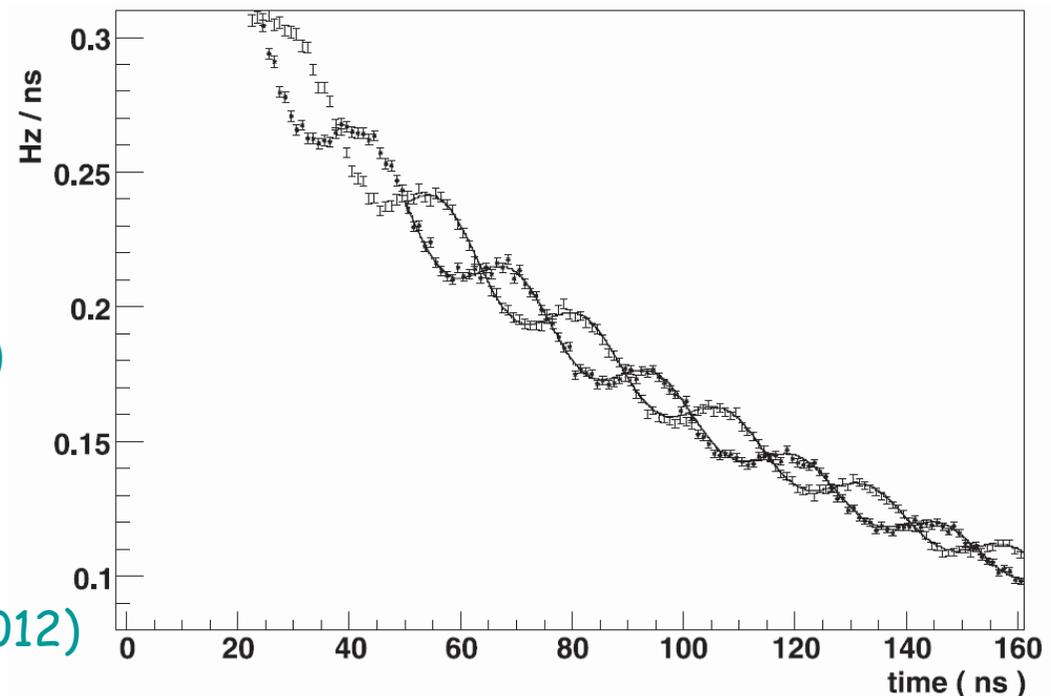
Physics Letters B **697**, 121 (2011)

Result : $\Delta_{\text{hfs}} = 203.324\ (39)$ GHz

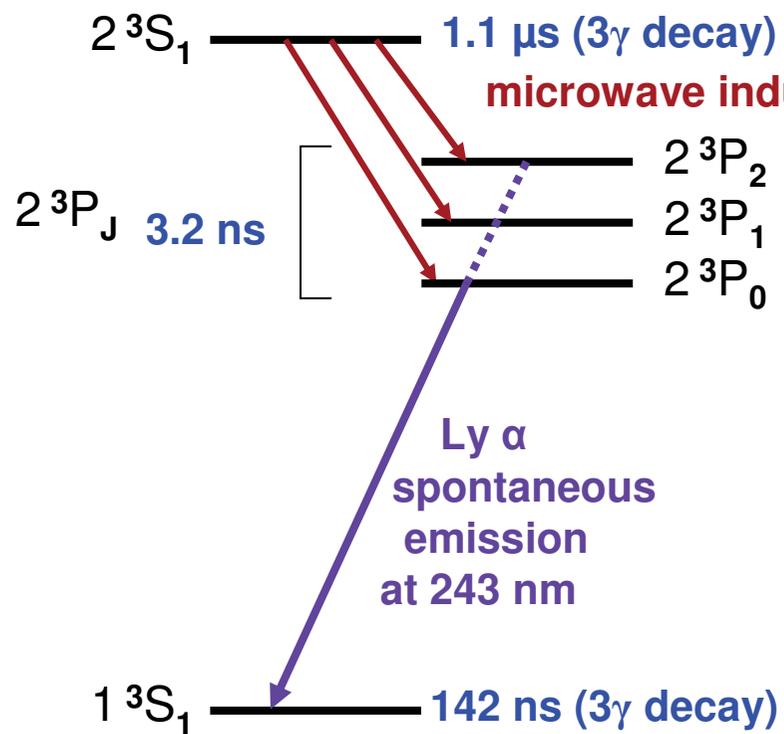
less precise but could be improved

- and also direct THz excitation

T. Yamazaki *et al.*, arXiv (5 Apr. 2012)



Measurements of the $n = 2$ fine structure intervals in positronium



microwave induced transitions

2^3S_1 and 2^3P_J states are all populated when positronium atoms are formed

The microwave transitions are detected through the increase of Ly α fluorescence

stat. syst.

The measured frequencies are :

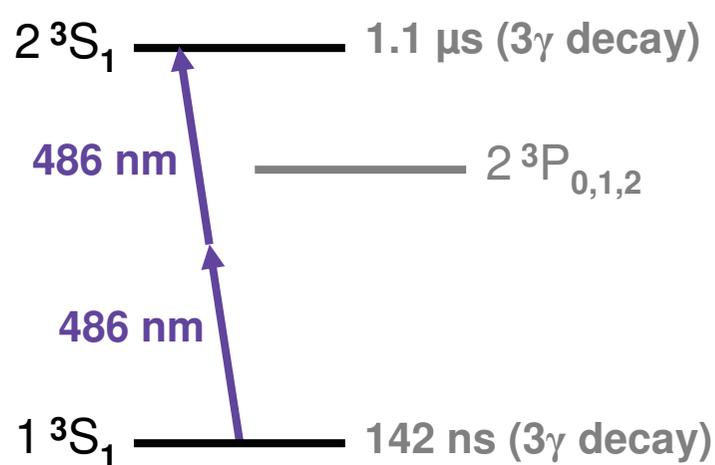
(Mainz)

- $\nu_0 = 18\,499.65 (1.20)(4.00) \text{ MHz}$
- $\nu_1 = 13\,012.42 (0.67)(1.54) \text{ MHz}$
- $\nu_2 = 8\,624.38 (0.54)(1.40) \text{ MHz}$

D. Hagen, R. Ley, D. Weil and G. Werth, Phys. Rev. Lett. 71, 2887 (1993)

Agreement with recent calculated QED corrections in $R_\infty \alpha^4 \ln \alpha^{-1}$

Frequency measurement of the $1\ ^3S_1 - 2\ ^3S_1$ transition of positronium



The most sensitive to QED effects since they roughly scale as $1/n^3$

Doppler-free two-photon spectroscopy

$n = 2$ population is detected through positrons counting after laser ionization

This transition was measured for the first time with a pulsed dye laser :

[S. Chu, A.P. Mills JR and J.L. Hall, Phys. Rev. Lett. 52, 1689 \(1984\)](#) (Stanford)

and later with cw excitation :

[M.S. Fee, S. Chu et al., Phys. Rev. A 48, 192 \(1993\)](#)

Result : **1 233 607 216.4 (3.2) MHz** accuracy 2.6 ppb

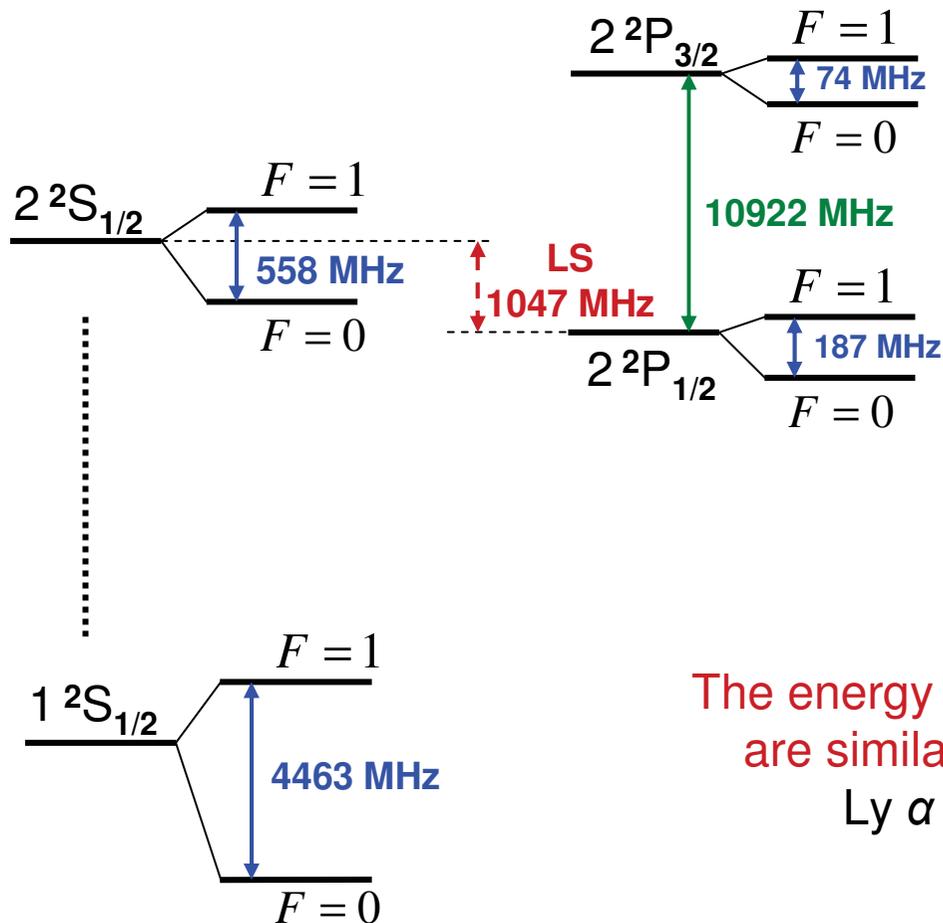
sufficient accuracy to test the $R_\infty \alpha^4$ QED corrections

Presently another group plans to perform a new experiment in order to improve this accuracy by a factor 5 and then check recent QED calculations

[P. Crivelli, C.L. Cesar and U. Gendotti, Can. J. Phys. 89, 29 \(2011\)](#) (Rio)

The muonium $\mu^+ - e^-$

Like electrons and positrons, muons are leptons,
but their mass is ~ 207 times larger



