QED & Quantum Vaccum, Low Energy Frontier, 03001 (2012)
DOI: 10.1051/iesc/2012qed03001

Experimental tests of QED in bound and isolated systems

Lucile JULIEN

QED 2012, Cargèse

3 lectures : 24, 25, 26 April 2012

Lucile JULIEN

- Université Pierre et Marie Curie (Paris 6)

  Professor lucile.julien@upmc.fr

  (Paris, campus Jussieu)

- Member of the laboratoire Kastler Brossel

In the group of Quantum metrology and simple atomic systems

with :
- François BIRABEN
- François NEZ
- Saïda GUELLATI
- Pierre CLADE

PhD students :
- Rym BOUCHENDIRA
- Sandrine GALTIER

Measurement of fundamental constants : R_∞ , α , r_p

Experimental tests of QED ?

Deviations from Dirac theory in atoms, ions and free particles

- Lamb shift measurements

more generally, precise determinations of atomic structures in simple systems

- Radiative decay measurements

- Gyromagnetic anomaly : determinations of g -2

We will restrict here to the domain of atomic physics (low energy tests)

Limited topics, but however quite wide…

General outline

Various "simple" systems

Lecture I

- Hydrogen atom (and experimental methods)

Lecture II

- Helium atom
- Exotic atoms : positronium, muonium, muonic hydrogen, …
- Highly charged ions

Lecture III

- g – 2 : ions, electron, muon
- Fine structure constant and h/M

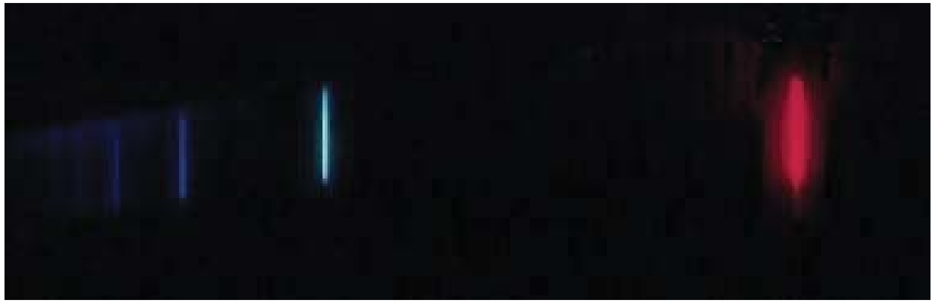
The hydrogen atom

The simplest atom in nature : proton + electron
involved in major advances of atomic physics and quantum mechanics

A brief history : 19th century

Optical spectrum
(Balmer 1885)

Hβ Hα



Balmer-Rydberg formula
(1889)

$$\frac{1}{\lambda} = R\left(\frac{1}{n^2} - \frac{1}{p^2}\right)$$

n and p integers

R : Rydberg constant

Balmer lines : $n = 2$

A brief history : 20th century

- Bohr's model (1913)
  - circular orbits
  - quantized angular momentum : $n\hbar$
  - quantum jumps

$$E = -\frac{E_I}{n^2} = -\frac{R_H\, hc}{n^2}$$

$$R_H = R_\infty \left(1 + m_e / m_p\right)^{-1}$$

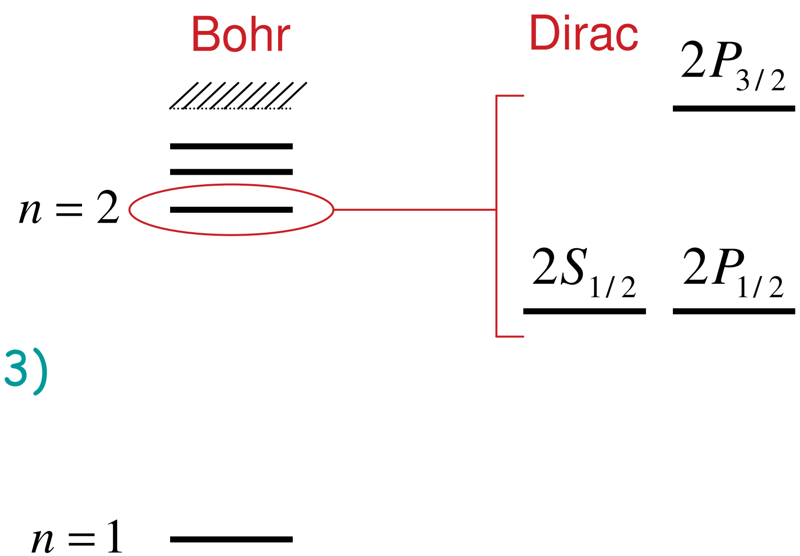
$$R_\infty = \frac{m_e\, e^4}{8\varepsilon_0{}^2 h^3 c}$$

The Rydberg constant is linked to other fundamental constants

$$R_\infty hc = \frac{1}{2} m_e c^2 \alpha^2$$

… quantum mechanics …

- Fine structure : Dirac equation (1928)



Small experimental discrepancies
(few percents) on the Hα line…

E.C. Kemble and R.D. Present <u>44</u>, 1031 (1933)
W.V. Houston, Phys. Rev. <u>51</u>, 446 (1937)
R.C. Williams, Phys. Rev. <u>54</u>, 558 (1938)

A brief history : 20th century

In 1938, the tests of discrepancies between theory and experiment on the spectroscopy of the Hα line were limited by the large Doppler effect

… second world war …

Great advances in microwave techniques → new experimental tools

Microwave spectroscopy of hydrogen :

- direct measurement of the hyperfine structure
- direct measurement of the Lamb shift

→ atomic clocks and redefinition of the second (1967)

In 1960, the first laser !

Laser spectroscopy of hydrogen (Doppler free optical spectroscopy) :

- precise determination of the Rydberg constant
- indirect measurements of the Lamb shift and the hyperfine structure

→ frequency combs and direct optical frequency measurements

The hyperfine structure of hydrogen

Hyperfine structure in atoms results from the small magnetic moment of the nucleus

The electron and the proton have both a spin,
that is an intrinsic angular momentum.

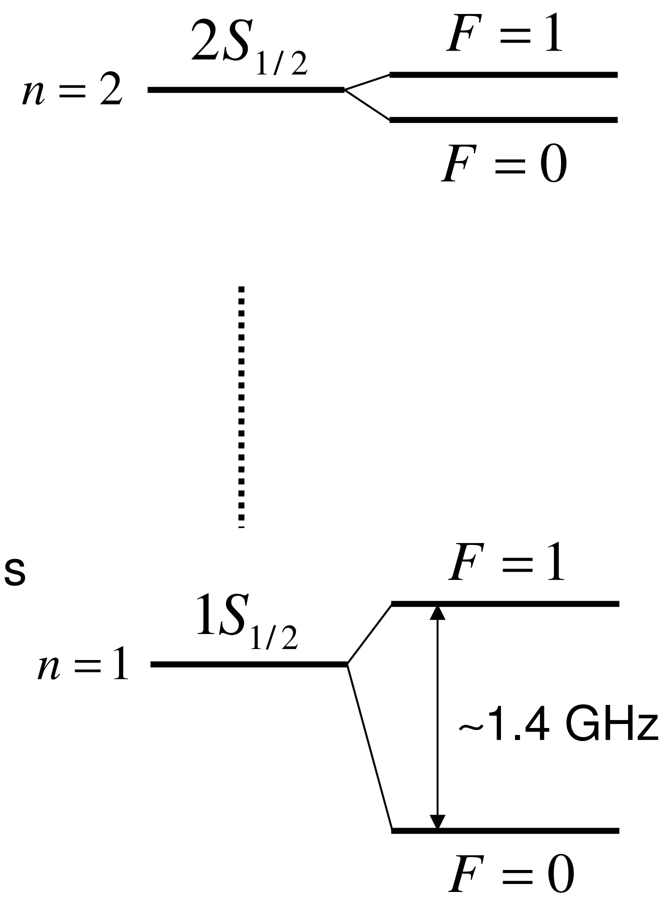
$S = 1/2$ for the electron

$I = 1/2$ for the proton

The total momentum is : $\vec{F} = \vec{S} + \vec{I}$

This leads to a splitting of all hydrogen energy levels

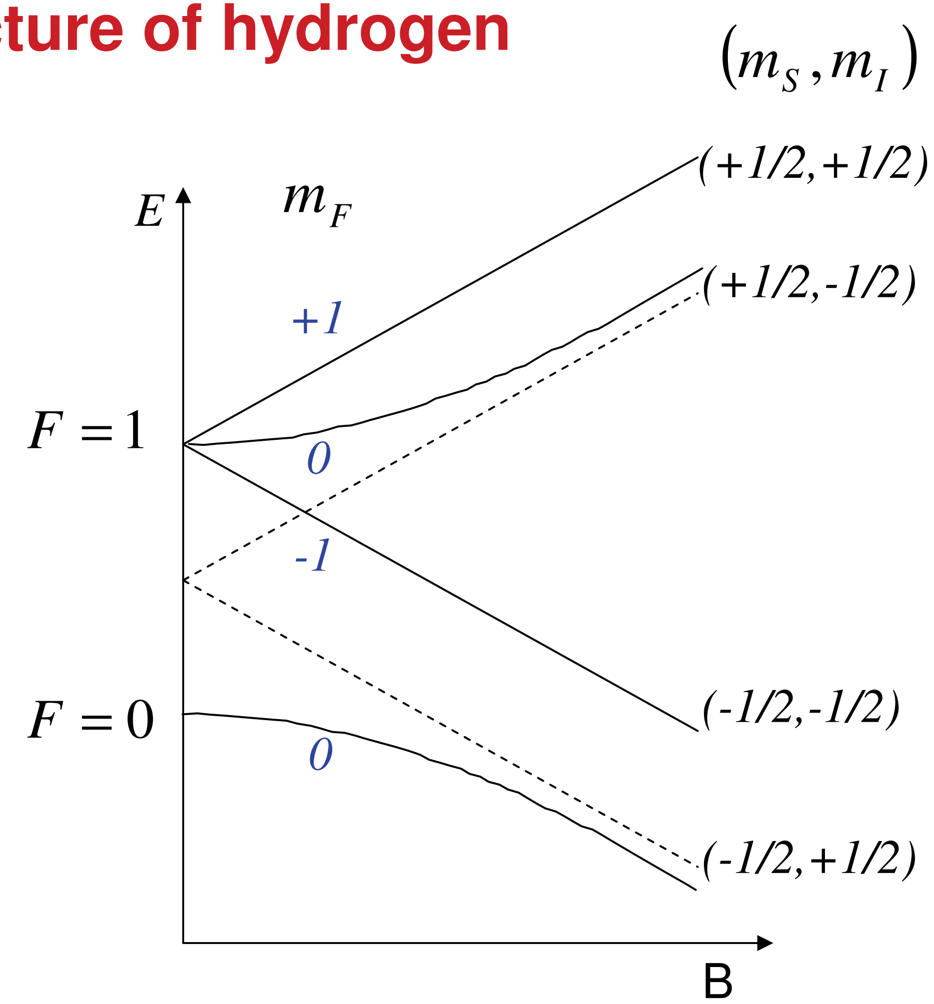
which varies roughly as $1/n^3$



The 1S hyperfine structure of hydrogen

In an applied magnetic field B,
the F = 1 hyperfine level is splitted
in three Zeeman sublevels.

In a high field, electronic and
nuclear momenta are decoupled.



In an inhomogeous magnetic field,
atoms undergo a force

→ deflection of an atomic beam

I.I. Rabi, J.M.B. Kellogg and J.R. Zacharias, Phys. Rev. 46, 157 and 163 (1934)

→ refocusing of an atomic beam (Columbia)

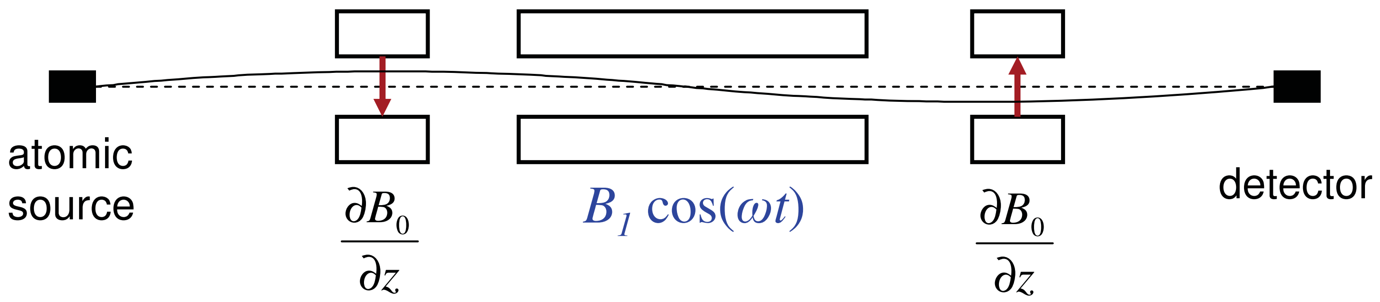
J.M.B. Kellogg, I.I. Rabi and J.R. Zacharias, Phys. Rev. 50, 472 (1936)

Atomic beam magnetic resonance

(after the war ...)

Principle of the method :

- two magnets to deviate the atomic beam and select a given state
- an oscillatory field to induce a transition between two states



First accurate measurement of the 1S hyperfine splitting :

J.E. Nafe, E.B. Nelson and I.I. Rabi, Phys. Rev. 71, 914 (1947); 73, 718 (1948); 75, 1194 (1949); 76, 1858 (1949).

in hydrogen $\Delta\nu_H$ = 1420. 410 (6) MHz
in deuterium $\Delta\nu_D$ = 327. 384 (3) MHz

Disagreement with theoretical predictions $\rightarrow$ $g \neq 2$

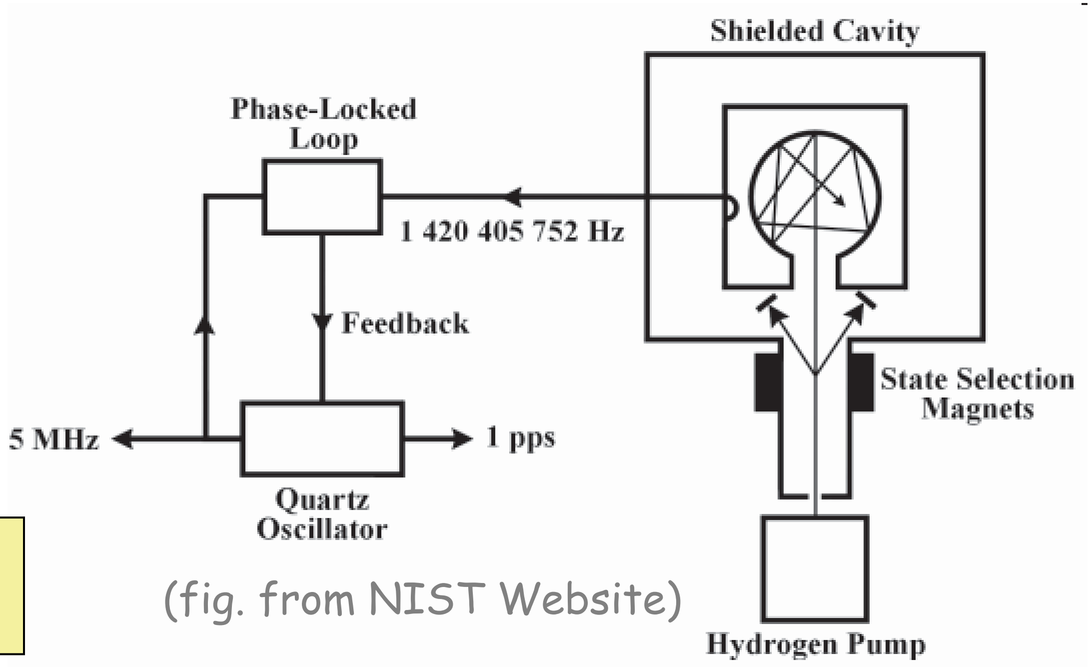
G. Breit, Phys. Rev. 72, 984 (1947)
J. Schwinger, Phys. Rev. 73, 416 (1948) and 76, 790 (1949)

The hydrogen maser

It gives by far the most accurate measurement of the hydrogen hyperfine structure

$F = 1$ states are selected and focused in a storage bulb

The atomic system oscillates at the $\Delta\nu_H$ frequency



(fig. from NIST Website)

First measurement by H.M. Goldenberg, D. Kleppner and N.F. Ramsey (1960)

Combined value of precise measurements : $\Delta\nu_H$ = 1420 405 751. 7667 (9) Hz
$\Delta\nu_D$ = 327 384 352. 5219 (17) Hz

For a review, see : N.F. Ramsey in Rev. Mod. Phys. 62, 541 (1990)

The 2S hyperfine structure of hydrogen

- It has been measured early by an atomic beam magnetic resonance method ...

J.W. Heberle, H.A. Reich and P. Kusch, Phys. Rev. 101, 612 (1956)
H.A. Reich, J.W. Heberle and P. Kusch, Phys. Rev. 104, 1585 (1956)

$\Delta\nu_H$ (2S) = 177 556.86 (5) kHz (Columbia)
$\Delta\nu_D$ (2S) = 40 924.439 (20) kHz

- and recently remeasured more precisely

N.E. Rothery and E.A. Hessels, Phys. Rev. A 61, 044501 (2000)

$\Delta\nu_H$ (2S) = 177 556. 785 (29) kHz (York)

It is not exactly equal to $\Delta\nu_H$ (1S) / 8

- The most accurate measurement is now an optical measurement !

N. Kolachevsky et al., Phys. Rev. Lett. 102, 213002 (2009)

$\Delta\nu_H$ (2S) = 177 556 834.3 (6.7) Hz (Garching)

The hyperfine structure of hydrogen : discussion

The 1S and 2S hyperfine structures in hydrogen and deuterium $\Delta\nu_H$ and $\Delta\nu_D$ have been measured very precisely (~ 10^{-12} for the 1S)

They are proportional to $\alpha^2 R_\infty$

An accurate value of α is crucial to allow to test all QED calculations

But, the comparison of their predicted and measured values cannot provide a competitive value of α because of the relative uncertainty of the theory (~ 10^{-6}) due to the internal structure of the proton (or deuteron)

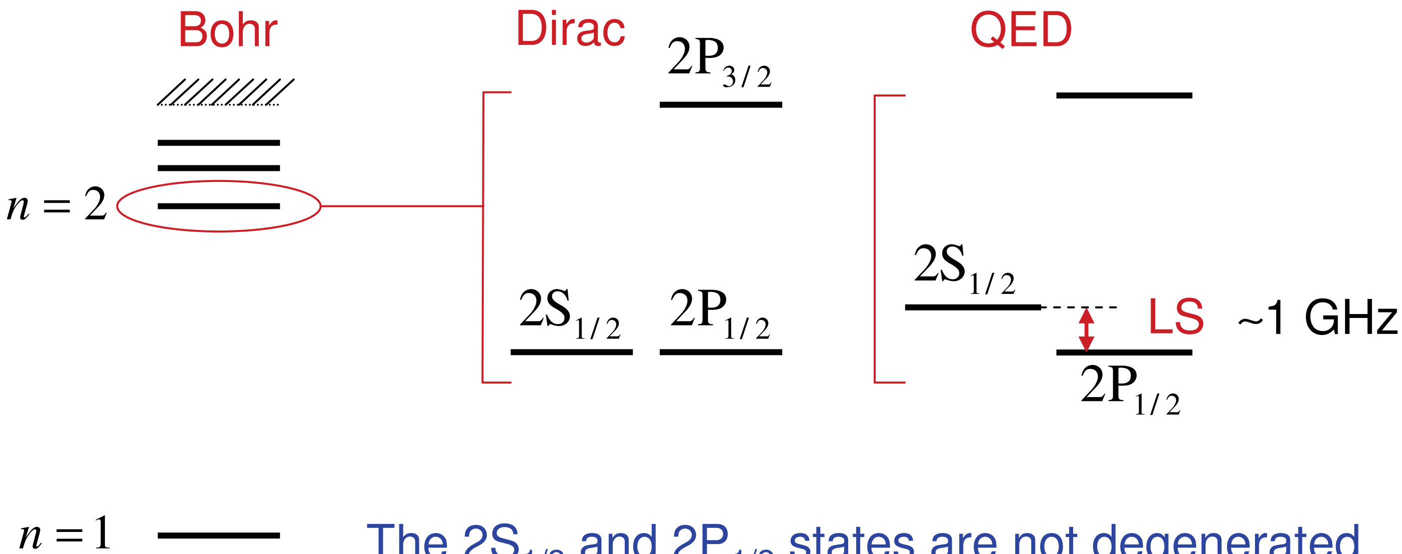
To overcome this limitation, a possibility is to study purely leptonic systems : positronium and muonium (see tomorrow)

The Lamb shift of hydrogen

We discuss here the 2S Lamb shift
which can be directly measured by a microwave method

$$E = -\frac{E_I}{n^2}$$

$$E_I \approx \frac{1}{2} m_e c^2 \alpha^2$$



The $2S_{1/2}$ and $2P_{1/2}$ states are not degenerated

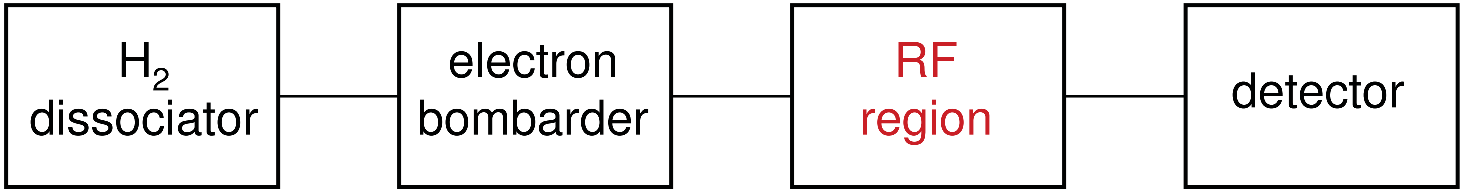
The $2S_{1/2}$ state is metastable

It was considered in 1937, and confirmed in 1947 :

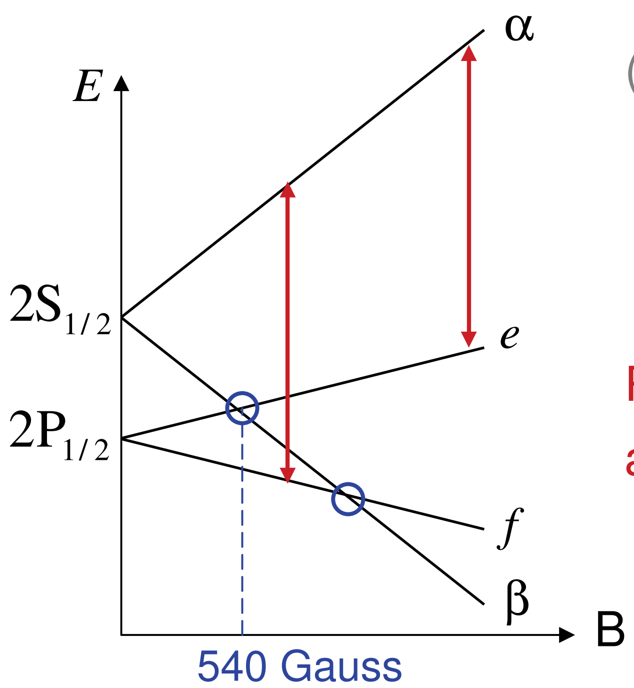
S. Pasternack, Phys. Rev 54, 1113 (1938)
W.E. Lamb Jr. and R.C. Retherford, Phys. Rev. 72, 241 (1947)

The Lamb and Retherford experiment (I)

Principle



- Thermal dissociation of molecular hydrogen in an oven $\rightarrow$ H (1S)
- Crossed electronic bombardment of the atomic beam $\rightarrow$ H (2S)
- Quenching of the metastable state by a RF in various magnetic fields
- Detection of the metastable atoms through electron ejection from a metal target



(the hyperfine splitting is omitted on the diagram)

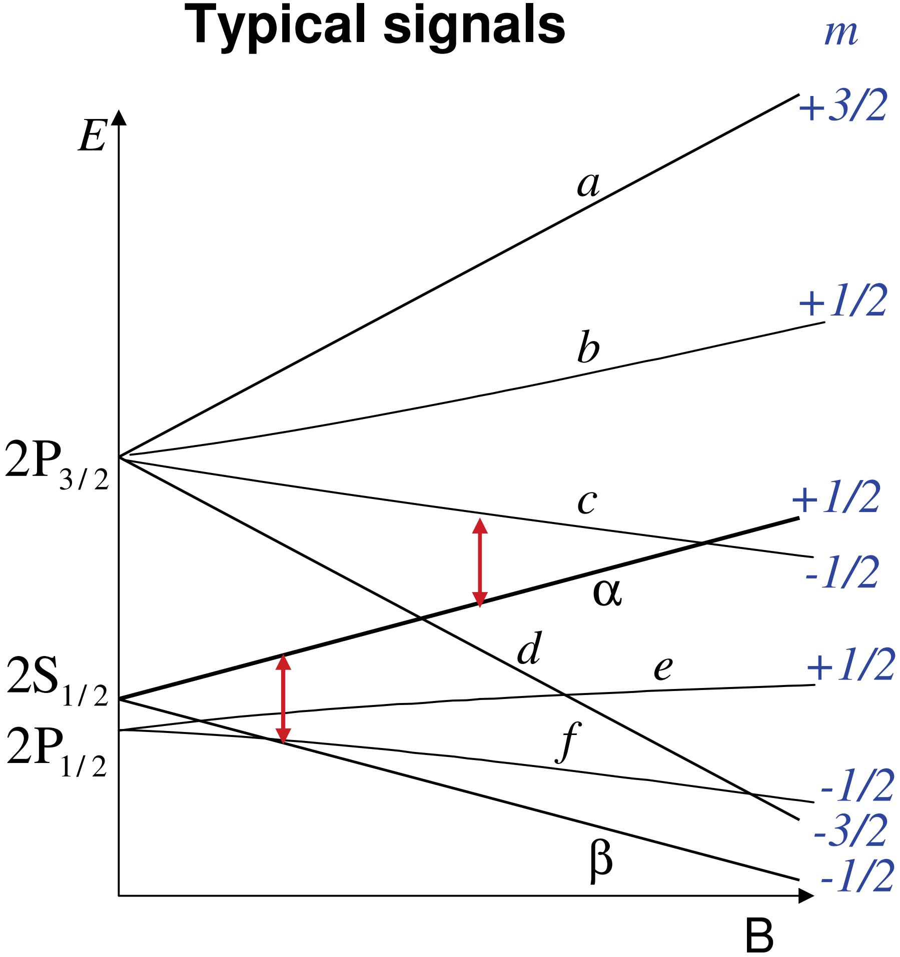
Due to βe and βf level crossings, and to the motional electric field seen by the atoms, the atomic beam is polarized

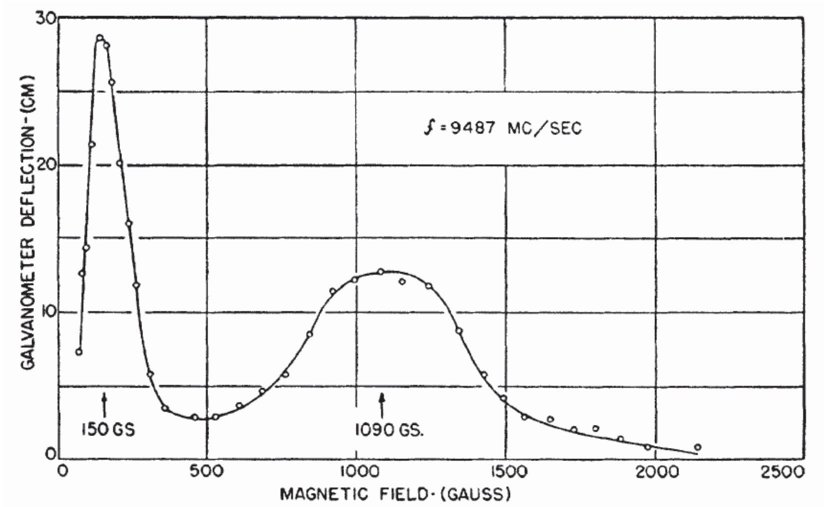
RF transitions are induced between $2S_{1/2}$ (α) state and the various sublevels of $2P_{1/2}$ and $2P_{3/2}$

They are detected through the decrease of the 2S beam intensity

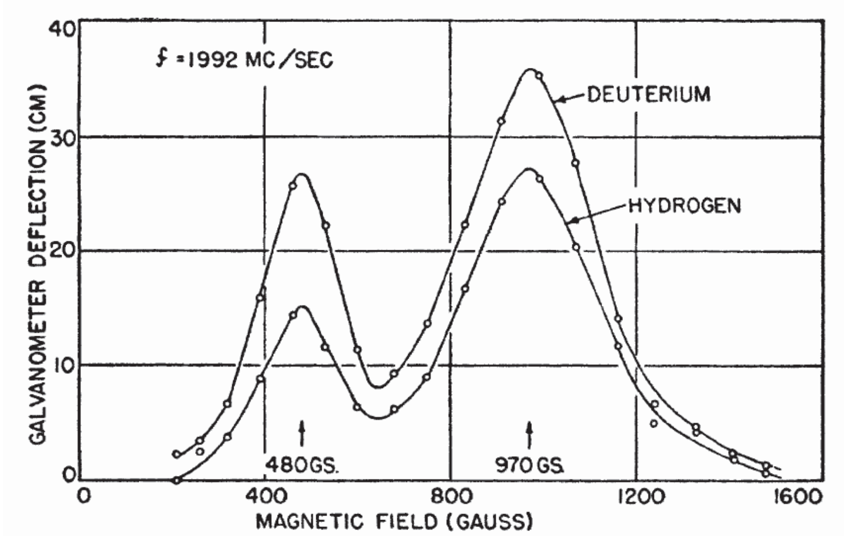
The Lamb and Retherford experiment (II)

Typical signals





Observed resonance curves

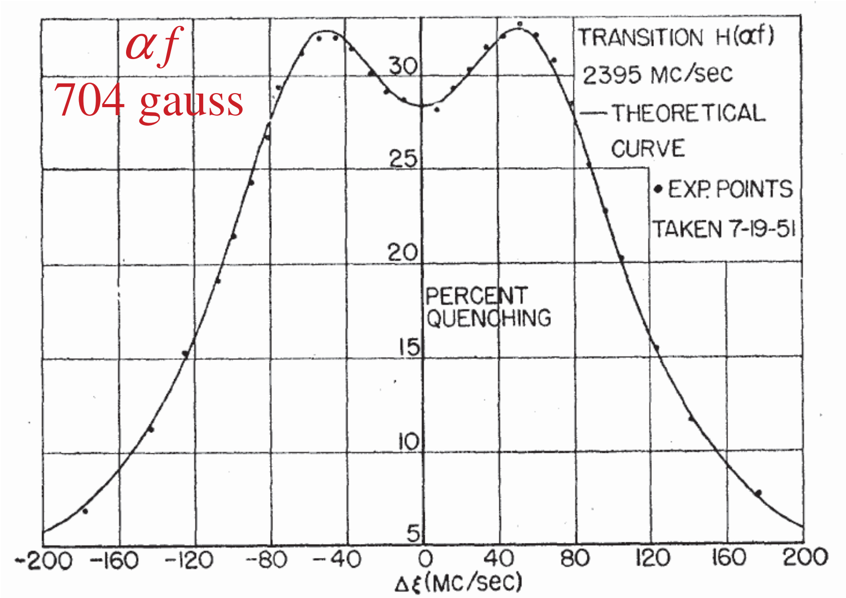


fixed RF frequency and various B field values

The Lamb and Retherford experiment (III)

Detailed analysis of the line profiles

(the hyperfine splitting is visible on the signal)



Results

for the $2S_{1/2} - 2P_{1/2}$ interval

in hydrogen and deuterium :

δ_H = 1057.77 (10) MHz

δ_D = 1059.00 (10) MHz

References

W.E. Lamb Jr. and R.C. Retherford, Phys. Rev. 79, 549 (1950)
Phys. Rev. 81, 222 (1951)
W.E. Lamb Jr, Phys. Rev. 85, 259 (1952)
W.E. Lamb Jr. and R.C. Retherford, Phys. Rev. 86, 1014 (1952)
S. Triebwasser, E.S. Dayhoff and W.E. Lamb Jr, Phys. Rev. 89, 98 (1953)
E.S. Dayhoff, S. Triebwasser and W.E. Lamb Jr, Phys. Rev. 89, 106(1953)

Separated oscillatory field measurement (I)

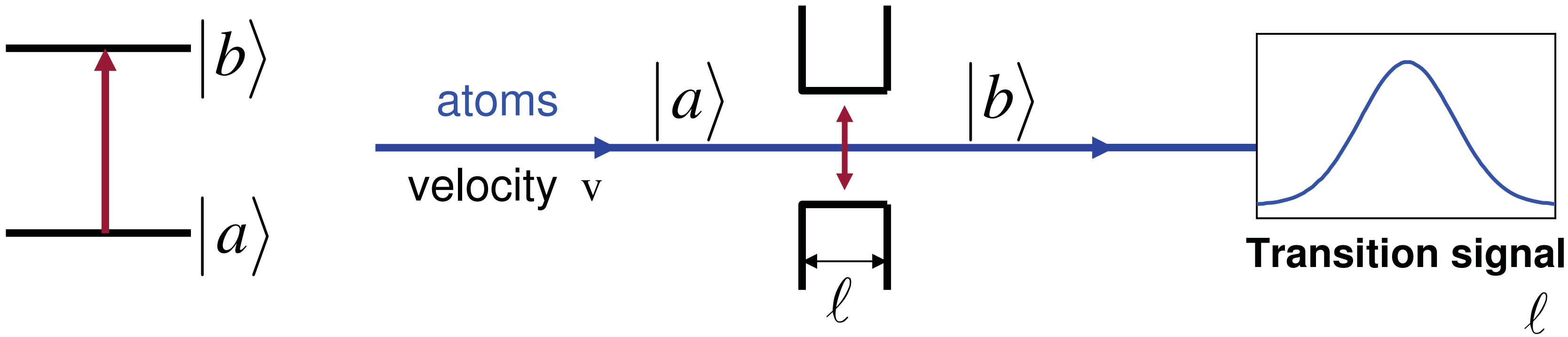
a more recent RF measurement of the 2S hydrogen Lamb shift