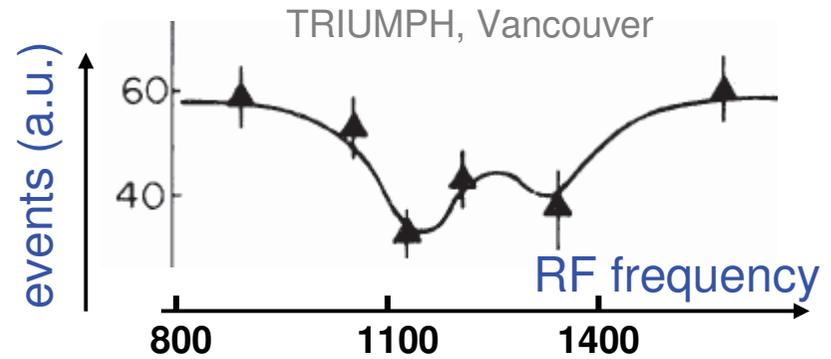
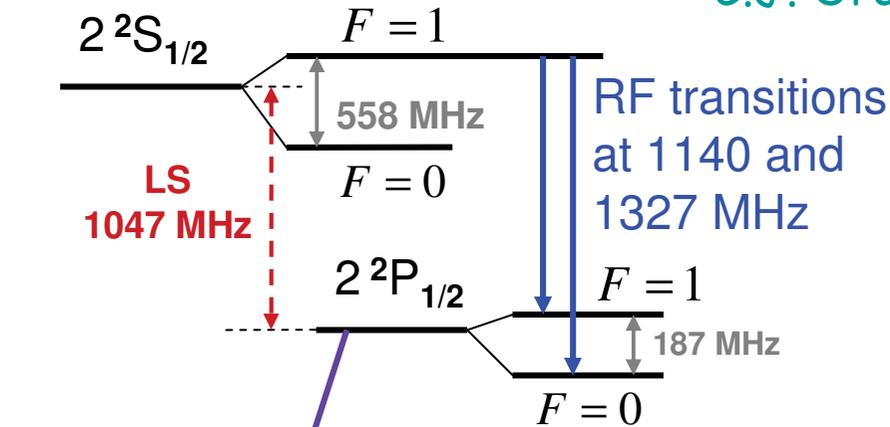
Production of muonium :

- high energy proton beam incident on a target (ex. Be)
- π^+ are created which decay in μ^+
- low energy muon beam incident on a gas target (ex. Ar) produces muonium atoms by electron capture

The energy levels of the muonium atom are similar to the ones of hydrogen
Ly α transition at 122 nm

RF measurement of the 2S Lamb shift in muonium

C.J. Oram et al., Phys. Rev. Lett. 52, 910 (1984)



Ly α
decay
at 122 nm

Results:

$$L_{2S-2P} = 1070^{+12}_{-15} \text{ (2) MHz}$$

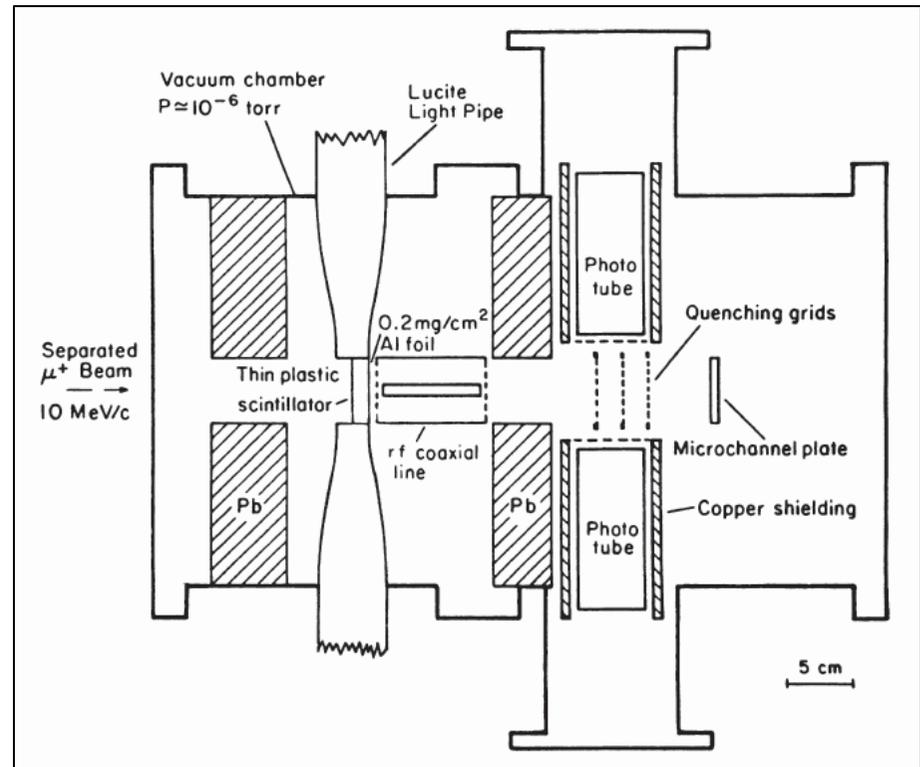
fit syst.

$$L_{2S-2P} = 1042^{+21}_{-23} \text{ MHz}$$

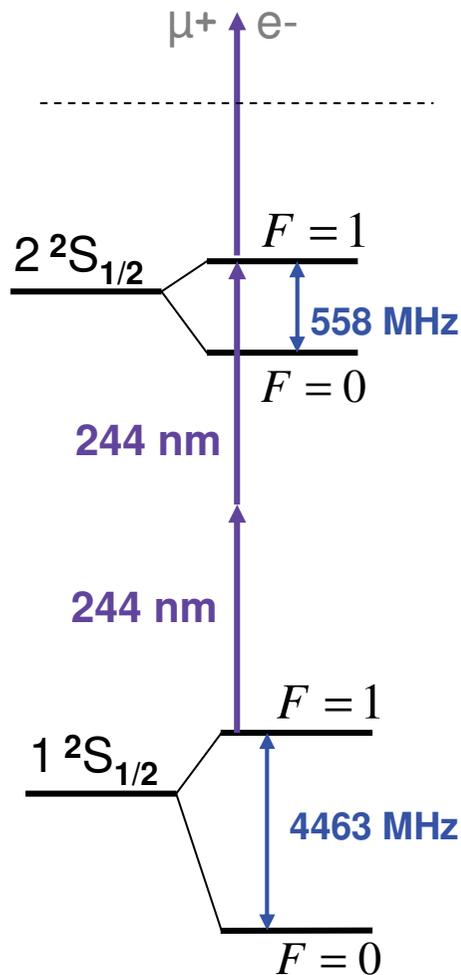
limited by the low density
of the muon beam

K.A. Woodle et al.,
Phys. Rev. A 41, 93 (1990)

LAMPF



Frequency measurement of the 1S-2S transition of muonium



First observation of the 1S-2S Doppler-free two-photon transition with a frequency doubled dye laser

S. Chu et al., Phys. Rev. Lett. 60, 101 (1988) (Stanford)

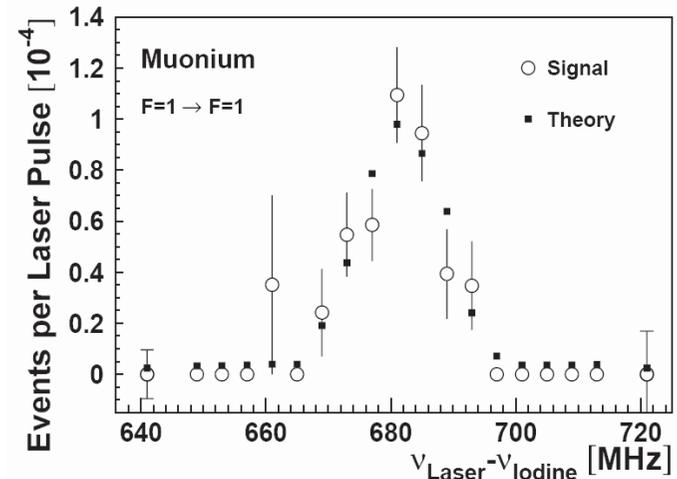
The 2S state is detected by photoionization

Frequency measurements :

*F.E. Maas et al.,
Physics Lett. A 187, 247 (1994)*

(Heidelberg, ...)

*V. Meyer et al.,
Phys. Rev. Lett. 84, 1136 (2000)*



Result : $\Delta\nu(1S-2S) = 2\,455\,528\,941.0 (9.8) \text{ MHz}$

in good agreement with theory

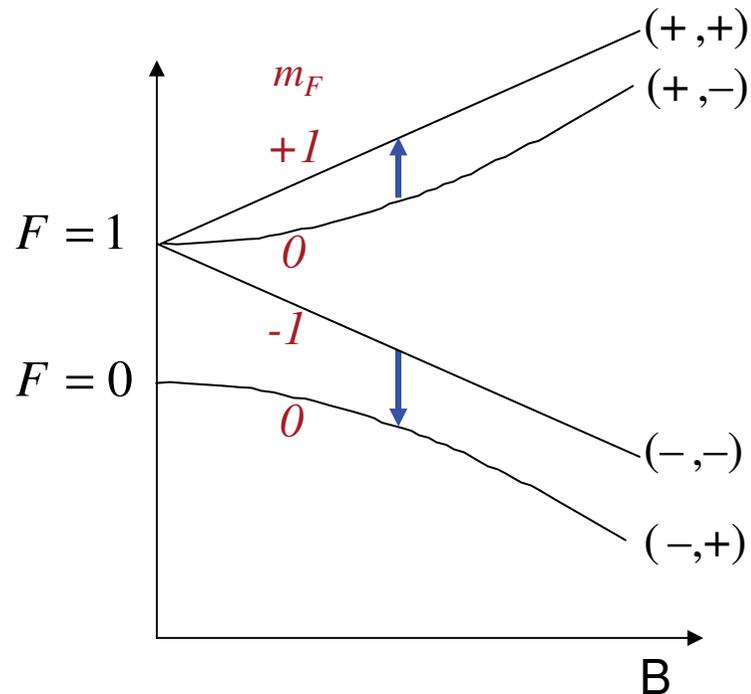
The Lamb shift contribution can be deduced :

$$\Delta\nu_{\text{LS}} = 7049.4 (9.9) \text{ MHz}$$

The exp./ theory comparison gives the mass ratio : $m_{\mu^+} / m_{e^-} = 206.768\,277\,5 (24)$
in agreement but less precise than the ratio deduced from the muon magnetic moment

The 1S hyperfine structure of muonium

Experiments have been performed either in low or in high magnetic field



Polarized muons are captured in a microwave cavity containing Kr gas inside a superconducting magnet

Muonium is formed in states $(+, -)$ and $(-, -)$

Microwave magnetic resonance induces muon spin-flip transitions

The transition signal is detected through the angular distribution of positrons produced by the μ^+ decay

Results : $\Delta\nu_{\text{hfs}} = 4\,463\,302\,765\ (53)\ \text{Hz}$ 12 ppb

$\mu_{\mu^+} / \mu_{e^-} = 3.183\,345\,13\ (39)$ 120 ppb

Most precise results obtained in muonium

W. Liu *et al.*, *Phys. Rev. Lett.* **82**, 711 (1999)

(Yale,...)
at LAMPF

Muonium / positronium : discussion

Natural widths in muonium are limited by muon lifetime (2.2 μs) since they are limited by $e^+ - e^-$ annihilation in positronium and are then shorter