Principle

- Fast atomic beam obtained by charge exchange from a 50-100 keV proton beam
- Detection of the metastable atoms through RF quenching and Lyman α decay

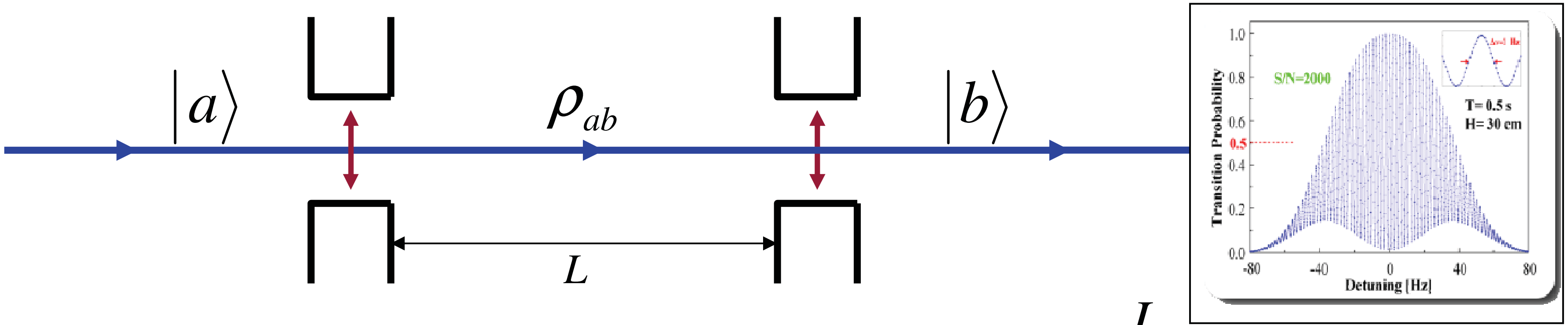
The separated oscillatory field method (Ramsey method)

- Conventional magnetic resonance or absorption RF spectroscopy in an atomic beam



For long-living states, the signal linewidth is limited by the interaction time : $t = \dfrac{\ell}{v}$

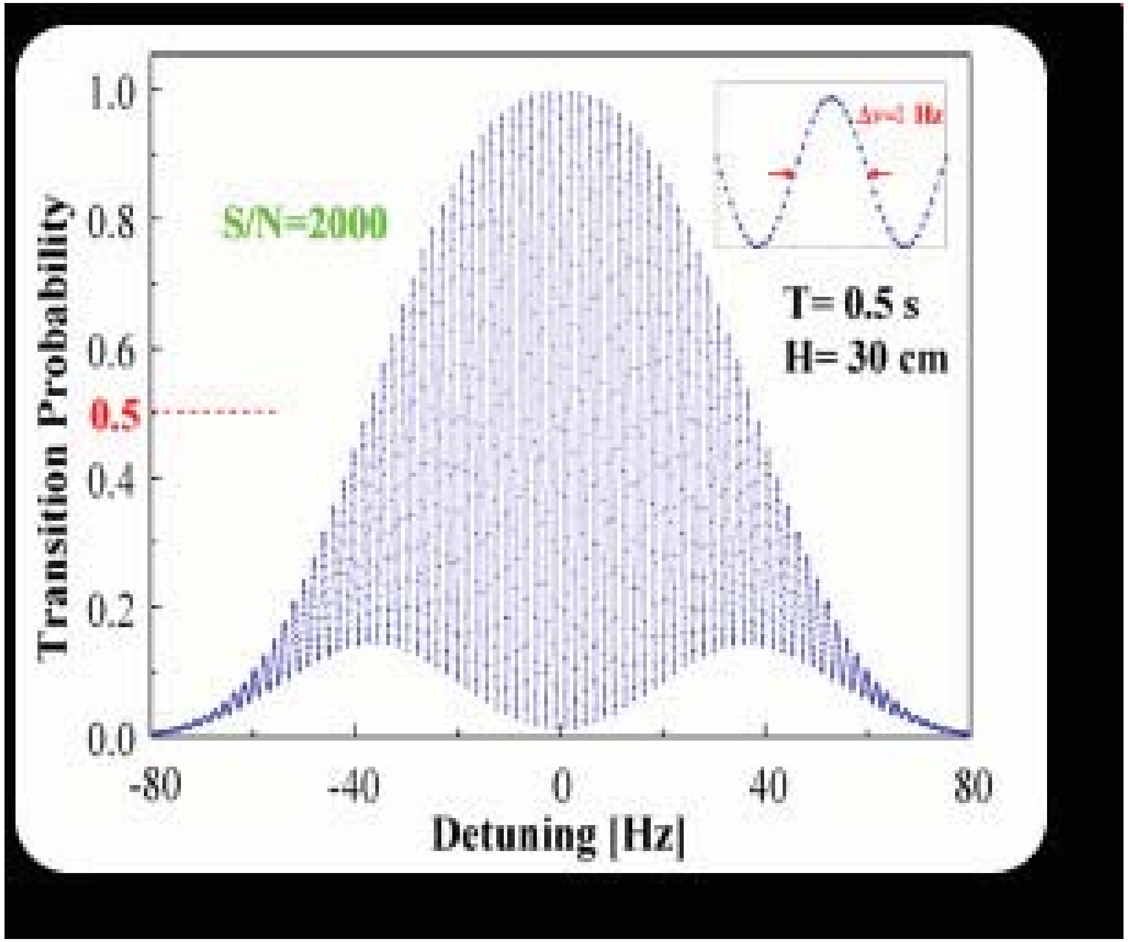
- Use of two separated excitation regions, where the same RF field is applied



Ramsey pattern

The central fringe is much narrower : its width is related to $T = \dfrac{L}{v}$

(see N.F. Ramsey Nobel prize lecture, 1989)



Ramsey pattern obtained in
a cold atoms clock
(Cs atomic fountain)
SYRTE

The central fringe width

is related to $T = \frac{L}{v}$

Here all atoms have
the same velocity
and the transition is between
long-living states

When the transition is induced in a non monokinetic beam,
since the fringe spacing depends on the velocity
only the central fringe, or few central fringes,
are visible with a good contrast

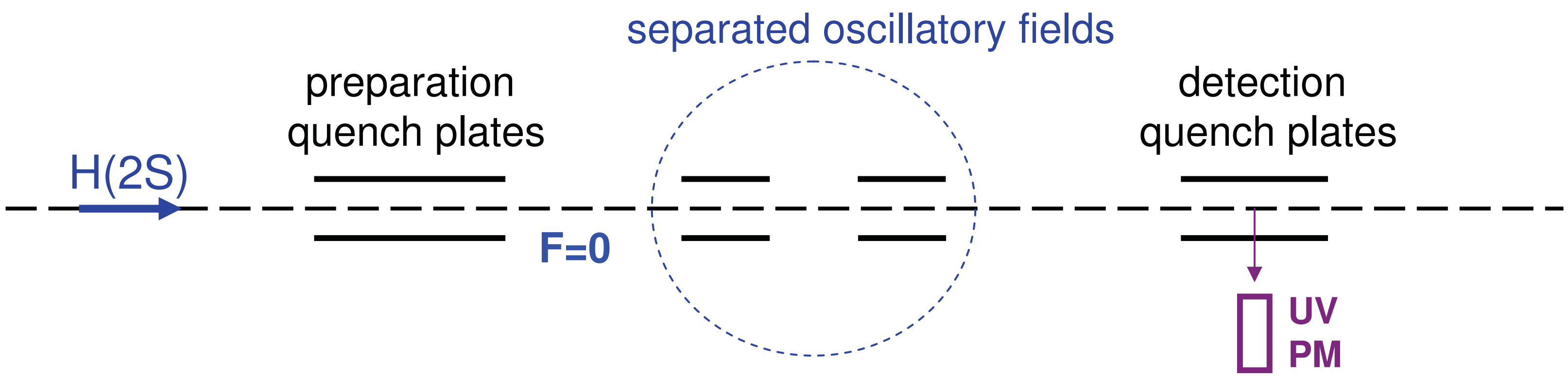
Separated oscillatory field measurement (I)

a more recent RF measurement of the 2S hydrogen Lamb shift

Principle

- Fast atomic beam obtained by charge exchange from a 50-100 keV proton beam
- Detection of the metastable atoms through RF quenching and Lyman α decay

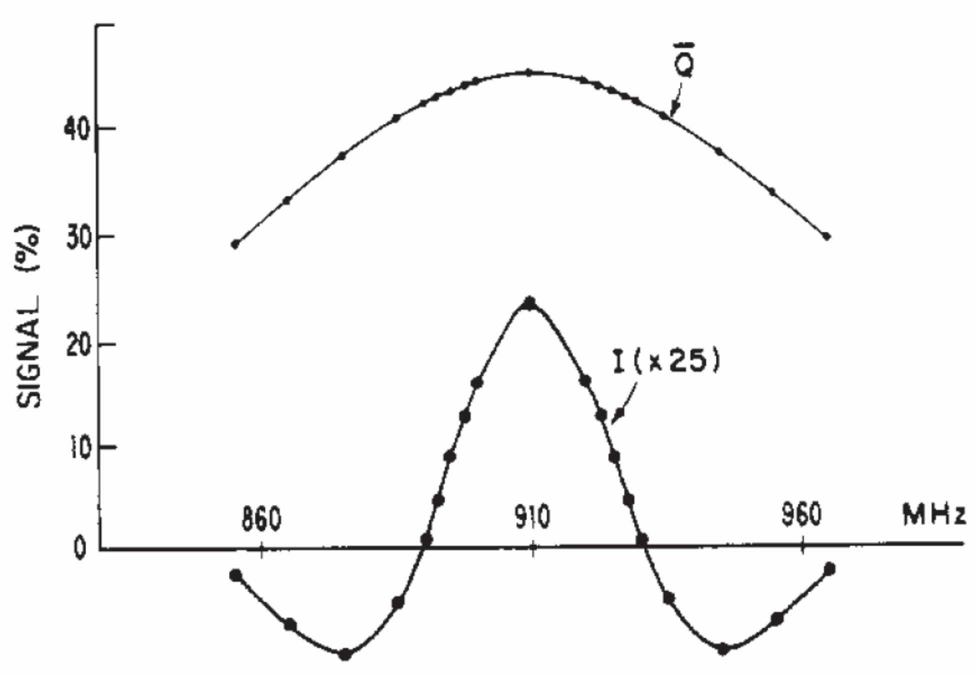
The interaction and detection region



By changing the relative phase of the two RF separated fields from 0 to 180°,
one can isolate an interference signal narrower than the natural linewidth (100 MHz)

Separated oscillatory field measurement (II)

Typical signal



$\overline{\mathrm{Q}}$: average quench signal

I : interference signal

The resonance is observed in zero magnetic field, by varying the frequency of the microwave field

Result : δ_H = 1057.845 (9) MHz

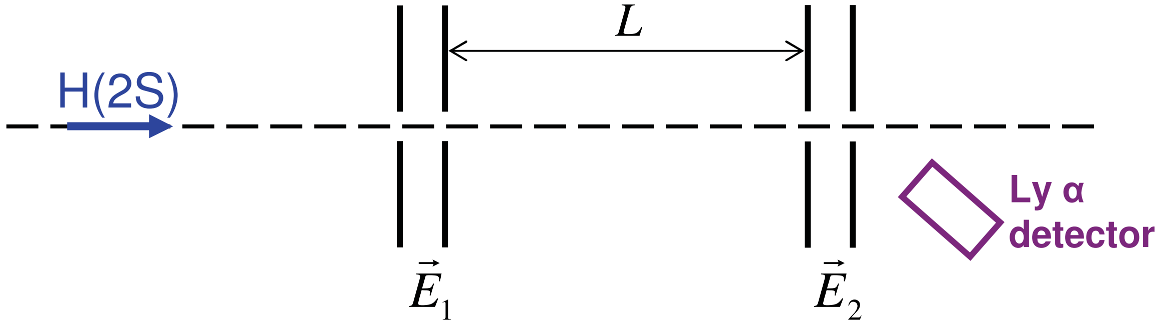
S.R. Lundeen and F.M. Pipkin, Metrologia 22, 9 (1986)

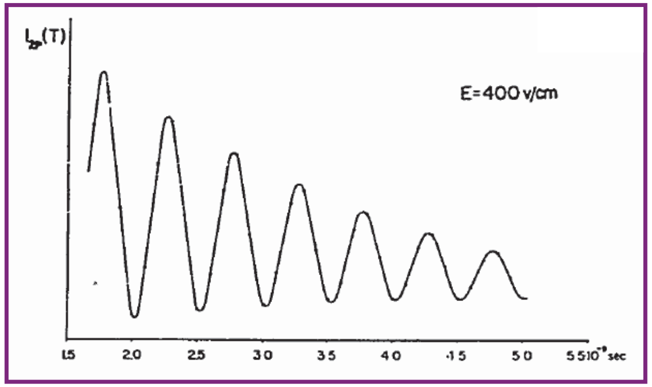
see also : E.W. Hagley and F.M. Pipkin, Phys. Rev. Lett. 72, 1172 (1994)

Other methods used to measure the 2S Lamb shift (I)

- **Atomic interferometer method**

Yu L. Sokolov and V.P. Yakovlev, Sov. Phys. JETP 56, 7 (1982)





The electric fields mix coherently the 2S and 2P states
One measures the yield of 2P state in fonction of the distance *L*,
through the spatial oscillations of the Lyman α fluorescence

This very accurate experiment measures the 2S Lamb shift in terms of the reciprocal of the lifetime of the $2P_{1/2}$ state.
It can be rather interpreted as a measurement of this lifetime

V.G. Pal'chikov, Yu L. Sokolov and V.P. Yakovlev, Physica Scipta 55, 7 (1997)
S.G. Karshenboim, Physica Scripta 57, 213 (1998)

Other methods used to measure the 2S Lamb shift (II)

- **Anisotropy method**

A. van Wijngaarden, F. Holuj and G.W.F. Drake, Can. J. Phys. 76, 95 (1998)

A fast beam of H(2S) atoms is subjected to a static electric field
The total Lyman α fluorescence is detected in two orthogonal directions

The measured anisotropy $R = (I_{||} - I_{\perp}) / (I_{||} + I_{\perp})$ is proportional to the Lamb shift

Result : δ_H = 1057.852 (15) MHz

This method, less accurate than the separated oscillatory one,
gives a result in good agreement with it.
This is a test of the validity of the method, which is also used in He^+

The 2S Lamb shift of hydrogen : discussion

It has been measured directly by microwave techniques

It provides a test of QED calculations
if both α and the charge proton radius are well known

As for the hyperfine structure, the best accuracy on the 2S Lamb shift
is now obtained by optical methods

Laser spectroscopy has opened the way to measure both 1S and 2S Lamb shifts
by comparing different optical transition frequencies.

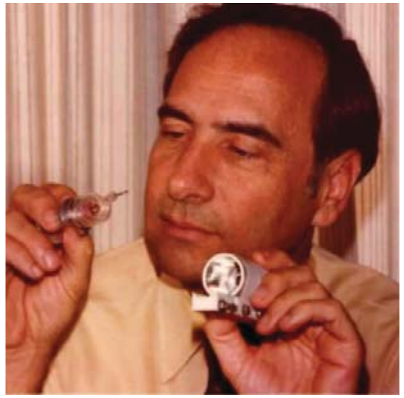
The 1S Lamb shift gives a more stringent test of QED

The Lamb shifts do not vary exactly as $1/n^3$

High resolution laser spectroscopy

In 1960 : the first laser !

pulsed ruby laser
694 nm



Theodore H. Maiman
Hughes Research Laboratories

Ali Javan, Bell Labs

In 1961, the first He-Ne laser

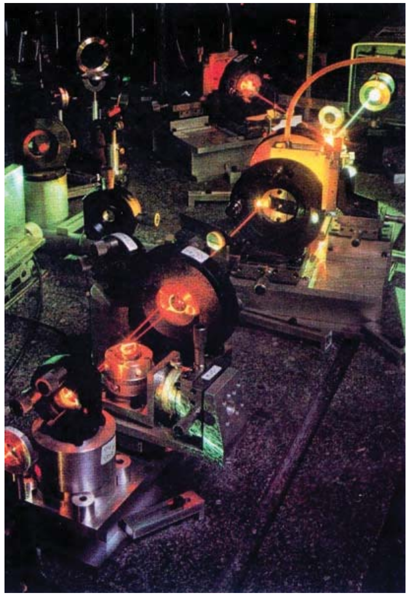
1.15 µm

In 1962 : 633 nm

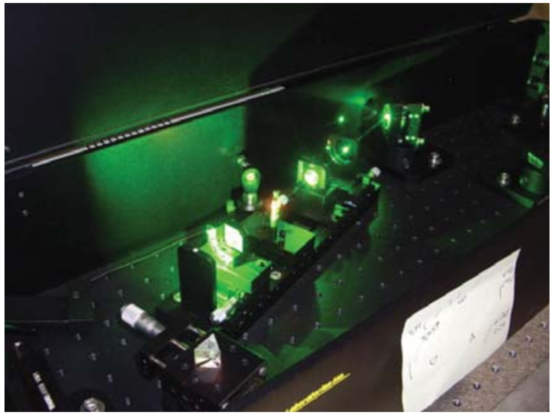
Atomic lines → fixed frequencies

Since the seventies, cw monomode tunable lasers are developped ...