Muonium is more sensible to QED corrections since its radius is smaller : vacuum polarization terms are more important

Compared to positronium, QED calculations in muonium are less complicated and generally more accurate

A good agreement is found for both atoms between experimental results and theoretical predictions but tests in muonium are more stringent

In particular, the study of hyperfine structure of muonium allows to deduce values either of m_{μ^+} / m_{e^-} or of the fine structure constant α , using other experimental data and/or theoretical predictions

$$m_{\mu^+} / m_{e^-} = 206.768\,267\,0\,(55) \quad 27 \text{ ppb}$$

$$1/\alpha = 137.035\,996\,3\,(80) \quad 58 \text{ ppb}$$

see discussion in : [W. Liu *et al.*, Phys. Rev. Lett. 82, 711 \(1999\)](#)

Outline of this lecture

previously : - hydrogen atom (2-body atomic system)

in this lecture :

- helium atom (3-body atomic system)
- positronium (2-body purely leptonic atomic system)
- muonium (2-body purely leptonic atomic system)

→

- muonic helium atom (3-body atomic system)
- antiprotonic helium atom (3-body atomic system)

- muonic hydrogen (2-body atomic system)
- He + (2-body atomic system)
- muonic He + (2-body atomic system)

- H-like highly charged ions (2-body atomic systems)
- He-like highly charged ions (3-body atomic systems)
- Li-like highly charged ions (4-body atomic systems)

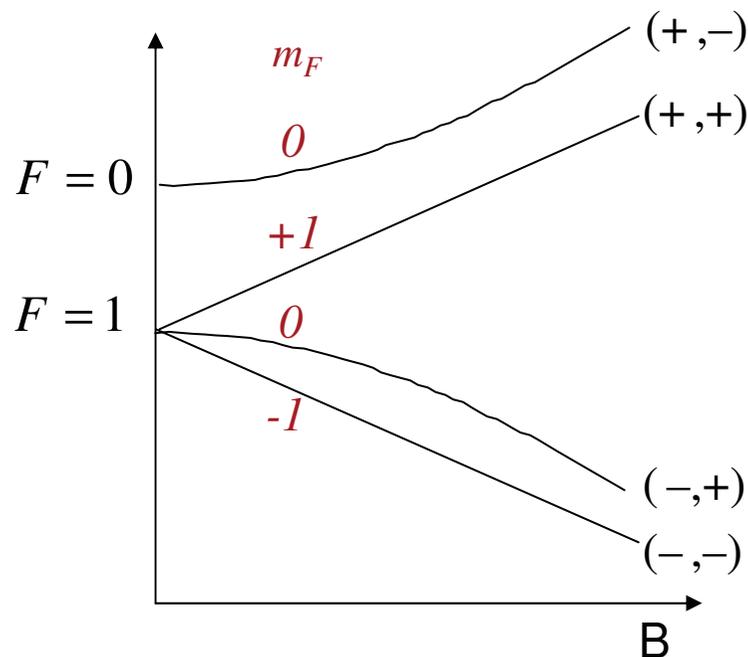
The muonic helium atom



One of the two electrons of an usual He atom is replaced by a negative muon

Because of their different masses, the radius of the e^- orbit is large compared to the one of the μ^-

This atom is similar to hydrogen with a pseudonucleus $({}^4\text{He}^{++} \mu^-)^+$



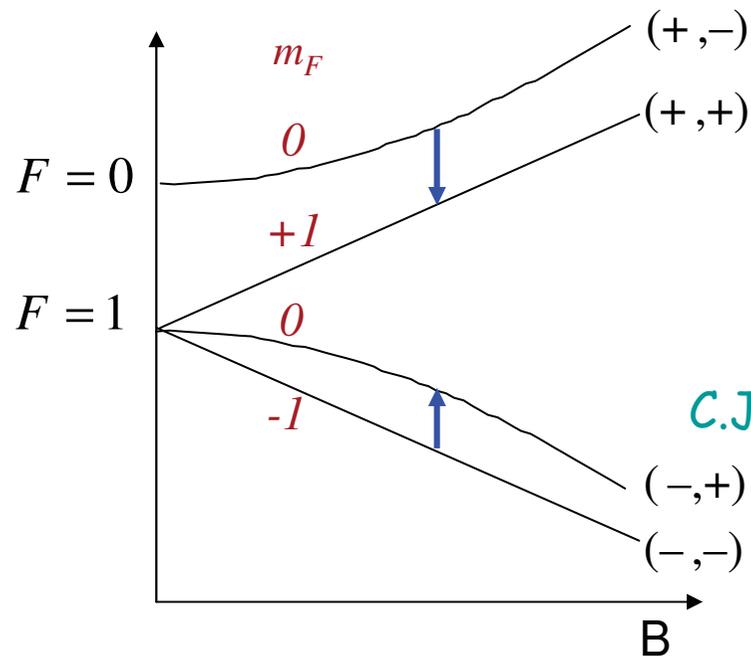
The ground state hyperfine structure is approximately equal to the one of muonium but inverted because μ^- and μ^+ magnetic moments have opposite signs

The finite size of the alpha particle has almost negligible influence on hyperfine structure

K. Pachucki *et al.*, *Phys. Rev. A* 63, 032508 (2001)

Spectroscopy of muonic helium atom

Muonic helium atoms are formed by stopping polarized μ^- in He gas mixed with some Xe : $(^4\text{He}^{++} \mu^-)^+$ is formed which radiates to its ground state and then captures an electron from a Xe atom



Precise measurements have been made both at weak and strong magnetic fields in a similar way as in muonium experiments

The muon spin-flip is detected through the angular distribution of electrons produced by the μ^- decay

C.J. Gardner et al., Phys. Rev. Lett. 48, 1168 (1982)
(Yale,...) at LAMPF

Results : $\Delta v_{\text{hfs}} = 4\,465.004 (29) \text{ MHz}$ 6.5 ppm

Recent theoretical prediction : $\Delta v_{\text{hfs}} = 4\,465.526 \text{ MHz}$
the difference lies in the range of theoretical error

A.A. Krutov and A.P. Martinenko, Phys. Rev. A 78, 032513 (2008)

and : $\mu_{\mu^-} / \mu_p = 3.183\,28 (15)$ 47 ppm

Spectroscopy of antiprotonic helium

$\bar{p}\text{He}^+$: helium atom where an electron is replaced by an antiproton

work carried out at CERN

The electron is in its ground state and the antiproton in a circular Rydberg state

$$n \approx \ell + 1 \approx 38$$

Two-photon transitions of the type $(n, \ell) \rightarrow (n-2, \ell-2)$ by two counter-propagating UV laser beams at 139.8 nm, 193.0 nm, and 197.0 nm

The comparison of the three measured frequencies with QED calculations allows to derive the antiproton-to-electron mass ratio 1836.1526736(23) in agreement with the proton-to-electron value

M. Hori et al., Nature [475](#), 484 (2011)

The study of hyperfine structure in antiprotonic helium leads also to the determination of the antiproton magnetic moment in agreement with that of the proton within 2.9×10^{-3}

T. Pask et al., Physics Letters B [678](#), 55 (2009)

Muonic hydrogen and the proton charge radius puzzle

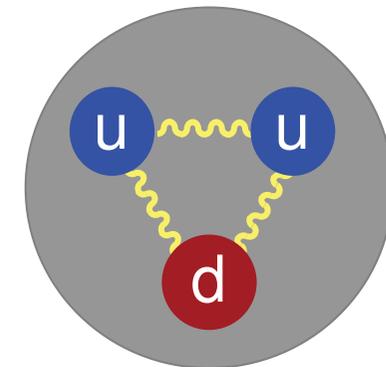
Exotic particles can be used :

- to eliminate proton size effects encountered in hydrogen atom
- but also to attempt to determine the proton size

We have seen that the measured Lamb shifts in hydrogen need the knowledge of the proton radius to be used to test QED predictions

The proton has a structure :

2 quarks **up** ($2/3 e$) + 1 quark **down** ($-1/3 e$)
+ strong interaction (**gluons**)



The proton charge radius is defined as :

$$r_p = \sqrt{\langle r^2 \rangle} \quad \text{where} \quad \langle r^2 \rangle = \frac{\int r^2 \rho(r) d^3 r}{\int \rho(r) d^3 r} \quad \text{and} \quad \rho(r) \text{ is the proton charge distribution}$$

$$r_p \sim 0.8 \text{ fm}$$

Determinations of the proton charge radius

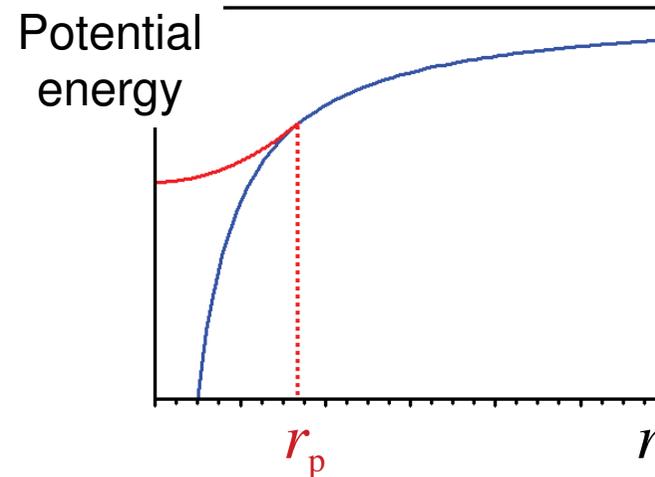
- From spectroscopic measurements

ex. : in hydrogen atom (e-p)

$$E(n,l,j) = \underbrace{\text{Dirac} + \text{recoil}}_{\text{exact}} + L(n,l,j)$$

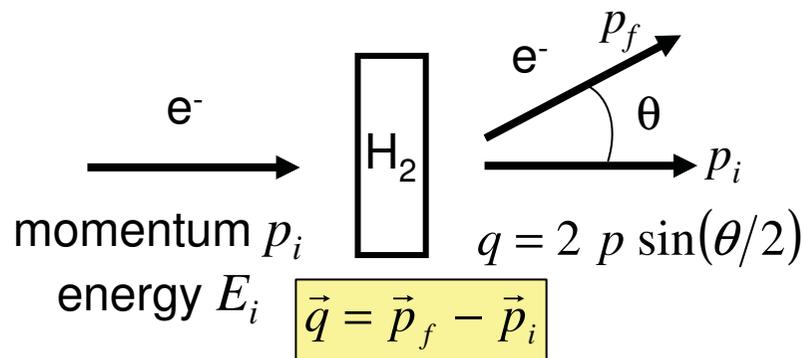
The Lamb shift $L(n,l,j)$ includes :