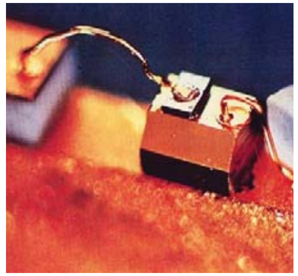
- Dye lasers ('70s)

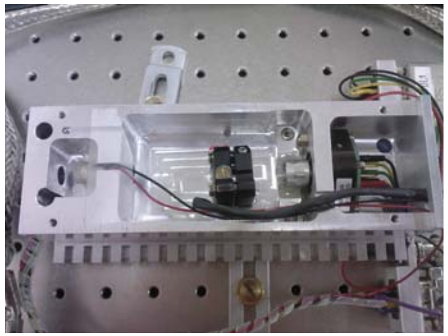


- Ti-sapphire lasers ('80s)



- Laser diodes
in extended cavities ('90s)

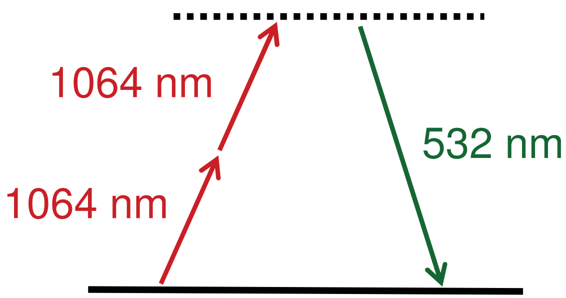


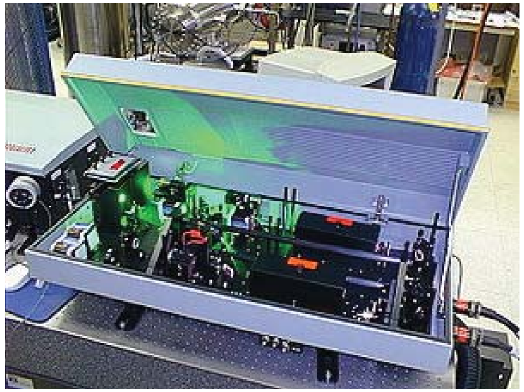


... with an ultrastable frequencies
in a wide range of wavelengths
in the optical domain

Generation of new frequencies in non-linear crystals

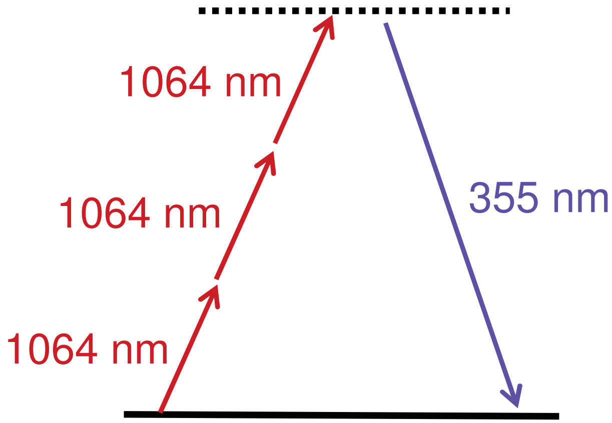
frequency doubling (SHG)



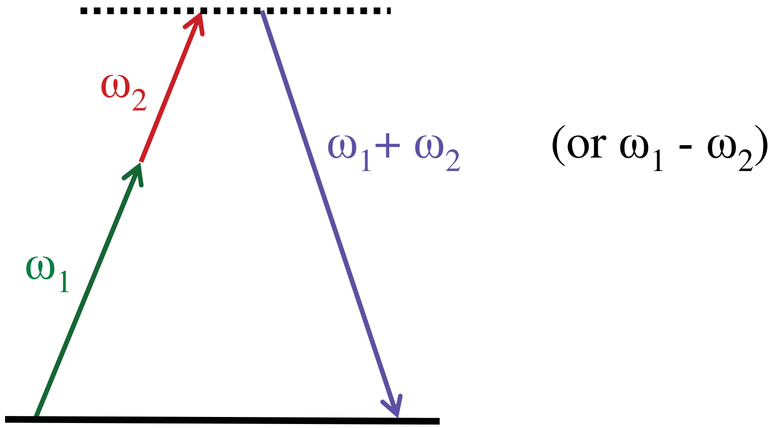


frequency doubled YAG laser

and also ... frequency tripling



... or frequency mixing

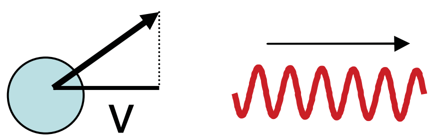


Doppler free spectroscopy

The Doppler effect

The standing atom emits or absorbs the ν_0 frequency such as :

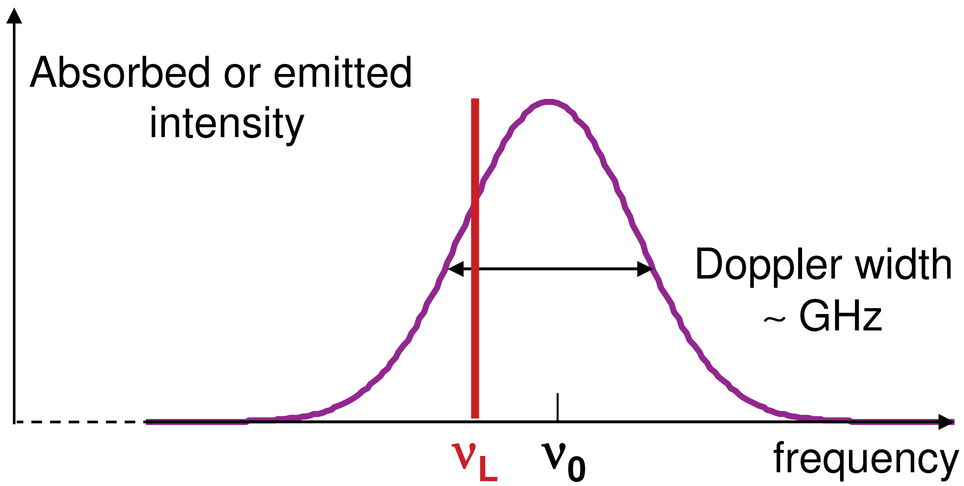
$$E_2 - E_1 = h\nu_0$$



The moving atom emits or absorbs the shifted frequency :

$$\nu = \nu_0 (1 + \mathrm{v}/c)$$

Due to the velocity distribution, the lines emitted or absorbed by atoms in a gas at room temperature are broadened by the Doppler effect



with a monomode tunable laser, one can select a given atomic velocity class in a gas

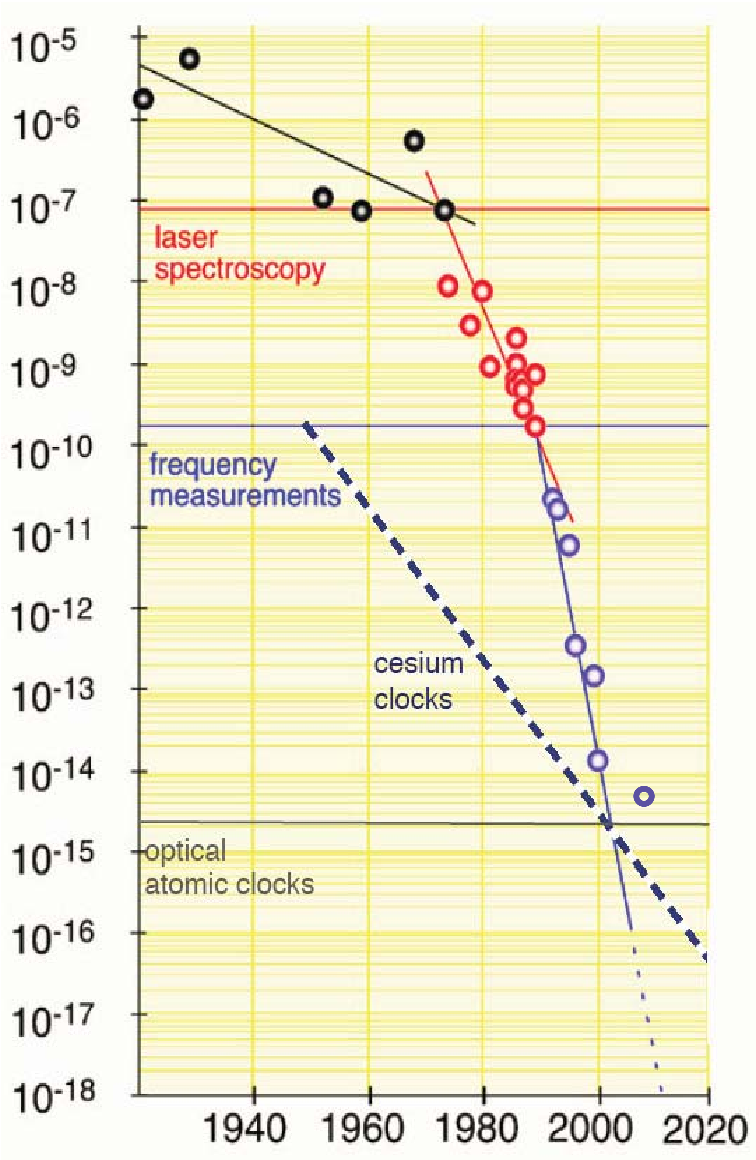
Doppler free spectroscopy in hydrogen atom

Tunable lasers have opened the way
to eliminate the Doppler effect

The precision of optical frequency measurements in hydrogen has been drastically improved

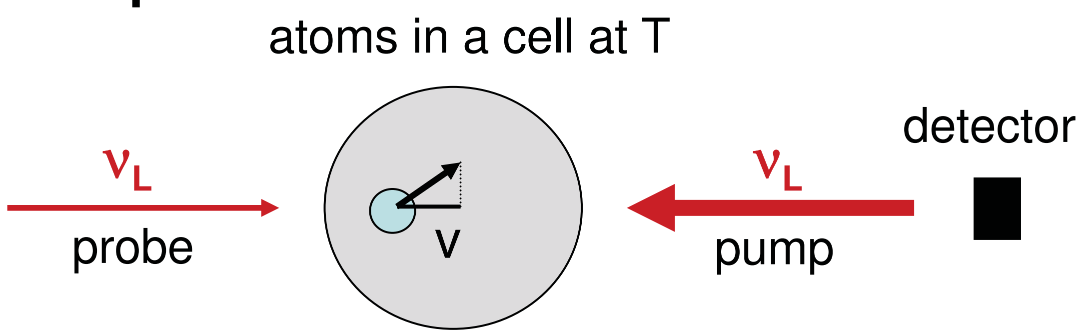
Two main methods are used,
depending on the studied transition :

- the saturated absorption
- the two-photon spectroscopy

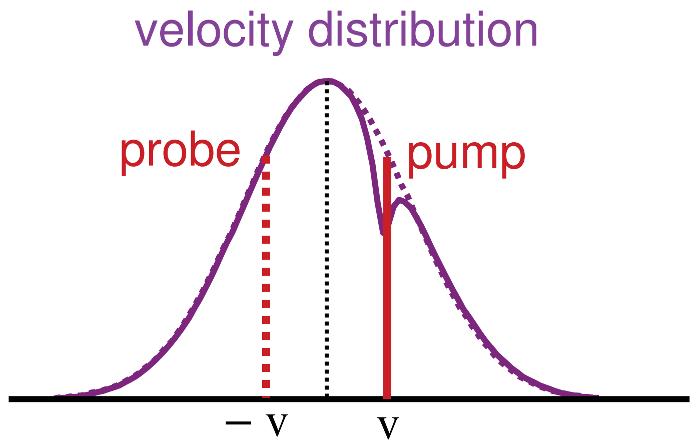


The saturated absorption method

Principle



- The velocity class $$\mathrm{v} = \frac{\nu_0 - \nu_L}{\nu_L} c$$ is depopulated by the pump beam

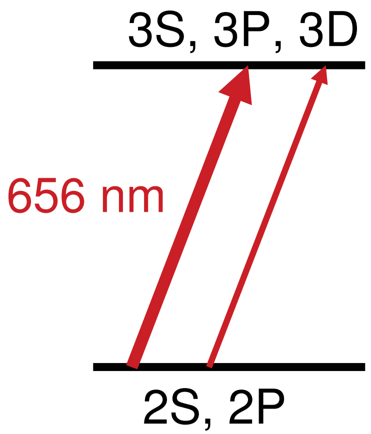


- The probe beam detects atoms having the velocity $-\mathrm{v}$

The detector compares the transmission of the probe beam with / without the pump beam

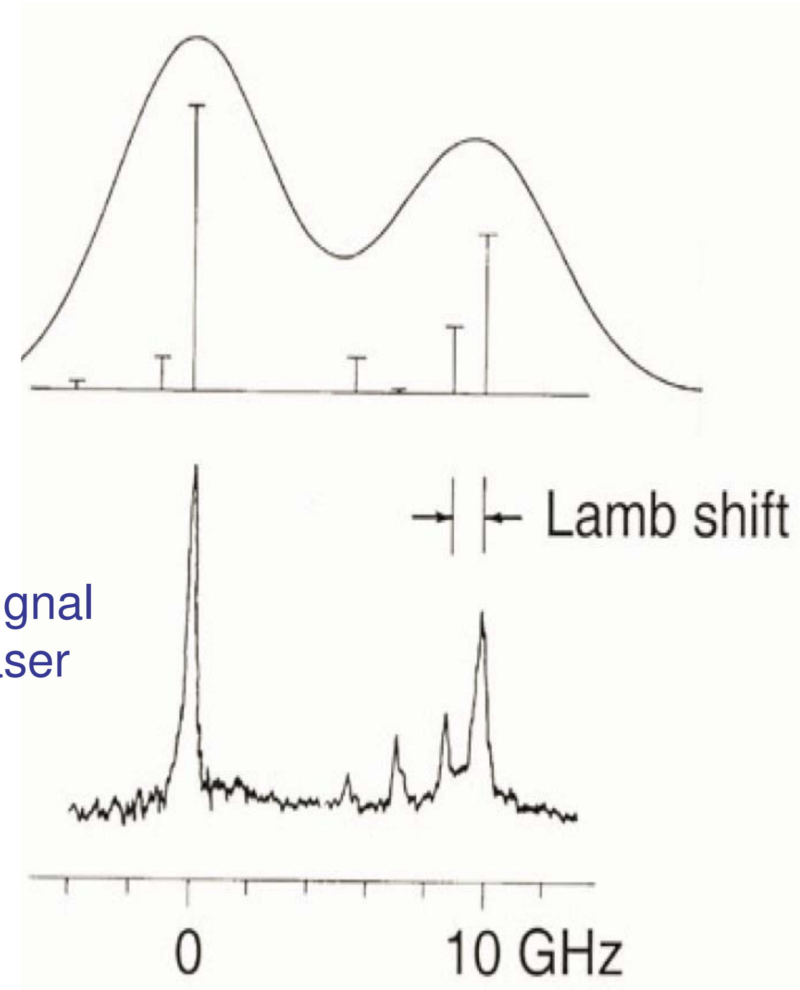
A Doppler-free signal is obtained when probe and pump beams are in resonance with the same atoms, that is for $\mathrm{v} = 0$ and $\nu_L = \nu_0$

Saturated absorption of the Balmer α transition



Doppler broadened
Balmer α line
(at 300 K)