- QED radiative corrections ($1/n^3$)
- relativistic recoil
- **nuclear size effect ($1/n^3$)**



$$f(1S-2S) = 2\,466\,061\,413\,187\,035 \text{ (10) Hz}$$

- From electron - proton scattering



$$\frac{d\sigma}{d\Omega}(E_i, \theta) \approx \underbrace{\left[\frac{a^2}{\sin^4(\theta/2)} \right]}_{\text{Rutherford}} G(q^2)$$

$$G(q^2) = \int d^3r e^{iqr} \frac{\rho(r)}{4\pi} \approx 1 - \frac{r_p^2}{6} q^2$$

r_p is deduced from an extrapolation to $q \rightarrow 0$

J.C. Bernauer, *Can. J. Phys.* **85**, 419 (2007)

The muonic hydrogen Lamb shift

It is much more sensitive to the proton radius than the hydrogen Lamb shift since the Bohr radius is only $a_0 / 207$

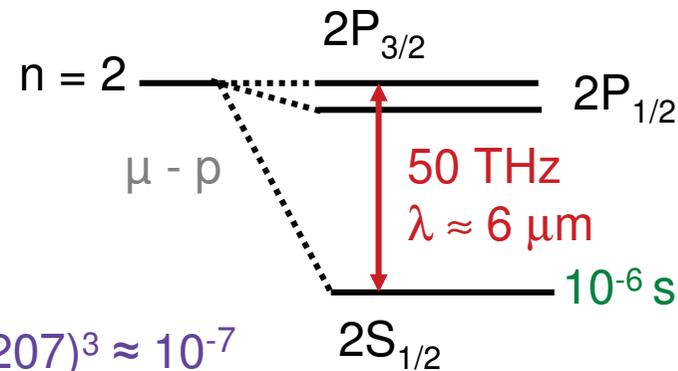
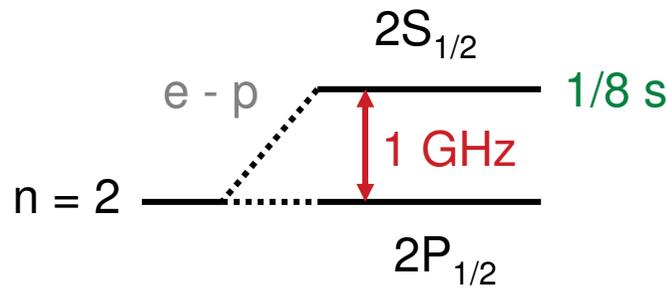
The various contributions to the 2S-2P Lamb shift :

2S-2P	self-energy	vacuum pol.	r_p	total
e - p	1085.8 MHz	-26.9 MHz	0.146 MHz	1057.8 MHz
μ - p	0.1 THz	-49.94 THz	0.93 THz	-49.05 THz

relative contribution of r_p

1.4×10^{-4}

2×10^{-2}



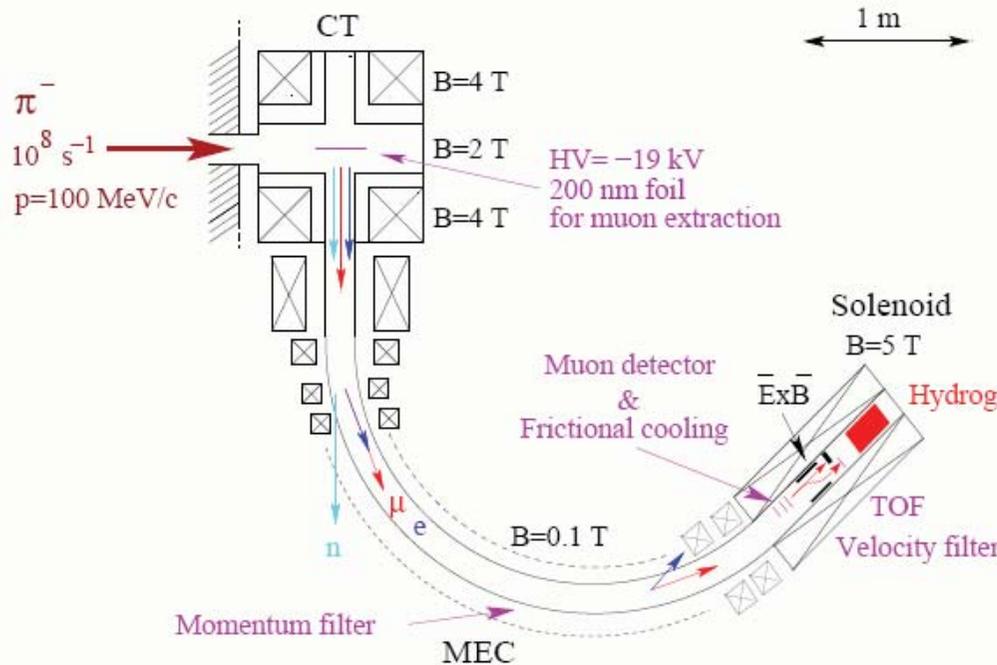
oscillator strengths ratio : $(f_{\mu-p}/f_{e-p}) \propto 1/(207)^3 \approx 10^{-7}$

$$\Delta E_{2S(F=1)-2P(F=2)} = 50.7702(12) - 1.2634 r_p^2 - 0.0084 r_p^3 = 49.8063 (149) \text{ THz}$$

with $r_p = 0.8760 (68) \text{ fm}$

Production of muonic hydrogen

CREMA
collaboration



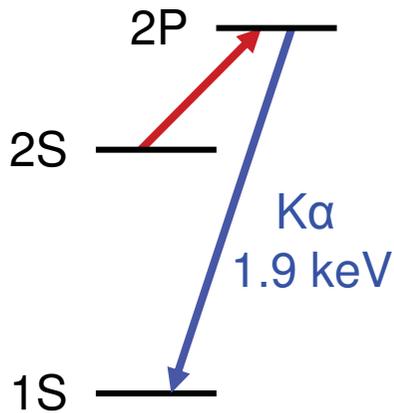
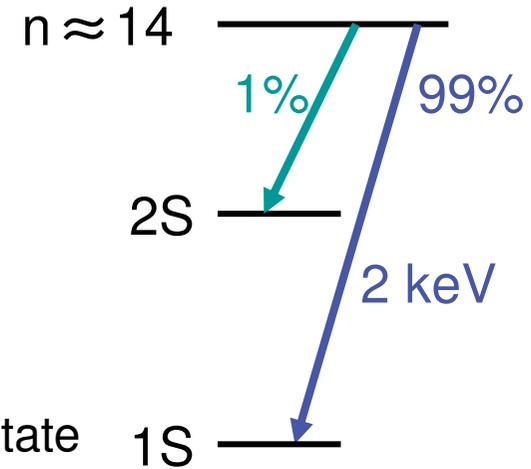
- π E5 line at PSI
- low energy muons are produced in a cyclotron trap
- they are transported in a curved solenoid (muon extracted channel)...

- detected by two fast detectors (stacks of ultra-thin carbon foils)
rate of muons : 300 /s
signal used to trigger the laser pulse
- and stopped in the H₂ target (1 mbar)



Production of muonic hydrogen in the 2S metastable state ...

- the muon is captured in a highly excited state which decays at 99 % to the ground state emitting a « prompt » X ray ($K\alpha$, $K\beta$,...)
- X rays are detected with LAAPDs (large area avalanche photodiodes) placed above and below the muon stop volume
- 1% of the stopped muons decay to the long-lived 2S state

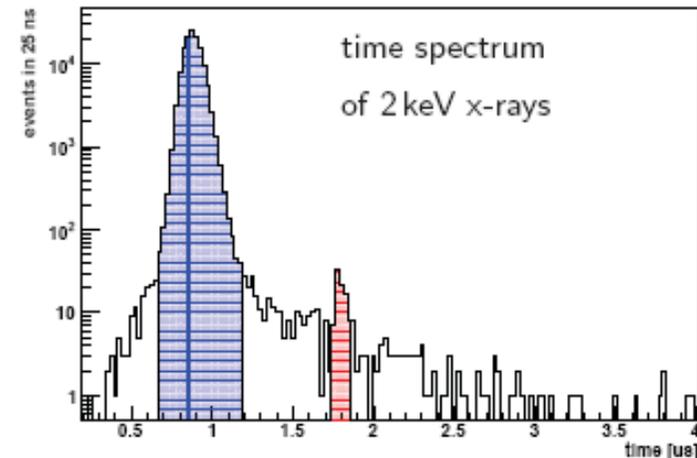


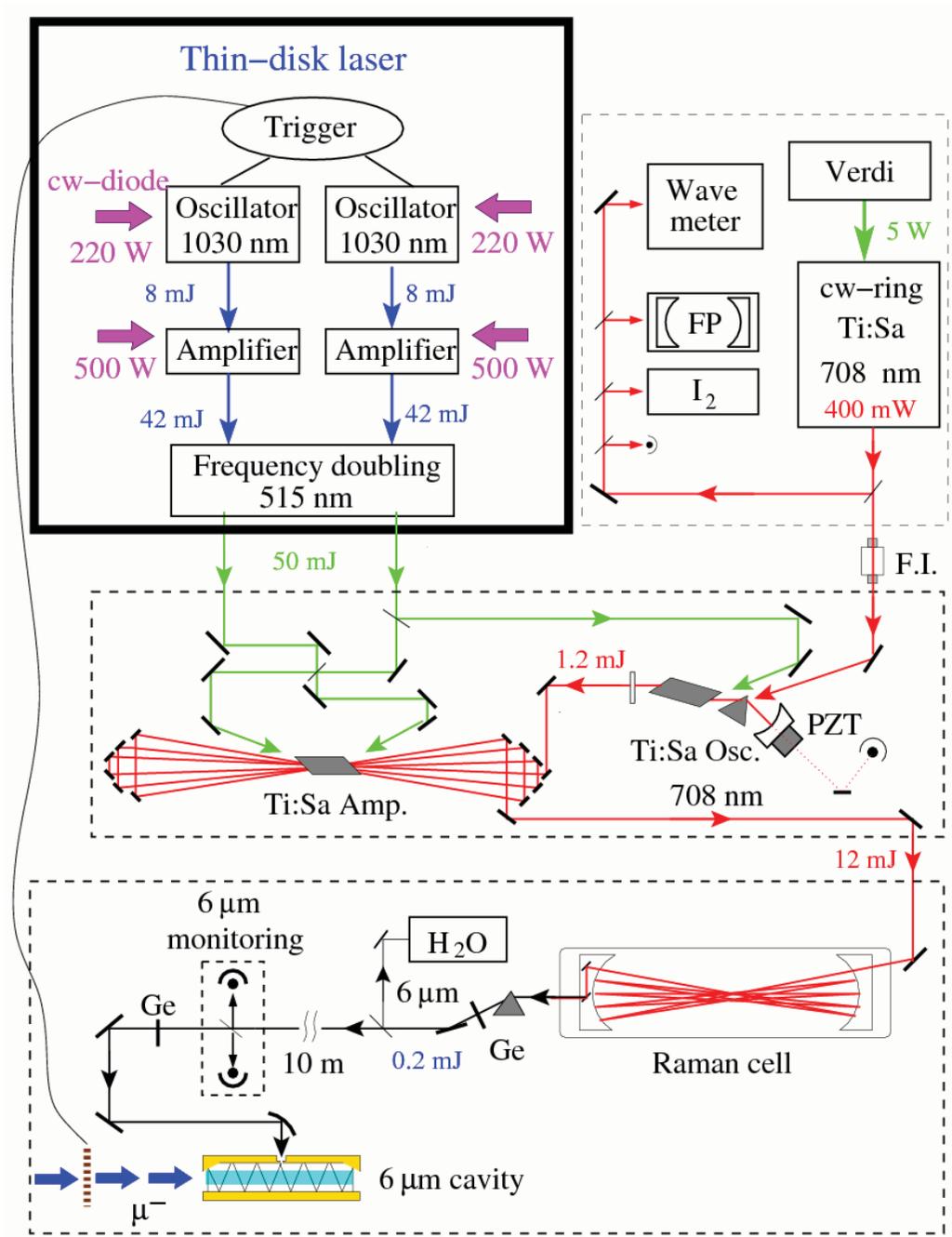
... and excitation of the transition

A short laser pulse at 6 μm drives the 2S-2P transition

- the transition is detected through the 1.9 keV $K\alpha$ decay of the 2P state (« delayed » X ray)

The signature of the signal is the detection of $K\alpha$, in time coincidence with the laser excitation, and of the electron originating from the muon decay (muon lifetime is 2.2 μs)

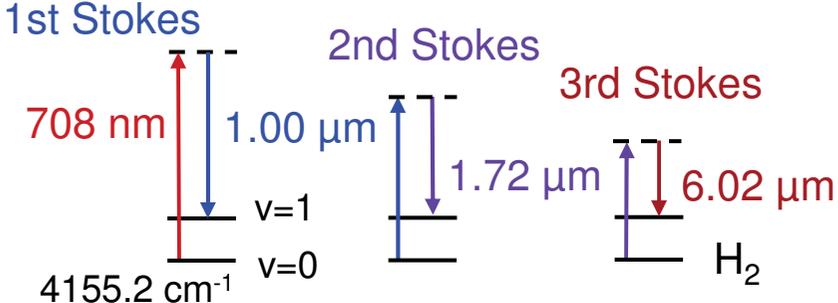




The laser chain

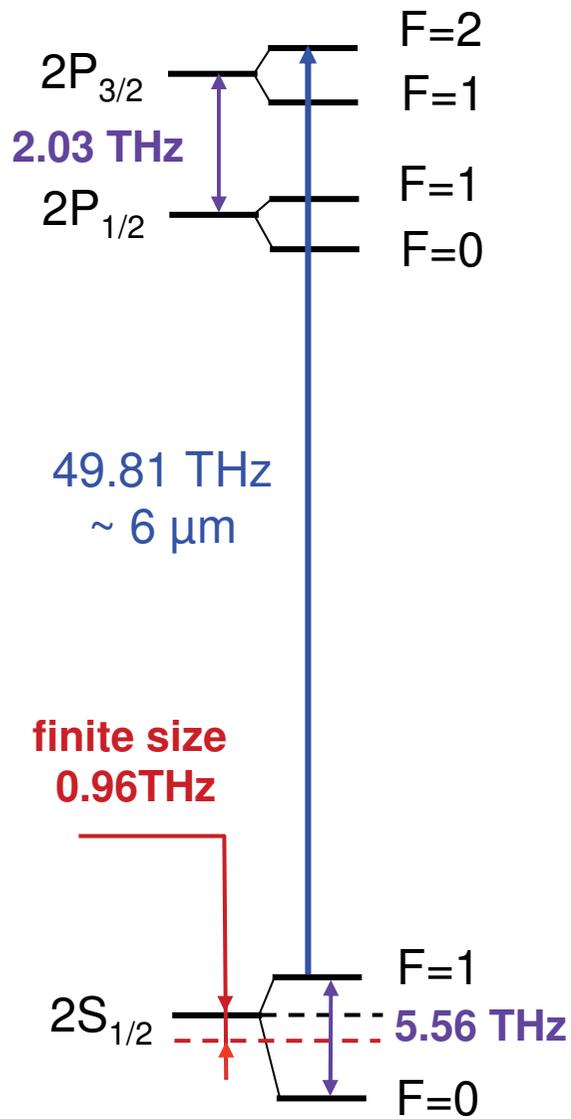
to produce a 6 μm tunable laser pulse

- thin disk laser (1030 nm)
 - μ⁻ triggered
 - + LBO (515 nm)
- pulsed TiSa oscillator + amplifier (cw TiSa seeded at 708 nm)
- Raman cell for frequency conversion

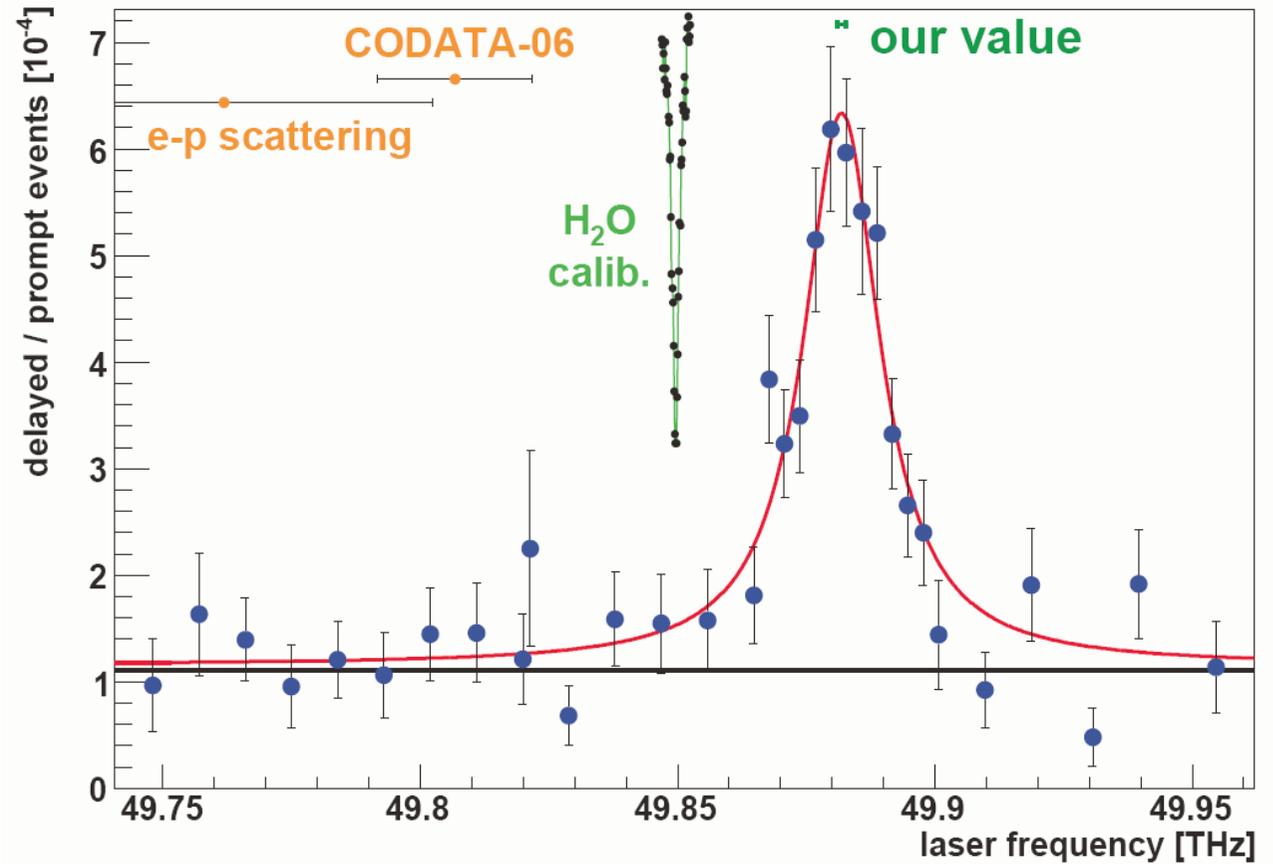


- Multipass cavity at 6 μm surrounding the H₂ target

2S_{1/2}(F=1) - 2P_{3/2}(F=2) transition observed in muonic hydrogen in 2009



- 550 events measured
- 155 backgrounds
- 31 FP fringes
- 250 hours



→ proton charge radius (~ 0.1%)

Result and comparison with other best determinations

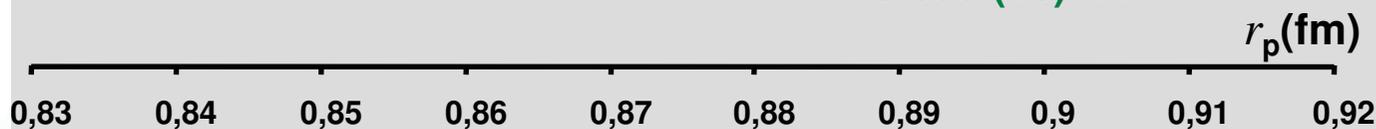
$$\left. \begin{aligned} \nu (\mu\text{-p} : 2S_{1/2}(F=1) - 2P_{3/2}(F=2)) &= 49\,881.88 (76) \text{ GHz} \\ \nu_{\text{theor}} &= 50772.43(118) - 1263.69 r_p^2 + 8.39 r_p^3 \text{ GHz} \end{aligned} \right\} r_p = 0.84184(36)(56) \text{ fm}$$

H/D spectroscopy + QED |-----●-----| 0.8760 (78) fm

$\mu\text{-p} (2S_{1/2}-2P_{3/2}) + \text{QED}$ ● 0.84184 (68) fm

New electron-proton scattering |-----●-----| 0.879 (8) fm

electron-proton scattering
re-analysis |-----●-----| 0.895 (18) fm



The result on $\mu\text{-p}$: [R. Pohl et al. , Nature 466, 213 \(2010\)](#)

New $e^- \text{-} p^+$ scattering : [J.C. Bernauer et al. , \(Mainz\) Phys. Rev. Lett. 105, 242001 \(2010\)](#)

$f_{1S-2S} = 2\,466\,061\,413\,187\,035 (10) \text{ Hz}$ (Garching meas.)

impact of the "muonic r_p " on this frequency : 80 kHz !!!