Saturated absorption signal
recorded with a dye laser



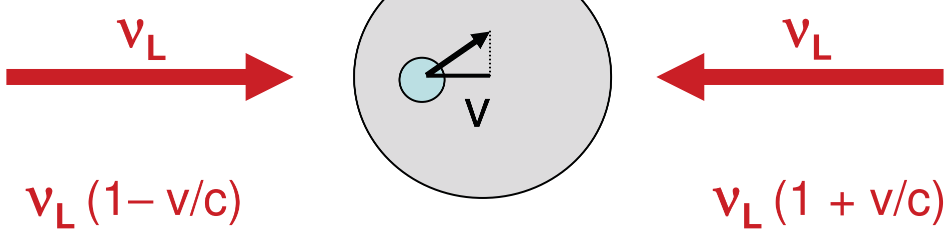
First observation of the 2S Lamb shift in the visible domain

T.W. Hänsch, I.S. Shahin and A.L. Schawlow, Nature 235, 63 (1972)

The two-photon spectroscopy method

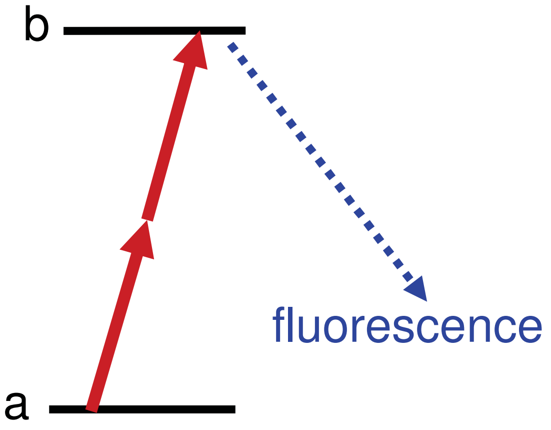
Principle

atoms in a cell or a beam



(in the laboratory frame)

(in the atom frame)



If the atom undergoes a two-photon transition by absorbing one photon from each beam, the Doppler shifts cancel and the resonance condition is :

$$E_b - E_a = h\nu_L\,(1 + v/c) + h\nu_L\,(1 - v/c) = 2\,h\nu_L$$

independent on the velocity

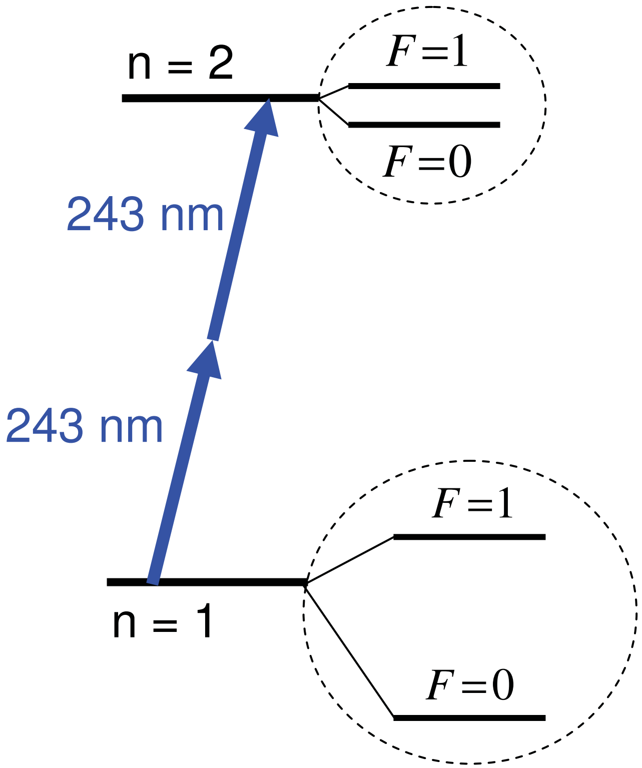
A Doppler-free signal is obtained, due to all the velocity classes
- either on the fluorescence of the b level
- or on the decrease of the a level population

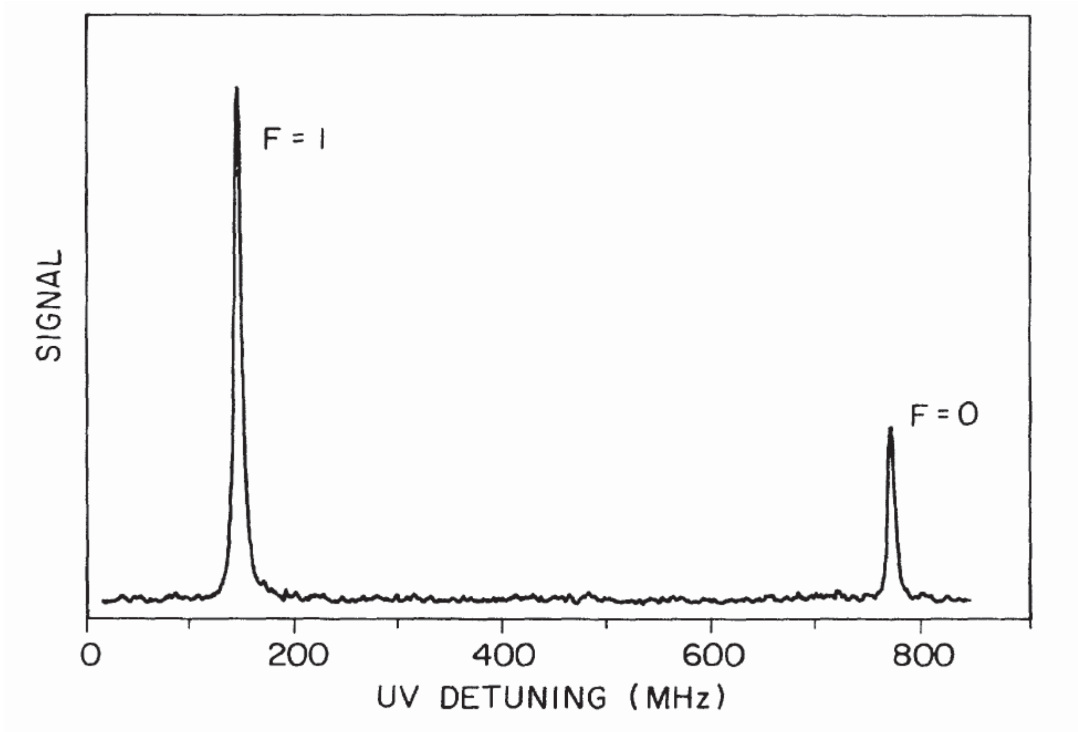
Two-photon spectroscopy of the 1S-2S transition

This transition has been extensively studied by the T.W. Hänsch's group
first in Stanford and then in Garching

natural width : 1.3 Hz !

First observation
with a cw laser : C.J. Foot, B. Couillaud, R.G. Beausoleil and T.W. Hänsch
Phys. Rev. Lett. 54, 1913 (1985)





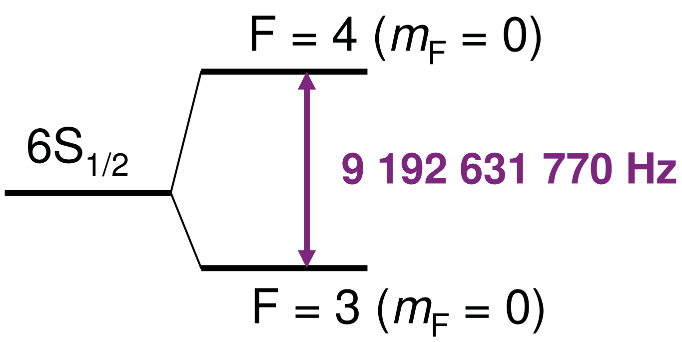
The metrology of optical frequencies

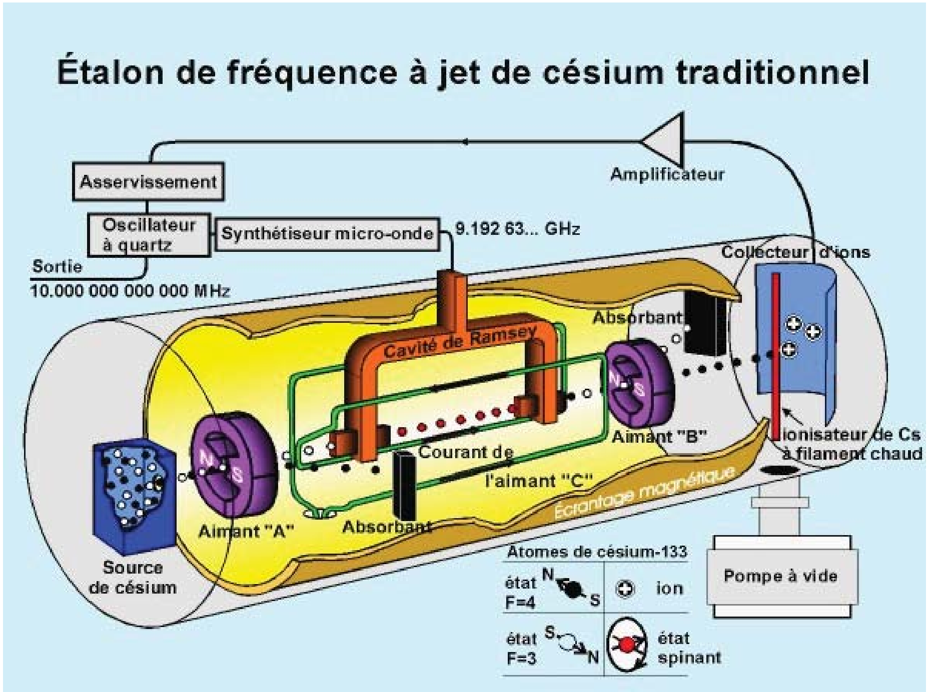
- The old method : interferometric comparison between two wavelengths
    - is a wavelength measurement (but time and length unities are connected)
    - needed a reference laser having a well known wavelength
    - was limited to few parts in 10^{10} due to interferometer mirrors

- The modern method : direct frequency comparison with the frequency standard

The Cs atomic clock

The definition of the second (1967) is derived from the hyperfine ground state transition frequency of ^{133}Cs





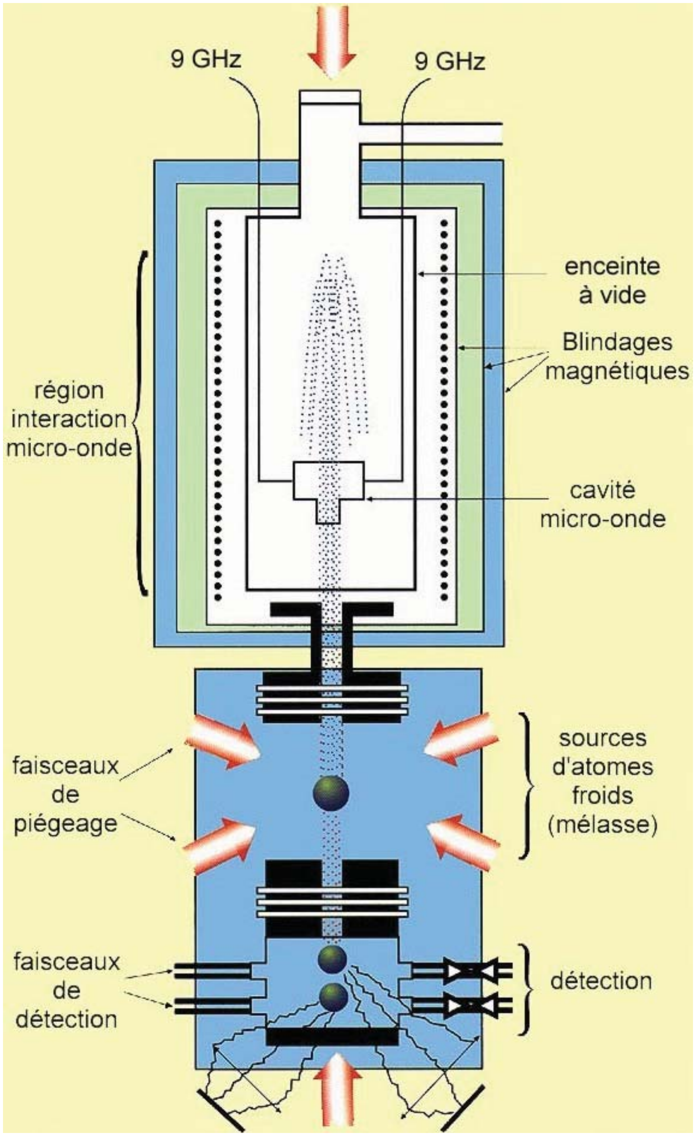
The Cs atomic fountain

Various laser beams are used :
- to select a given sublevel
- to detect the transition
- for cooling and trapping of atoms

Improvements thanks to cold atoms :
- increase of the interaction time between atoms and microwave
- reduction of the Doppler effect

Stability and accuracy are improved
by a factor of the order of 100
$\rightarrow \sim 10^{-16}$

J. Guena et al. , Phys. Rev. Lett. 106, 130801 (2011)
[and quoted references]

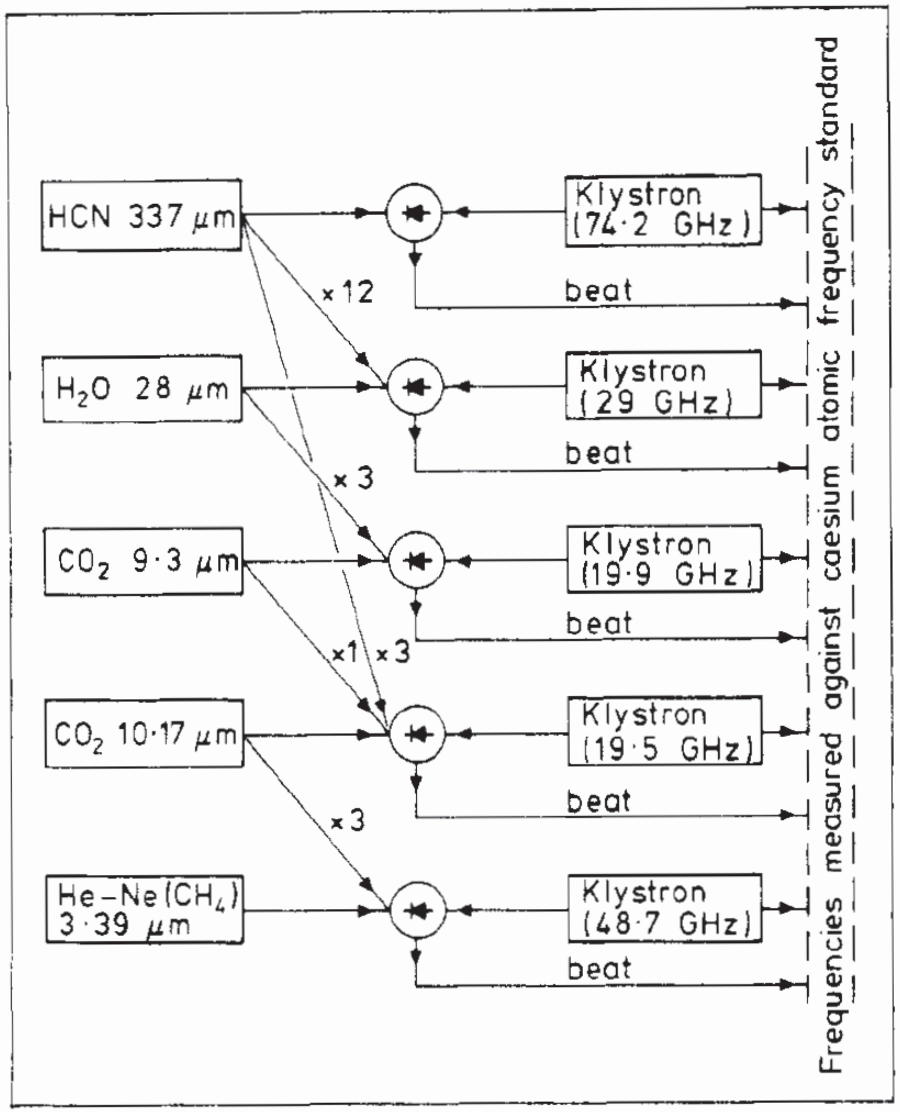


The measurement of an optical frequency

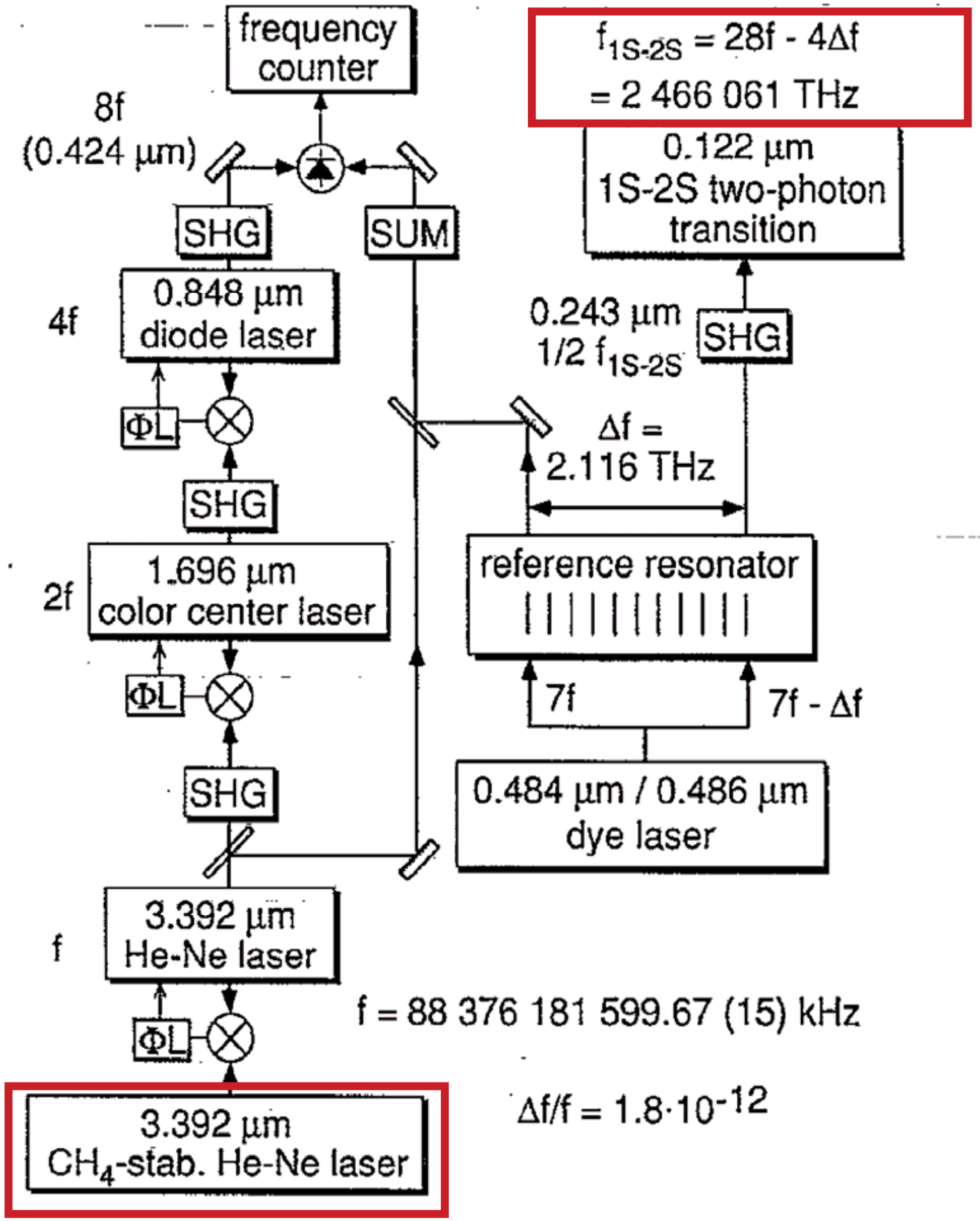
by comparison with the Cs clock at 9.2 GHz

Until the '90s, sophisticated frequency chains were used including :
- a lot of lasers at intermediate frequencies
- a lot of non linear devices for frequency multiplication and comparison

Scheme of a frequency chain used in 1980 to measure the frequency of the CH_4 stabilized He-Ne laser at 88 THz



Frequency measurement of the 1S-2S hydrogen transition (I)



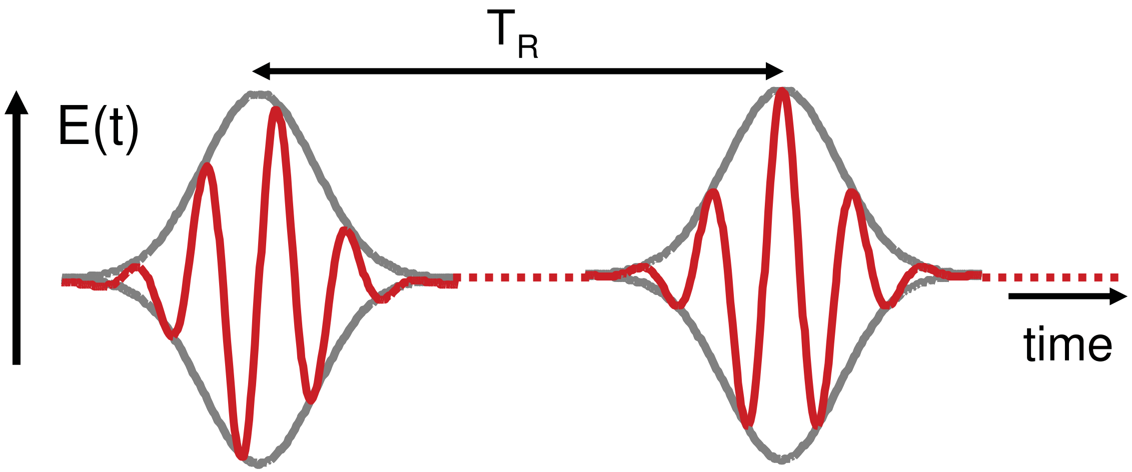
two large rooms experiment !

T. Andreae *et al.*
Phys. Rev. Lett. **69**, 1923 (1992)

Frequency measurement with an accuracy of 1.8 x 10^{-12}

The measurement of an optical frequency with a frequency comb

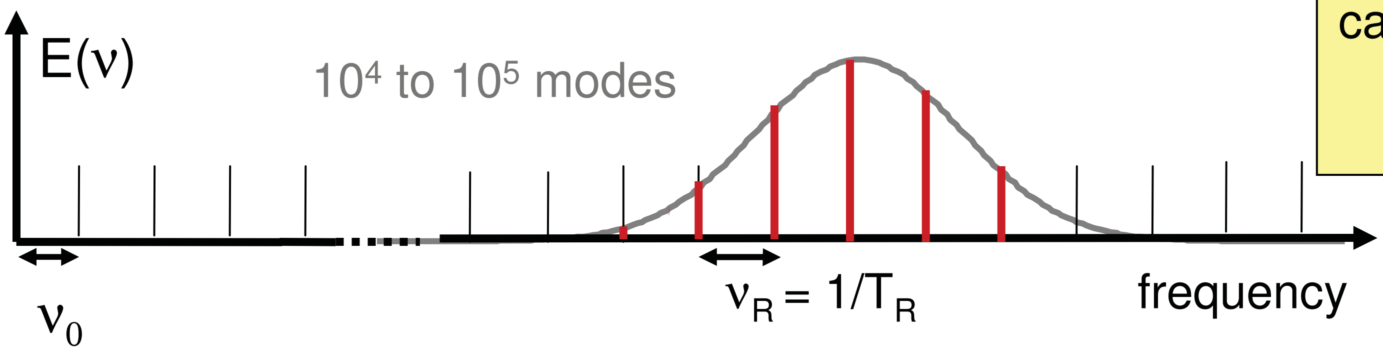
A mode locked fs laser delivers a train of short pulses



λ = 600 nm
ν = 500 THz
T = 2 fs

only a few oscillations per pulse

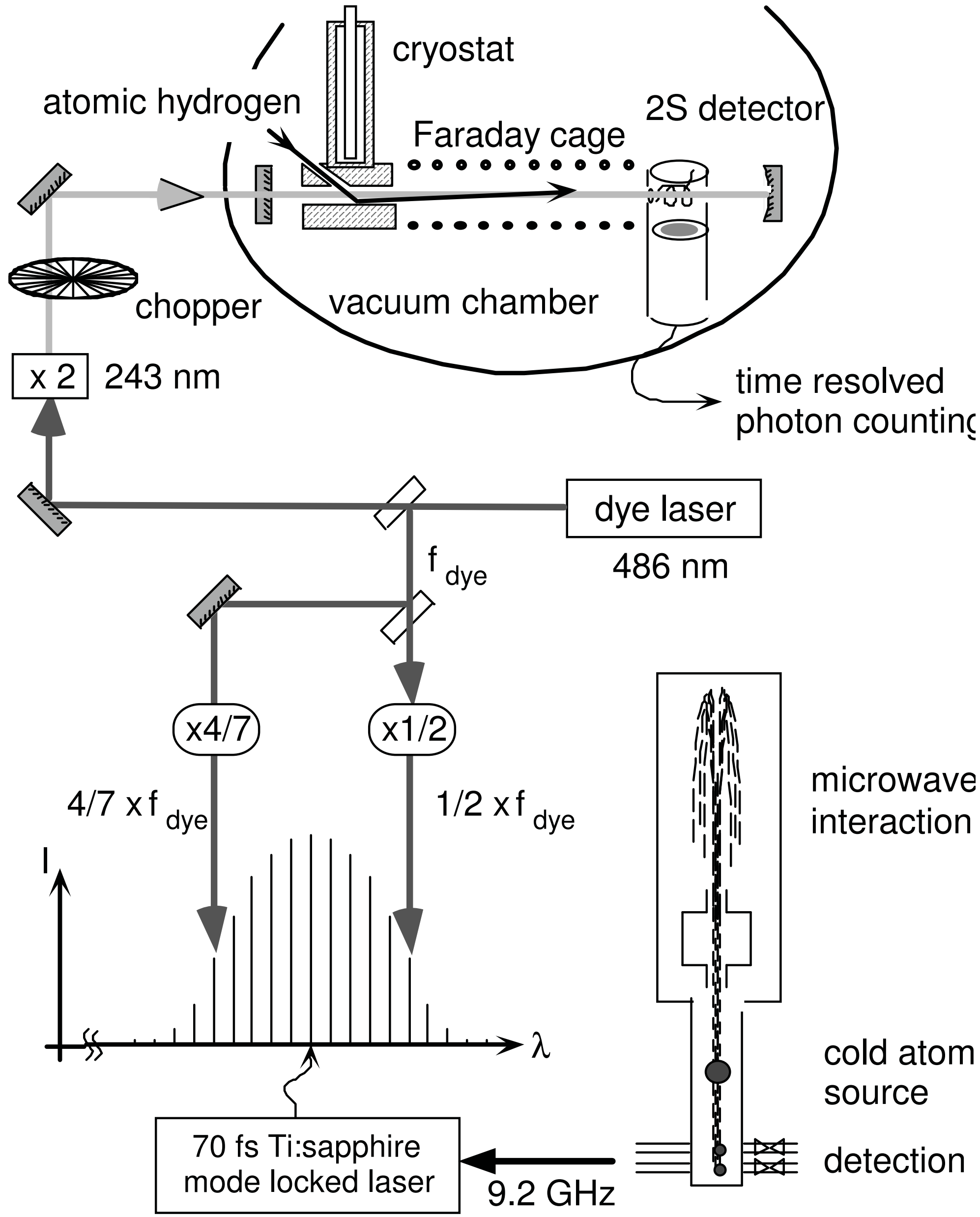
To produce short pulses, the laser must oscillate on a very large number of coherent modes ; in the frequency domain, one obtains a comb of regularly spaced modes :



cavity length 30 cm
T_R = 1 ns
ν_R = 1 GHz

The spacing of the comb (repetition rate) is locked to the Cs clock

Absolute frequency measurement of the 1S-2S hydrogen transition (II)



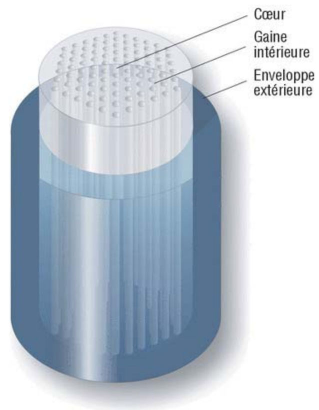
« Measurement of the hydrogen 1S-2S transition frequency by phase coherent comparison with a microwave cesium fountain clock »

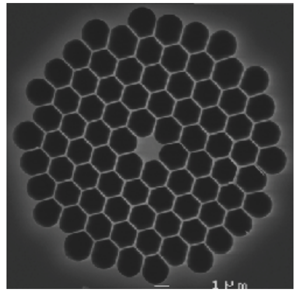
M. Niering *et al.*,
Phys. Rev. Lett. **84**, 5496 (2000)

Frequency measurement with an accuracy of 1.9×10^{-14}

Spanning of the frequency comb ...

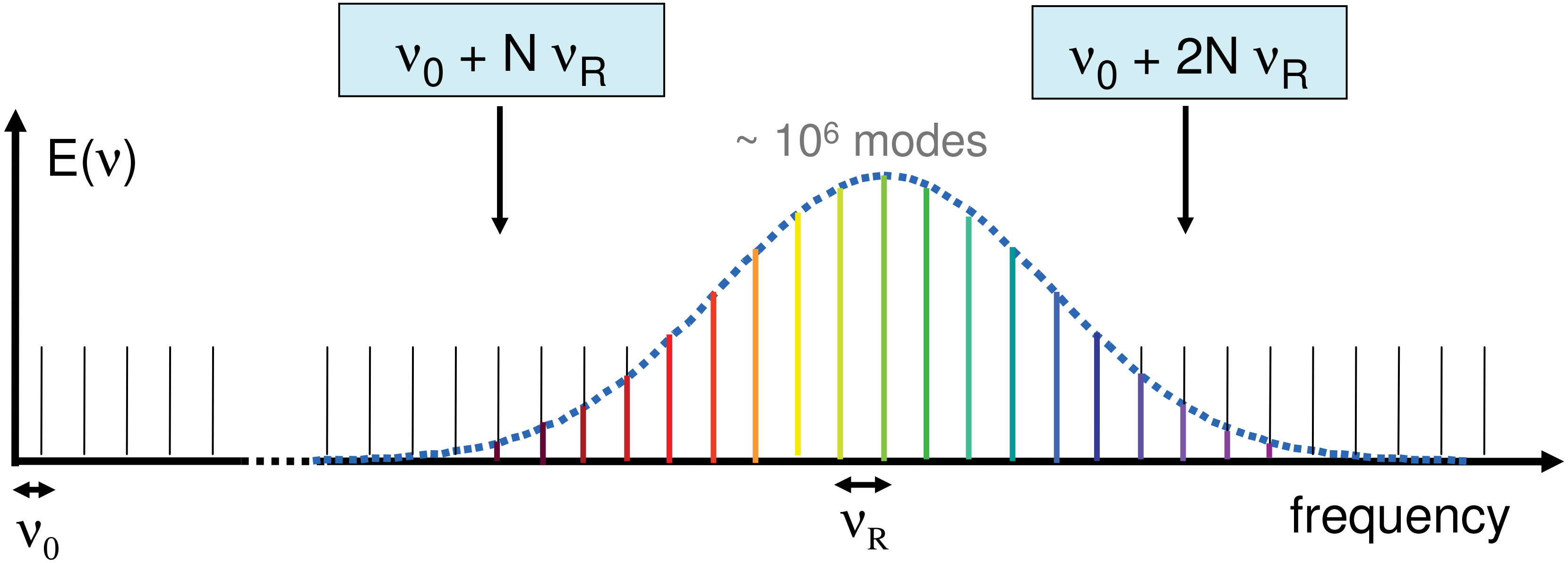
The laser spectrum is broadened
over more than one octave
by propagation in a crystal photonic fiber





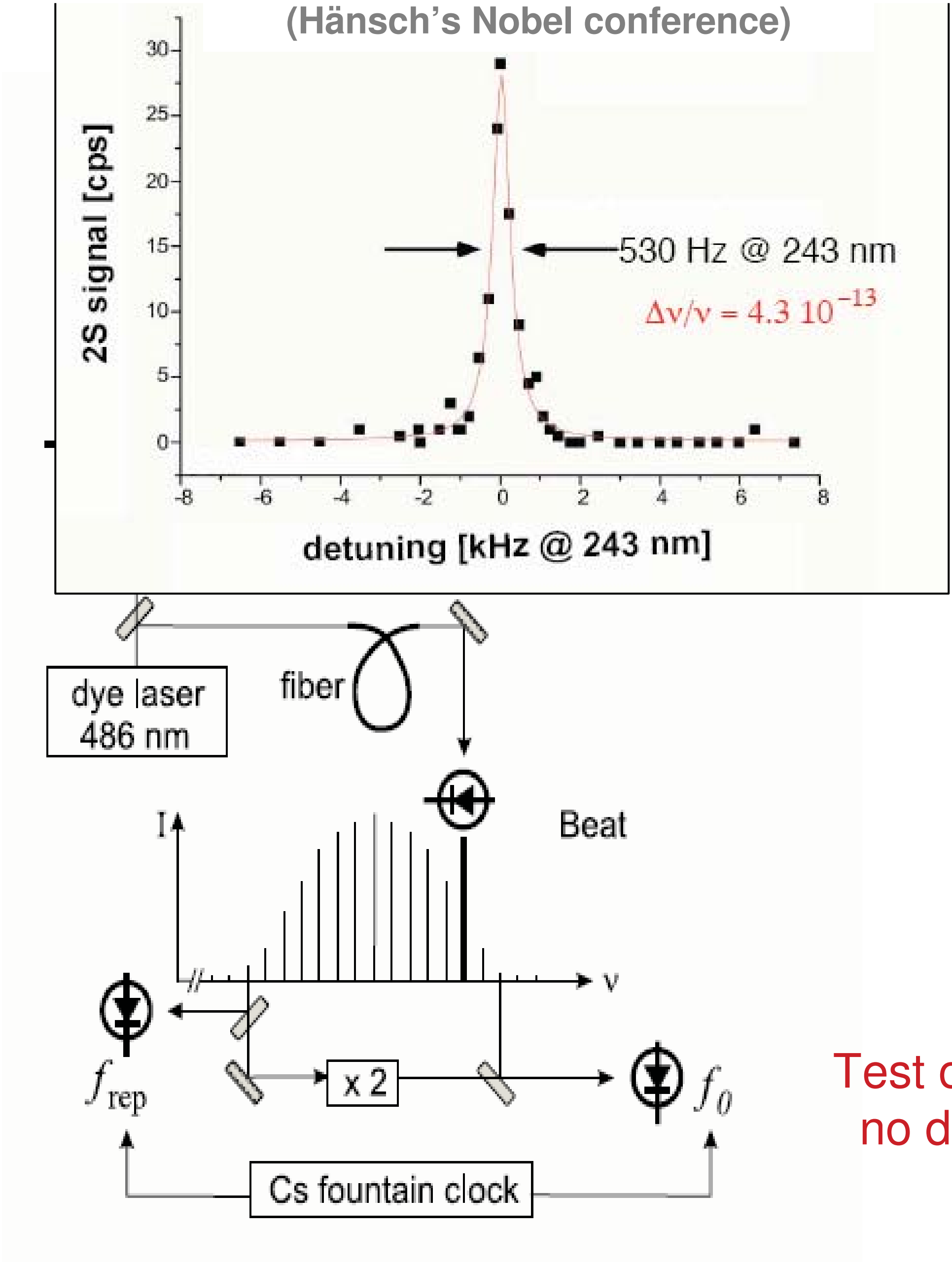
fiber core 1.7 μm

... and self-referencing



The IR part of the comb is frequency doubled and compared to the blue part
by beat note : the offset frequency ν_0 is then determined

Absolute frequency measurement of the 1S-2S hydrogen transition (III)



Nobel Prize 2005 half awarded jointly to J. L. Hall and T.W. Hänsch

"for their contributions to the development of laser-based precision spectroscopy, including the optical frequency comb technique"

table experiment !