$f_{1S-2S} = 2\,466\,061\,413\,1\text{xx}\, \text{xxx}\, \text{Hz}$



Other transitions observed at PSI

- in μ -p atom : $2S_{1/2}(F=0) - 2P_{3/2}(F=1)$ at 5.5 μm

The hyperfine structure will also give the Zeemach radius of the proton

- in μ -d atom : $2S_{1/2}(F=3/2) - 2P_{3/2}(F=5/2)$ at 5.9 μm

$$2S_{1/2}(F=1/2) - 2P_{3/2}(F=3/2)$$

$$\text{and } 2S_{1/2}(F=1/2) - 2P_{3/2}(F=1/2) \quad \text{at } 5.7 \mu\text{m}$$

Analysis is in progress ...

Conclusion

- Since 2010, QED calculations have been checked in e-p and μ -p but the results obtained in the two systems remain inconsistent

- It cannot be explained by the proton structure

R.J. Hill and G. Paz, Phys. Rev. Lett. 107, 160402 (2011)

- Is it the signature of "new physics" ?

V. Barger et al., Phys. Rev. Lett. 106, 153001 (2011)

Spectroscopic measurements of various lines in hydrogen and in other simple atomic systems must be pursued

On going experiments in hydrogen atom ...

- 1S-2S hydrogen spectroscopy in MPQ Garching
hydrogen cooled with magnetic field
- 1S-3S hydrogen picosecond spectroscopy in MPQ Garching
- 1S-3S (and 1S-4S) hydrogen cw spectroscopy in LKB Paris
Frequency mixing in place of frequency doubling
to efficiently produce 205 nm radiation (in progress)
- Hydrogen 2S-nS/nD spectroscopy at NPL

Improved version of the previous Paris experiment : $n > 4$, frequency comb,
optical production of metastable atoms

J.L. Flowers et al., IEEE Trans. Instrum. Meas. 56, 331 (2007)

... and in He⁺

As for hydrogen, the charge rms radius of its nucleus (the alpha-particle ⁴He⁺⁺) is derived from scattering data : $r = 1.681 (4) \text{ fm}$

I. Sick, *Phys. Rev. C* 77, 041302(R) (2008)

and the Lamb shift determination in this H-like ion provides a test of QED calculations

Scaling factors in H-like ions

- Energy $\sim Z^2$
- Dipole $\sim Z^{-1}$
- 1 photon decay $\sim Z^4$
- 2 photon decay $\sim Z^6$ → 2S lifetimes : in H : 0.14 s
in He⁺ : 1.9 ms
- 1 photon excitation $\sim Z^{-6}$
- 2 photon excitation $\sim Z^{-12}$ → very low probability for the excitation of the 1S-2S transition !

M. Haas *et al.*, *Phys. Rev. A* 73, 052501 (2006)

Prospects in hydrogen-like He⁺ ions

- 1S-2S He⁺ femtosecond spectroscopy at MPQ Garching

1S-2S two-photon transition at 64 nm excited in trapped ions with a frequency comb

M. Herrmann et al., Phys. Rev. A 79, 052505 (2009)

- 2S-2P Lamb shift measurement in muonic helium ion (He⁺⁺ μ⁻)⁺

Planned in 2013 at PSI by the CREMA collaboration

The transition is in the 800 - 960 nm range

The contribution of the finite size effect to the 2S-2P Lamb shift is ~ 20%

The frequency measurement will lead to a determination of the charge radius (limited by the polarisability contribution) to be compared to scattering value

→ solve the "proton puzzle" ?

A Antognini et al., Can. J. Phys. 89, 47 (2011)

These both measurements in He and muonic helium ions will help to clarify the present discrepancy in hydrogen and to test bound states QED at a more sensitive level (two-loop effects, B_{60} and B_{71} terms)

Measurement of the 2S Lamb shift in He⁺

by the anisotropy method, as already discussed in H atom

The two-loop correction to the self energy of the electron scales as Z^5
contribution : -1.339 MHz in He⁺ compared to -0.0418 MHz in H

A fast beam of He⁺(2S) atoms is subjected to a static electric field
The total Lyman α fluorescence is detected in two orthogonal directions

The measured anisotropy $R = (I_{\parallel} - I_{\perp}) / (I_{\parallel} + I_{\perp})$ is proportional to the Lamb shift

The measured Lamb shift is 14041. 13(17) MHz

A. van Winjngaarden, J. Kwela and G.W.F. Drake, *Phys. Rev. A* 43, 3325 (1991)
A. van Winjngaarden, F. Holuj and G.W. Drake, *Phys. Rev. A* 63, 012505 (2000)

Excellent agreement with the theoretical value 14041.18(13) MHz

Outline of this lecture

previously : - hydrogen atom (2-body atomic system)

in this lecture :

- helium atom (3-body atomic system)
- positronium (2-body atomic system)
- muonium (2-body atomic system)
- muonic helium atom (3-body atomic system)
- antiprotonic helium atom (3-body atomic system)
- muonic hydrogen (2-body atomic system)
- He + (2-body atomic system)
- muonic He + (2-body atomic system)

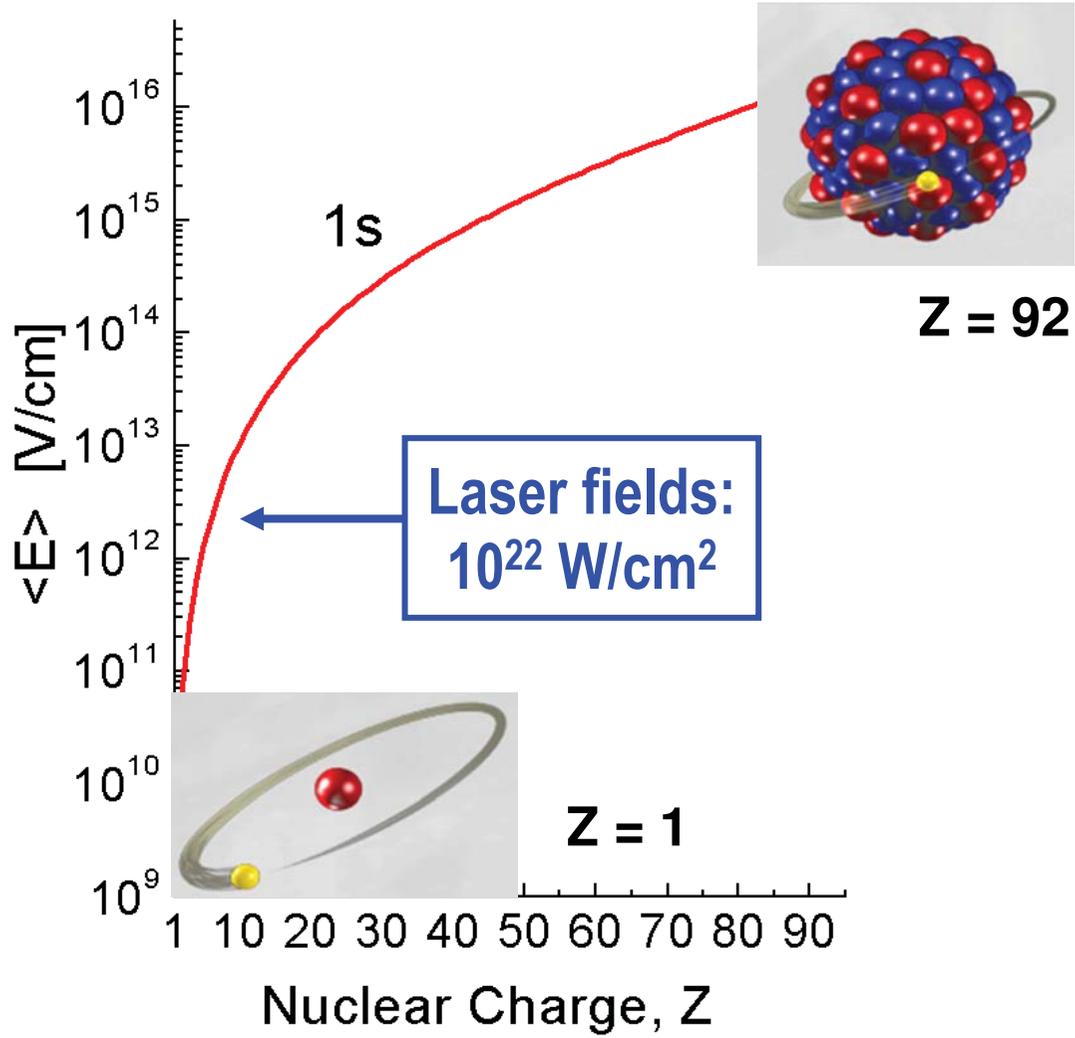
→

- H-like highly charged ions (2-body atomic systems)
- He-like highly charged ions (3-body atomic systems)
- Li-like highly charged ions (4-body atomic systems)

Test QED with highly charged ions (HCI)

Other hydrogen-like atomic systems with $Z \gg 1$
 ($Z\alpha \sim 10^{-2} \rightarrow Z\alpha \sim 1$)

$$E_n = mc^2 \frac{(Z\alpha)^2}{2n^2}$$



An electron in an extremely strong Coulomb field

$$r_0 = \frac{\hbar}{Z\alpha mc}$$

Increased :

- relativistic corrections
- QED effects
- nuclear structure effects

in HCI series expansion in $Z\alpha$ is not appropriate

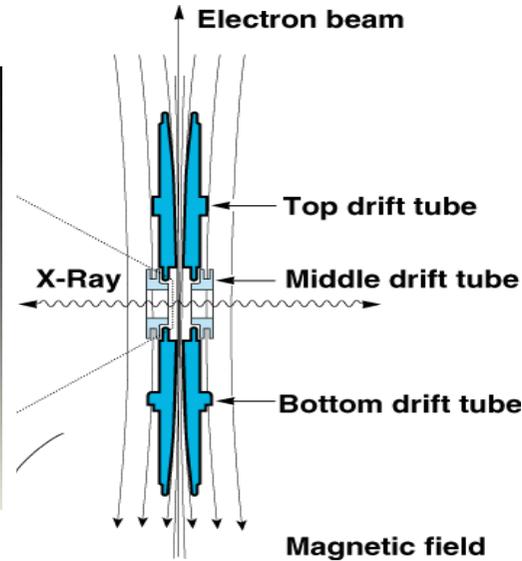
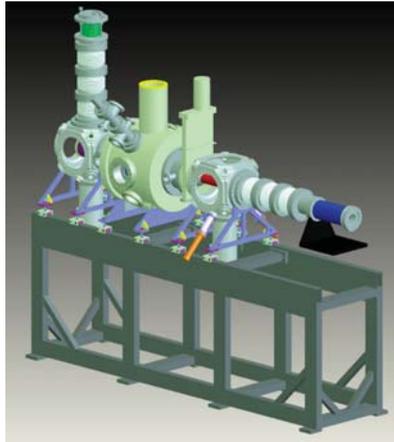
1S Lamb shift in H-like highly charged ions