Frequency measurement with an accuracy of 1.4×10^{-14}

M. Fischer *et al.*
Phys. Rev. Lett. 92, 230802 (2004)

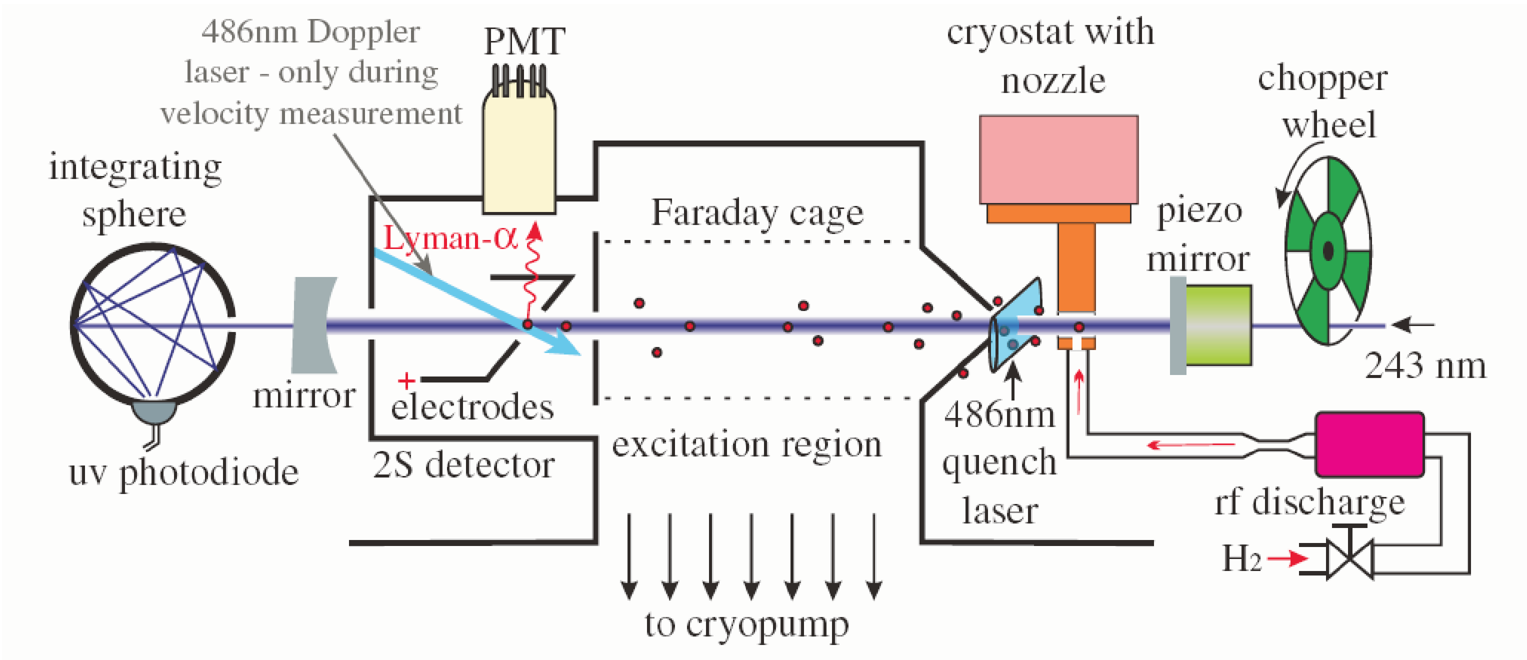
Test of the stability of fundamental constants : no drift observed at a level of 10^{-15} per year

Absolute frequency measurement of the 1S-2S hydrogen transition (IV)

Recent improvements of the experiment

→ frequency measurement with an accuracy of 4.2×10^{-15}

- improved laser stability
- better control of systematic effects (2^{nd} order Doppler, ac and dc Stark shifts)

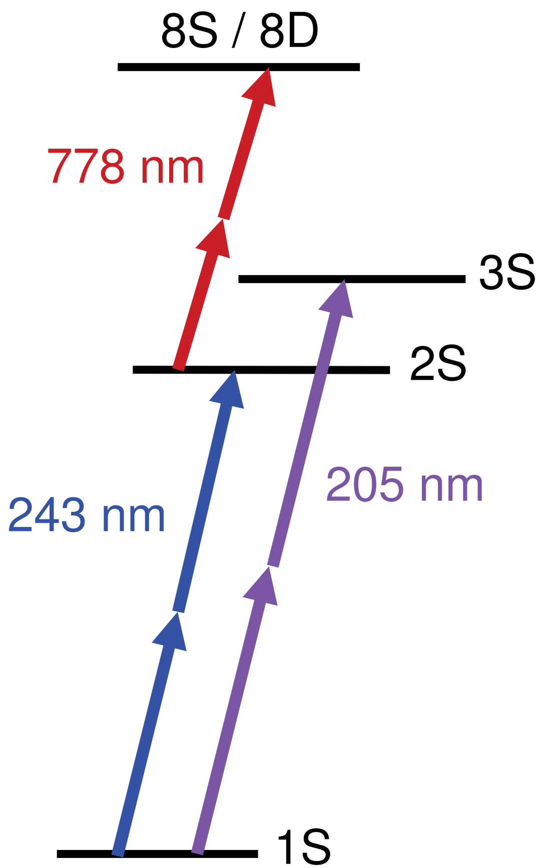


C.G. Parthey *et al.*, Phys. Rev. Lett. 107, 203001 (2011)

f (1S-2S) = 2 466 061 413 187 035 (10) Hz

Two-photon spectroscopy in hydrogen

The 1S-2S two-photon transition has been measured at a very high level of precision
but the determination of the Rydberg constant and of the Lamb shifts
needs the comparison of different optical frequencies



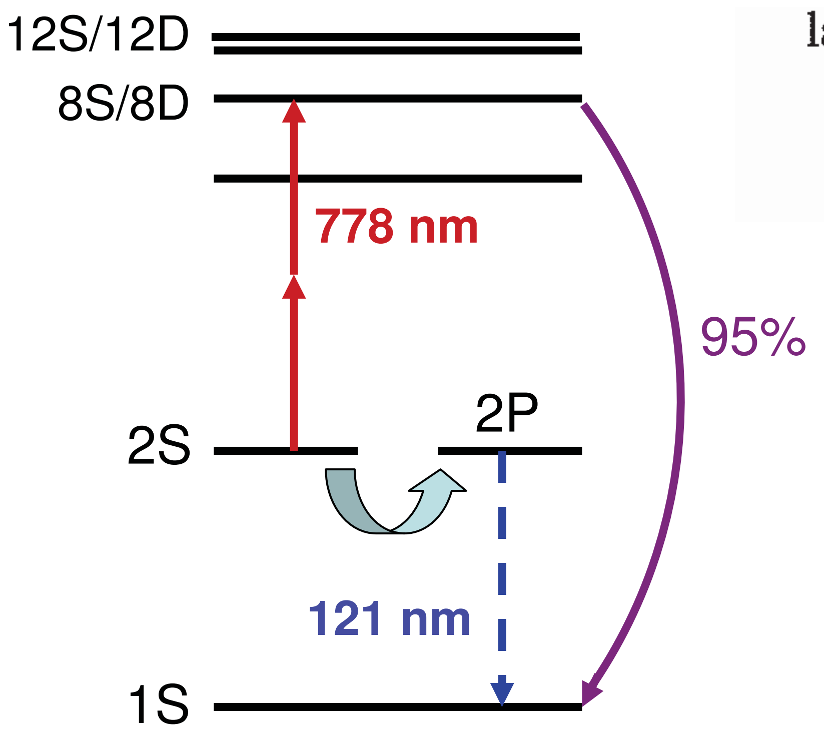
Our group in Paris has studied :

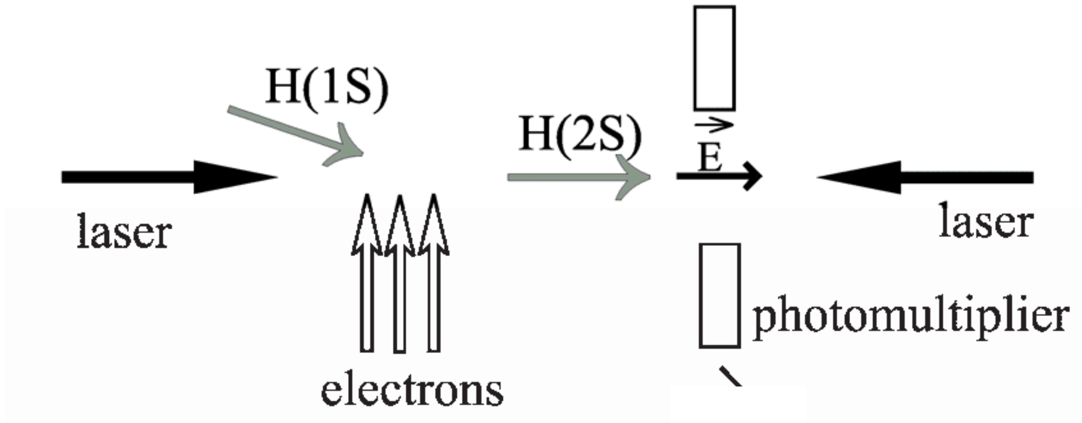
the 2S-nS et 2S-nD two-photon transitions
from the metastable state
towards the n = 8 and 12 levels

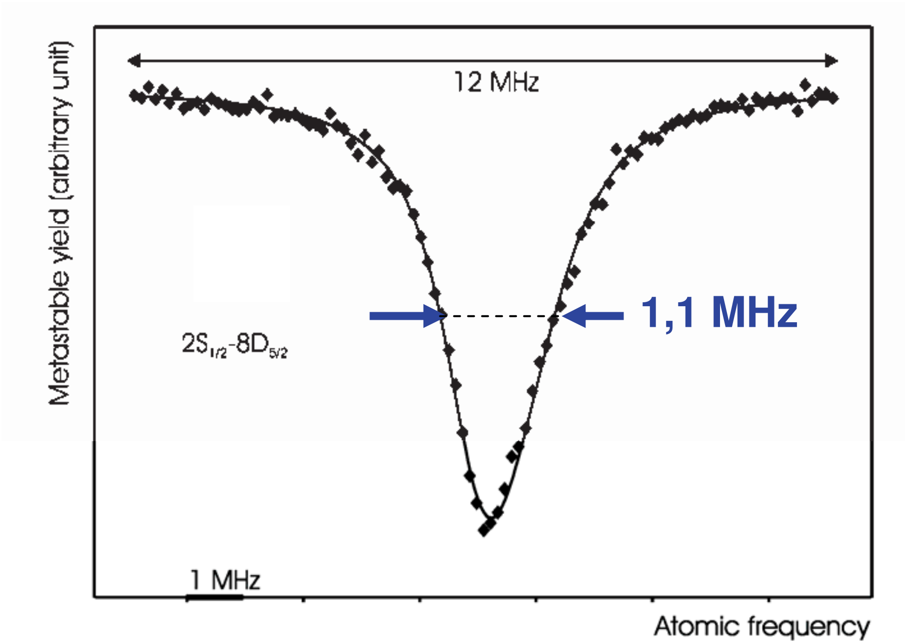
and the 1S-3S two-photon transition
from the ground state

The 2S-nS and 2S-nD transitions

studied in Paris (LKB)