The binding energy varies as Z^2 and QED corrections as Z^4

		Frequency domain
• $Z = 1$: hydrogen (-13.6 eV)		
Lamb shift	3.5×10^{-5} eV	RF
Vacuum polarization	2.5 %	
Nuclear size	0.01 %	
• $Z = 14$: H-like silicon (-2.6 keV)		
Lamb shift	0.48 eV	optical
Vacuum polarization	5.9 %	
Nuclear size	0.5 %	
• $Z = 92$: H-like uranium (-177 keV)		
Lamb shift	462 eV	X-Rays
Vacuum polarization	13.8 %	
Nuclear size	30.4 %	

Production, storage and cooling of HCl

Electron Beam Ion Trap

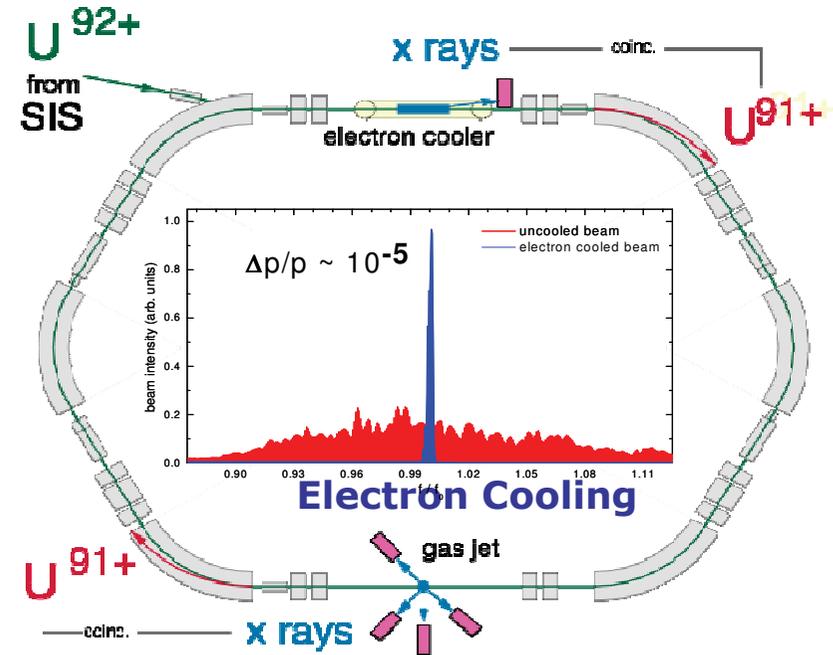


Cooling in traps

- resistive cooling
- evaporative cooling
- laser cooling
- electron cooling



Storage Ring



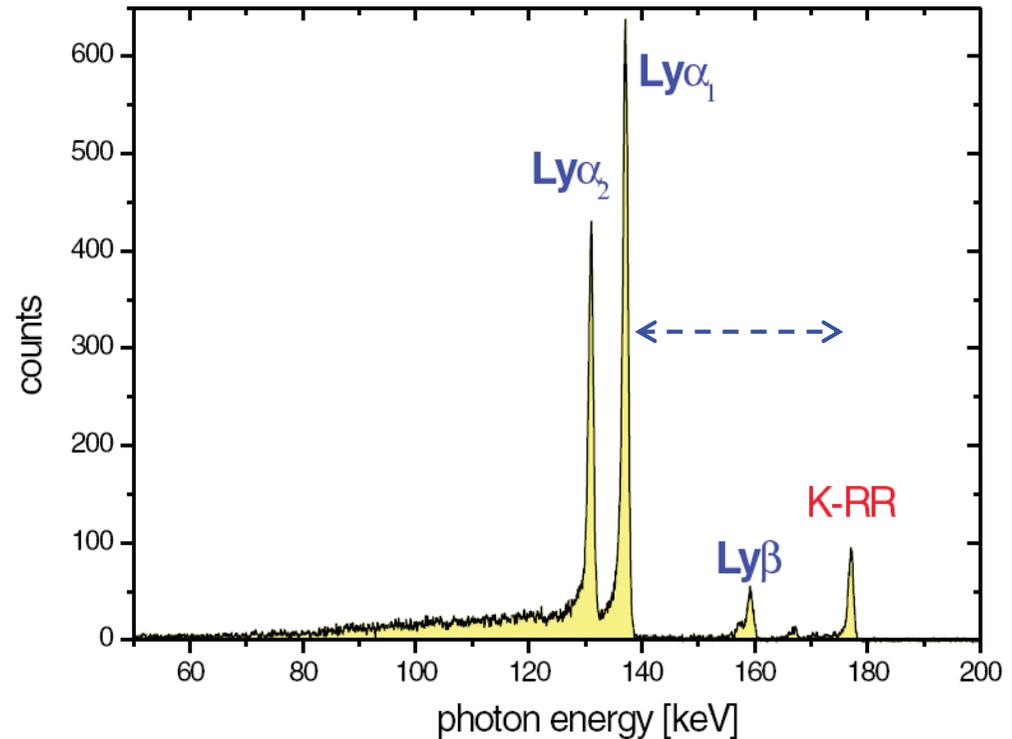
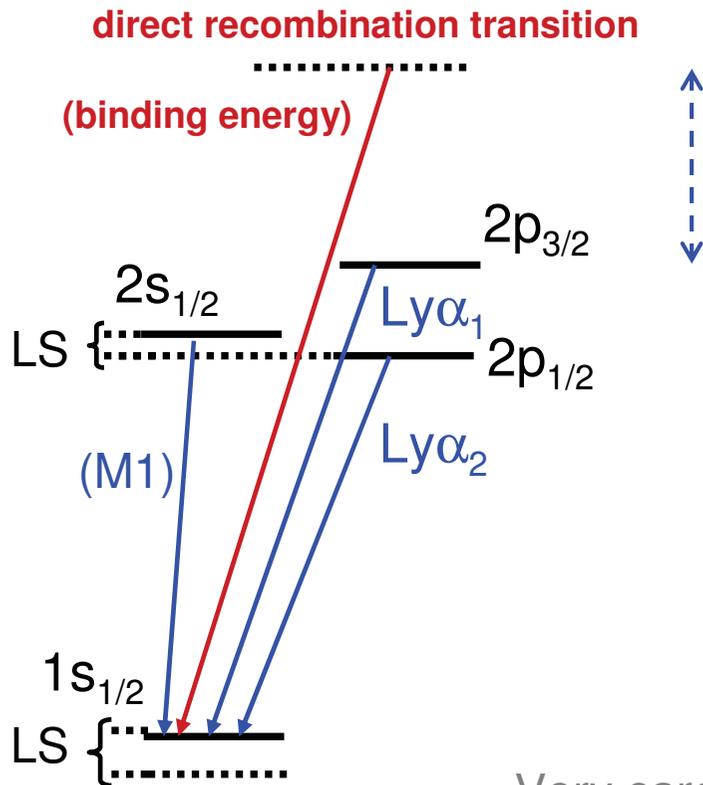
Cooling in Storage Rings

- electron cooling
- stochastic cooling
- laser cooling

Highly-charged ions up to bare, H-like and He-like uranium can be produced

1S Lamb shift spectroscopy in H-like uranium

Measurement performed at the ESR storage ring in GSI (Darmstadt)



Very careful energy calibration and Doppler correction

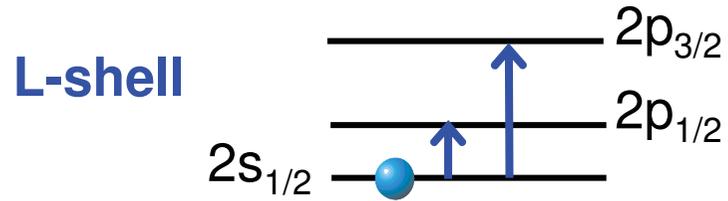
A. Gumberidze *et al.*, *Phys. Rev. Lett.* 94, 223001 (2005)

Result : $L_{1S} = 459.8 (4.6) \text{ eV}$

Theoretical prediction : 463.95 eV

Lamb shift in Li-like ions

Li-like ion : 3 e⁻



- simple theoretical description :
only 1 valence electron
→ small contributions from e⁻ - e⁻ interaction
- intra-shell transitions ($\Delta n = 0$)
→ lower energy



Comparison of H-like and Li-like data are required
to disentangle QED and nuclear effects
(*Shabaev, Jentschura, Glazov, Yerokhin, et al.*)

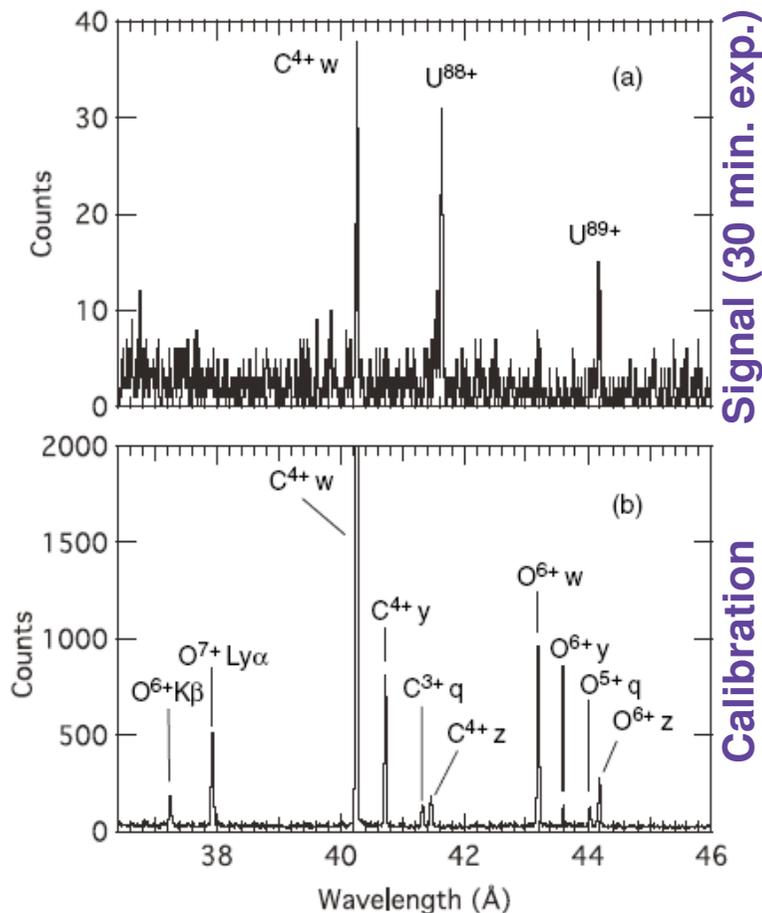
The contribution of two-loop QED terms in L_{1S} of H-like uranium is -1.27 eV.
It cannot be directly tested since it is smaller than the experimental accuracy (4.6 eV)

The calculation of high order QED terms in Li-like ions is more complex
but the experimental accuracy is more than one order of magnitude better
than the estimated size of the two-loop corrections

Measurement of the $2s_{1/2} - 2p_{1/2}$ splitting in Li-like HCl (1)

- Measurement performed at Livermore using the superEBIT electron beam ion trap
P. Beiersdorfer et al., Phys. Rev. Lett. 95, 233003 (2005)

- Extreme ultraviolet emission spectroscopy of the $2s_{1/2} - 2p_{1/2}$ transition in U^{89+} with a special grazing incidence spectrometer



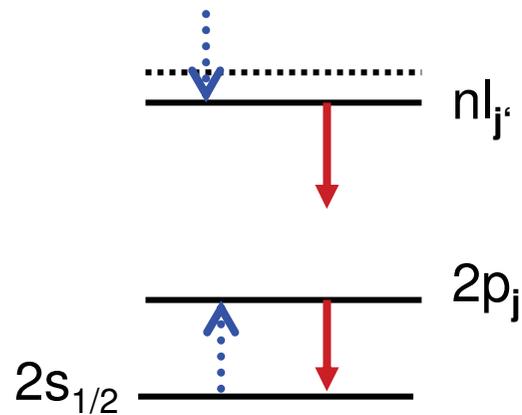
Result in Li-like U^{89+} : **280.645 (0.015) eV**

This result can be used to determine the two-loop Lamb shift in U^{89+} since other contributions are well calculated

The value of the 1s two-loop Lamb shift in H-like U^{91+} can be inferred : **-1.27 eV**
in excellent agreement with the calculated value :
-1.26 (0.33) eV

V.A. Yerokhin et al., X-Ray Spectrom. 32, 83 (2003)

Measurement of the $2s_{1/2} - 2p_{1/2}$ splitting in Li-like HCl (2)



Non conventional method \neq spectroscopy

- Measurement performed in the electron cooler of the experimental storage ring at GSI
- Low energy dielectronic recombination (time reversed autoionization)

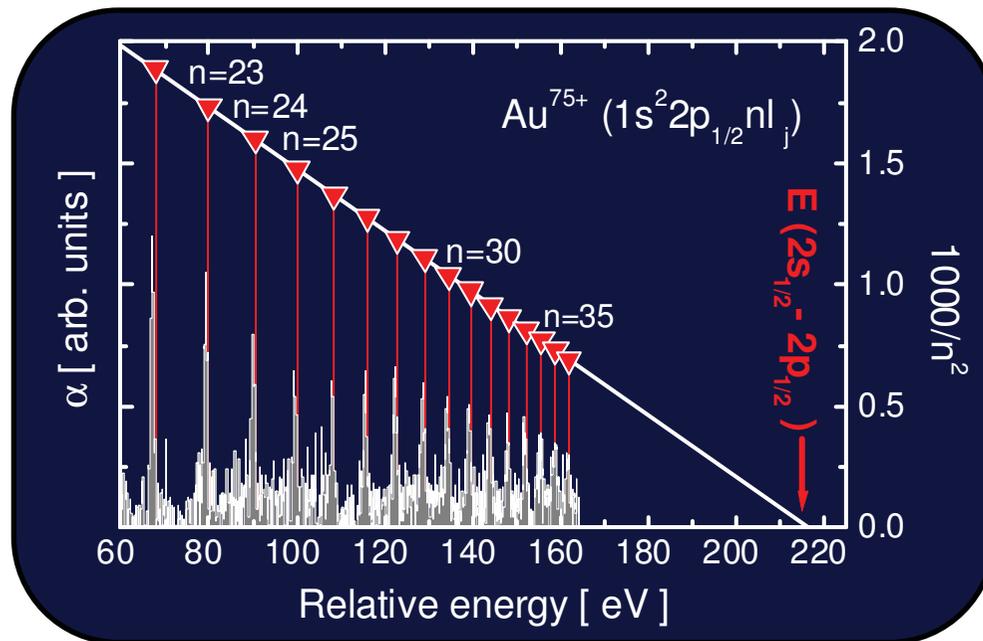
Capture of free electrons into doubly excited high Rydberg states followed by radiative decay

Signal : resonant dielectric capture

The $2s_{1/2} - 2p_{1/2}$ splitting is obtained by extrapolation from the $2p_{1/2} n l_j$ resonance peak frequencies

Result in Li-like U^{89+} :
280.516 (0.099) eV

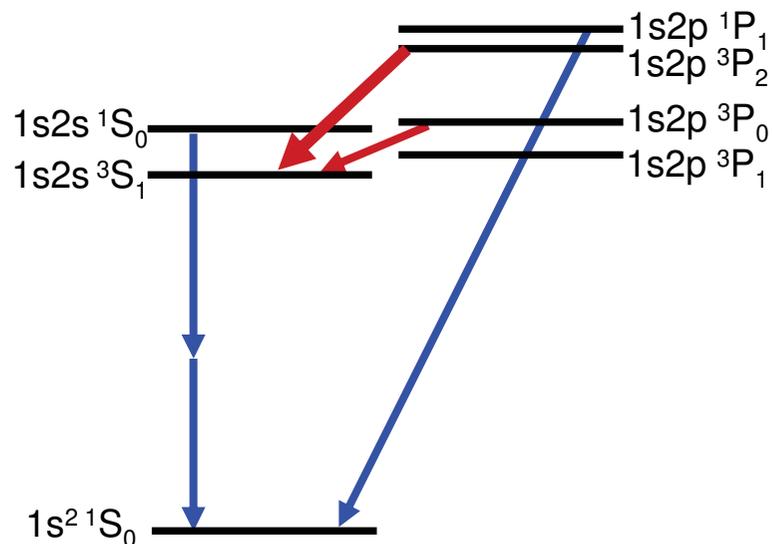
The results obtained in Au^{76+} , Pb^{79+} and U^{89+} are in good agreement with QED calculations



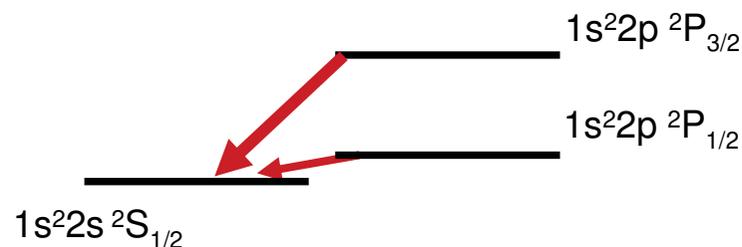
C. Brandau *et al.*, Phys. Rev. Lett. 91, 073202 (2003)

Spectroscopy of highly charged He-like ions

Heavy He-like ions



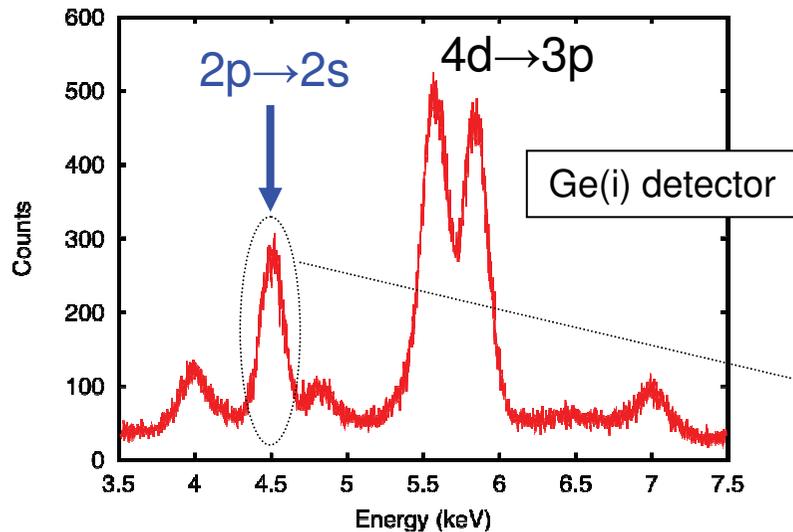
Heavy Li-like ions



Intra-shell transition spectroscopy in He-like and Li-like ions

- same nucleus
 - same one-electron QED contribution
- reduction of the systematic uncertainties
in the theoretical predictions

Spectroscopy of highly charged He-like ions



First clear identification of the $1s2p\ ^3P_2 - 1s2s\ ^3S_1$ transition in He-like uranium

Precise measurement of the transition

$$\delta E/E = 2 \times 10^{-4}$$

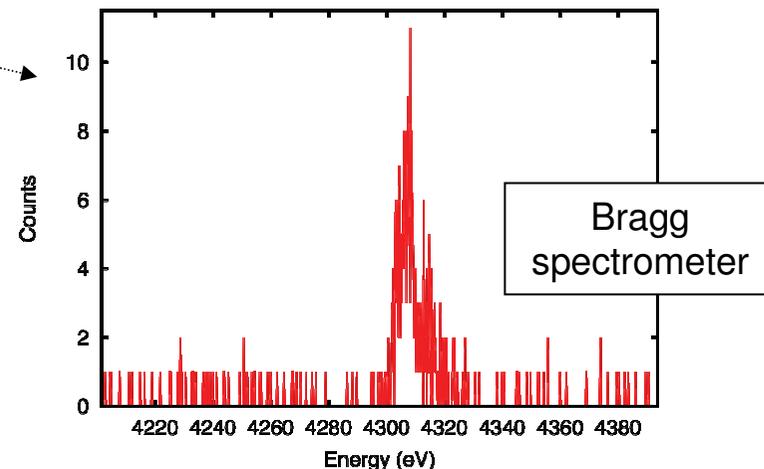
M. Trassinelli *et al.*, *Europhys. Lett.* 87, 63001(2009)

result : 4509.71 (0.99) eV

theory: 4509.86 (0.07) eV Kozhedub and Shabaev, unpublished (2008)

4510.03 eV Indelicato, unpublished (2008)

- Measurement performed at the ESR in GSI
- Helium-like ions created by electron capture in a nitrogen gas jet target
- Detection with a Bragg crystal spectrometer (resolution few eV at 4-5 keV)



First test of two-electron QED terms in excited levels of heavy He-like ions

Spectroscopy of HCl : discussion

QED predictions can be tested in heavy ions

- 1S Lamb shifts have been measured in H-like ions up to $^{238}\text{U}^{91+}$
They give a test of one-loop QED effects (all orders in $Z\alpha$)
and are in agreement with theoretical predictions
 - The $2s_{1/2} - 2p_{1/2}$ splitting determination in Li-like ions
allows to derive the uncalculated two-loop contributions to the Lamb shifts
 - Lamb shifts in He-like ions give a test of two-electron QED terms
- Each type of these measurements provide a specific test of QED

Not discussed here : the hyperfine structure of HCl

- Hyperfine splitting in H-like ions up to $^{209}\text{Bi}^{82+}$ has been measured
Is not an efficient test of QED because of the nuclear magnetization
distribution correction (Bohr-Weisskopf) which is not well known
- Hyperfine splitting in Li-like ions combined with hfs of corresponding
H-like ion is a way to overcome this limitation

Another QED test : the g-factor !