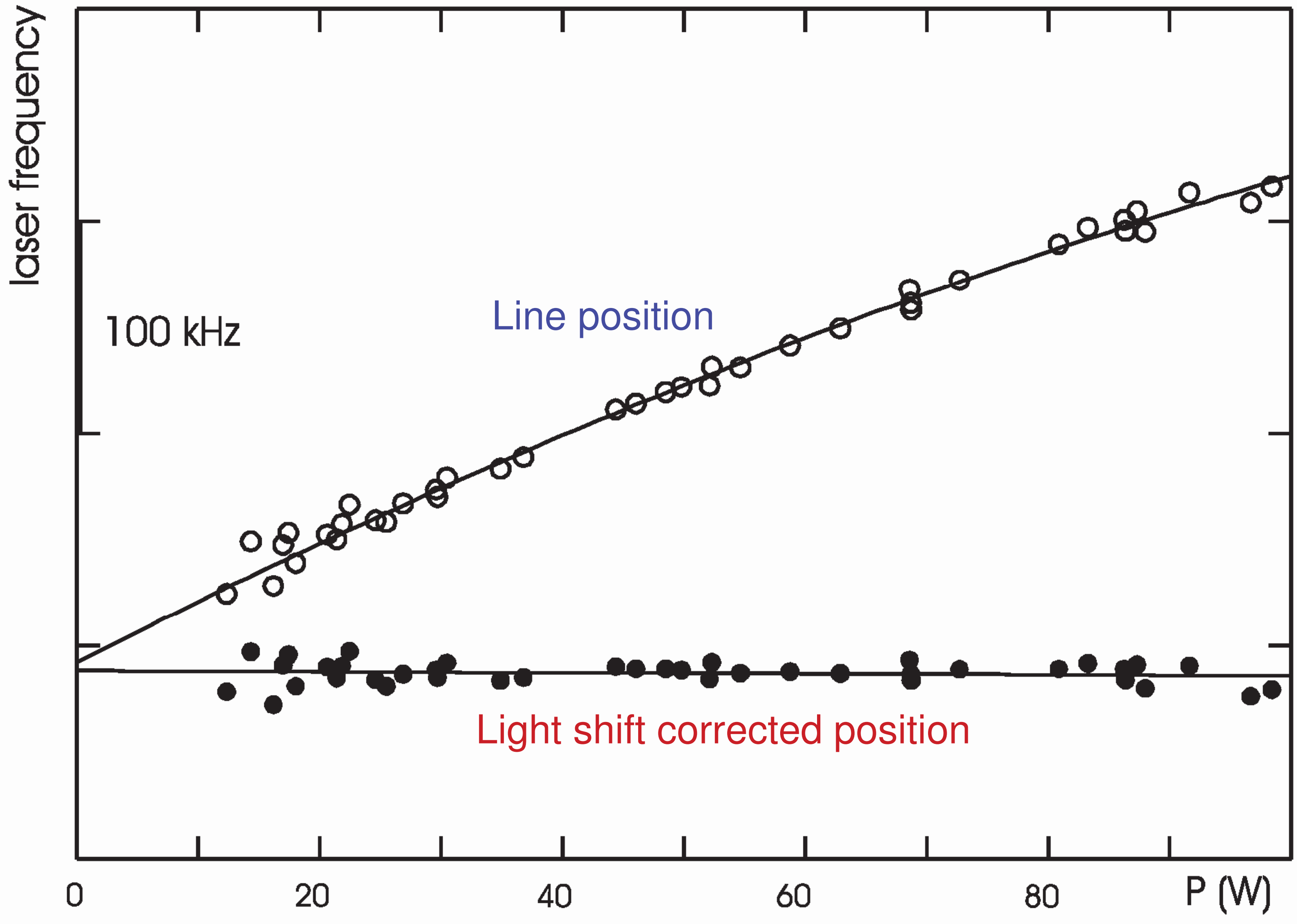


Natural width : 100 - 500 kHz

Typical line width : 1 MHz

mainly due to inhomogenous light shift

Light shift of the $2S_{1/2}$-$8D_{5/2}$ transition

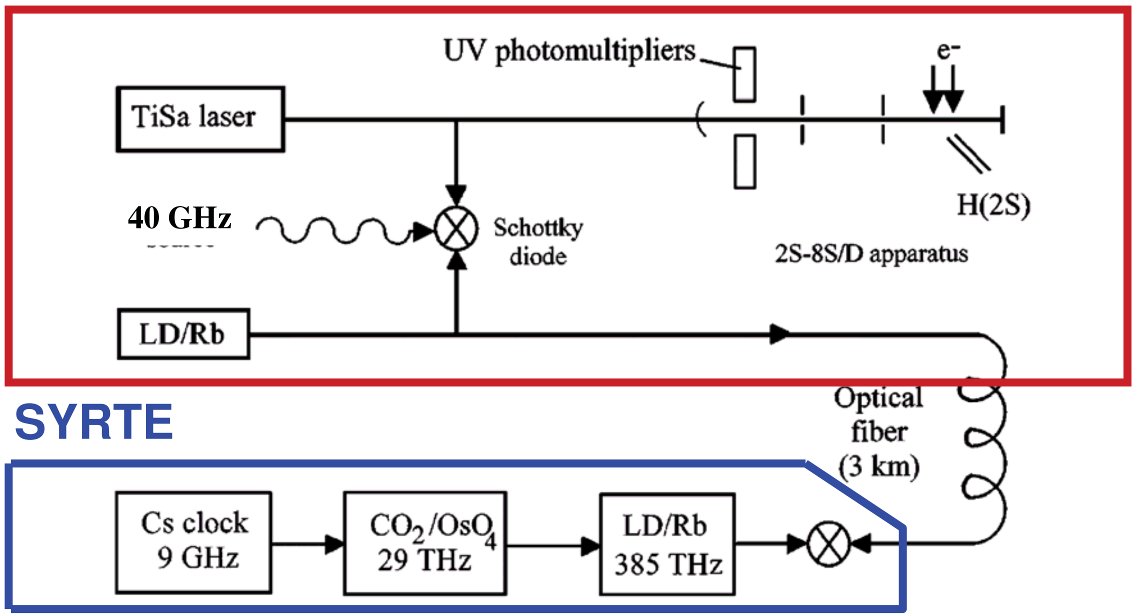


Measurement of the 2S-8S and 2S-8D frequencies (SYRTE-LKB)

First pure frequency measurement in 1993

Use of a two-photon rubidium frequency standard
3 km long optical fiber between the two labs

LKB



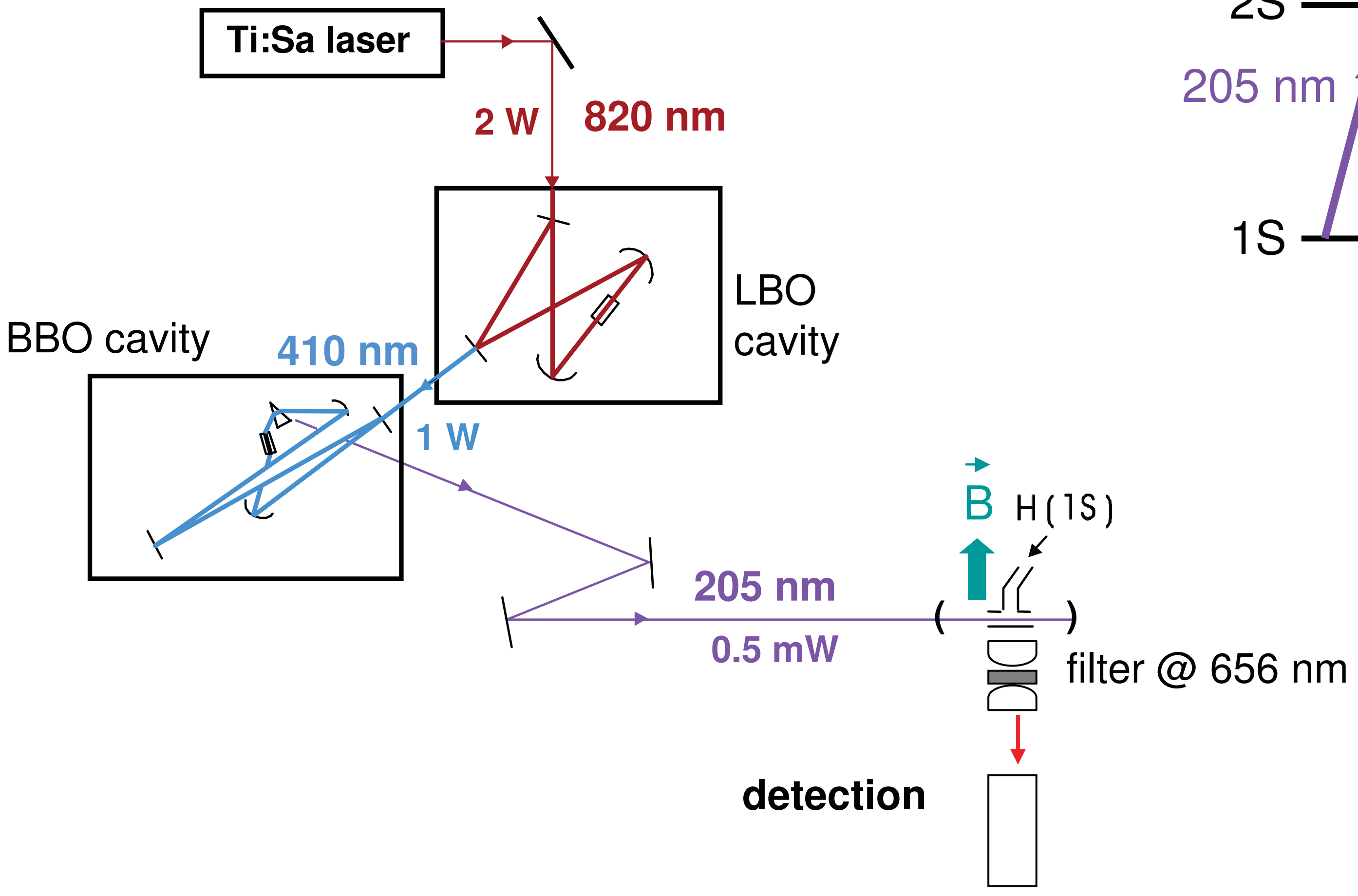
B. de Beauvoir *et al.*, Phys. Rev. Lett. 78, 440 (1997)
and Eur. Phys. J. D 12, 61 (2000)

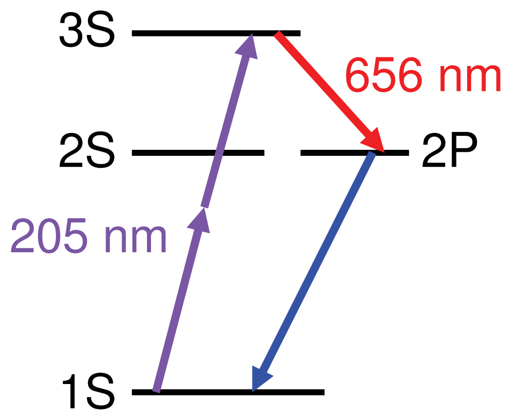
$f\,(2S_{1/2}\text{-}8D_{5/2})$ = 770 649 561 581.1 (5.9) kHz

relative uncertainty 7.6×10^{-12}

Study of the 1S - 3S transition

Need of a tunable source at 205 nm





Study of the 1S - 3S transition

Signal First observation by our group in 1995

The frequency comparison of the 1S-3S and 2S-6S/6D transitions led to a determination of the 1S Lamb shift

S. Bourzeix et al., Phys. Rev. Lett. **76**, 384 (1996)

Compared to the 2S - nD transitions

- much larger number of atoms (1S in place of 2S) ~ 10^8
- negligible light shift (low laser power)
- but large second-order Doppler effect (thermal beam)

$$\Delta\nu_{Doppler} = -\frac{v^2}{2\,c^2}\,\nu_0$$

~ 146 kHz

Original method

A transverse magnetic field is applied which is responsible for a motional electric field

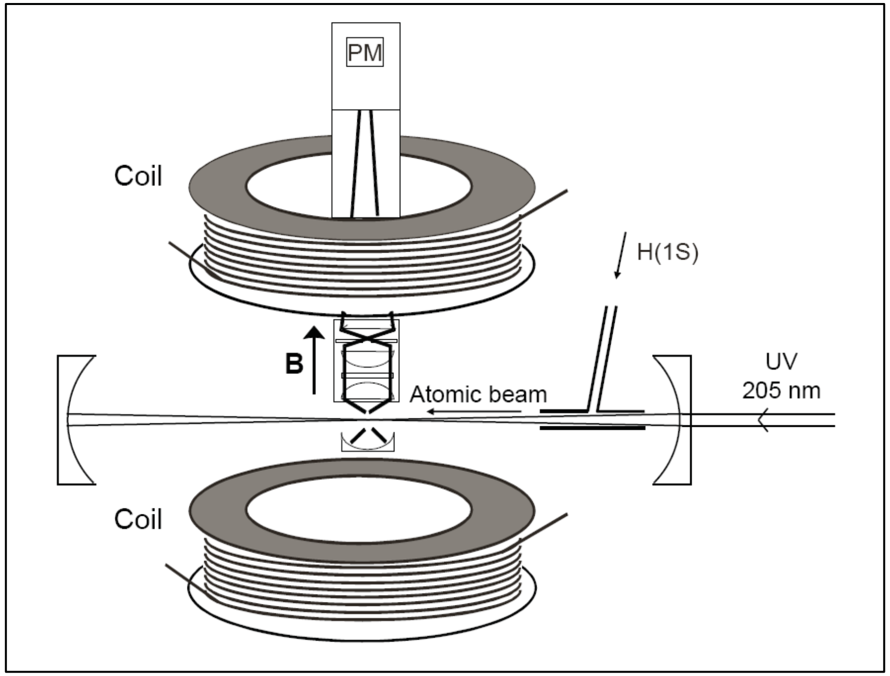
$$\vec{E} = \vec{v} \times \vec{B}$$

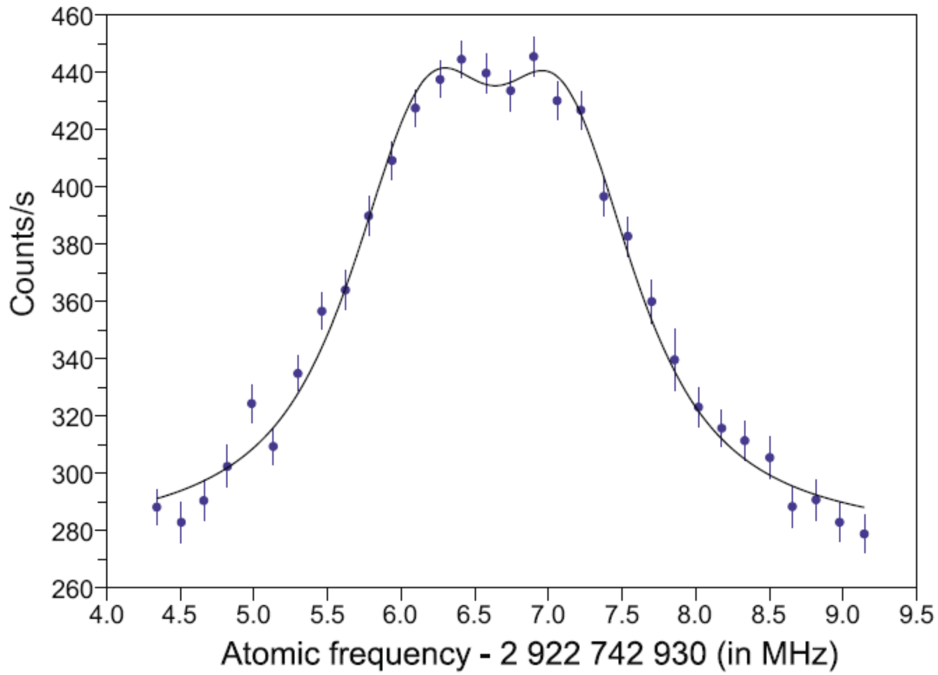
The quadratic Stark shift induced is v^2 dependent and used to partially compensate the second order Doppler effect

F. Biraben, L. Julien, J. Plon and F. Nez, Europhys. Lett. 15, 831 (1991)

G. Hagel et al., Phys. Rev. Lett. 89, 203001 (2002)

Experimental arrangement





Fit of the theoretical profile to the data

Frequency measurement using a frequency comb

f (1S-3S) = 2 922 742 936.729 (13) MHz

relative uncertainty 4.5 x 10^{-12}

After the 1S-2S frequency, this is the 2nd most precisely known frequency in hydrogen atom

O. Arnoult, F. Nez, L. Julien and F. Biraben, Eur. Phys. J. D 60, 243 (2010)

The optical transitions of hydrogen : discussion

Can we test QED predictions with the measured frequencies of atomic hydrogen ?

The energy levels of hydrogen can be written as the sum of four terms :

- The Dirac energy depending on n and j
- The first recoil term (varying as $1/n^4$) depending on n (and j)

These both terms are exactly known and can be expressed as

$$h\,c\,R_\infty\; f\left(\alpha, \frac{m_e}{m_p}; n, l, j\right)$$

- The Lamb shift depending on n, l and j

It includes :

- QED radiative corrections
- relativistic recoil
- nuclear size effect

} ← to be tested

- The hyperfine structure

The optical transitions of hydrogen allow to determine the Rydberg constant and the Lamb shifts

The Rydberg constant

There are several way to deduce the Rydberg constant from the 1S-2S interval, the 2S-nD interval or from their combination

B. de Beauvoir *et al*, Eur. Phys. J. D 12, 61 (2000)

F. Biraben et al., in « The Hydrogen atom : precision physics of simple atomic systems » Springer (2001)

F. Biraben, Eur. Phys. J. Special topics 172, 109 (2009)

- From the 1S-2S frequency, using Lamb shifts deduced from QED calculations
  → uncertainty ~1.8×10^{-11} mainly limited by the proton size

- From the 2S-nD frequencies, using the 2S Lamb shift from QED calculations
  → uncertainty ~1.1×10^{-11} mainly limited by the frequency measurement and the proton size

- From the 2S-nD frequency, using the measured 2S Lamb shift
  → uncertainty ~1.2×10^{-11} mainly limited by 2S Lamb shift mesurement
  independent from the proton size

The Rydberg constant

- From the 1S-2S and 2S-nD intervals, using the scaling law of the Lamb shift

The Lamb shifts vary aproximatively as $1/n^3$; the deviation from this law is given by Δ_2

$$\Delta_2 = L_{1S_{1/2}} - 8L_{2S_{1/2}}$$

This quantity has been calculated very precisely and is independent from the proton size

S.G. Karshenboim, J. Phys. B 29, L29 (1996) ; Z. Phys. D 39, 103 (1997)

A. Czarnecki, U.D. Jentschura and K. Pachucki, Phys. Rev. Lett. 95, 180404 (2005)

It is then possible to form a linear combination to eliminate these Lamb shifts

$$7f\left(2S_{1/2} - 8D_{5/2}\right) - f\left(1S_{1/2} - 2S_{1/2}\right) \approx \left(\frac{57}{64}\right) c R_\infty + 7L_{8D_{5/2}} + \Delta_2$$

This method needs neither the 2S Lamb shift nor the proton charge radius and is applicable to hydrogen and deuterium

The result in hydrogen is : $R_\infty = 10\,973\,731.568\,528\,(94)\ \mathrm{cm}^{-1}$

with a relative uncertainty of 8.6×10^{-12}

The Rydberg constant

- From a least square adjustment
  - it can be done using only the hydrogen data

    the values of α and m_e/m_p being given a priori
  - or including data concerning all the fundamental constants

Since 1998, the CODATA (Committee on Data for Science and Technology) uses this method to determine the Rydberg constant

P.J. Mohr and B.N. Taylor, Rev. Mod. Phys. 72, 351 (2000)
Rev. Mod. Phys. 77, 1 (2005)
P.J. Mohr, B.N. Taylor and D.B. Newell, Rev. Mod. Phys. 80, 633 (2008)

The result obtained in the 2010 CODATA adjustment is :

$$R_\infty = 10\,973\,731.568\,539\,(55)\ \mathrm{cm}^{-1}$$

with a relative uncertainty of 5.0×10^{-12}

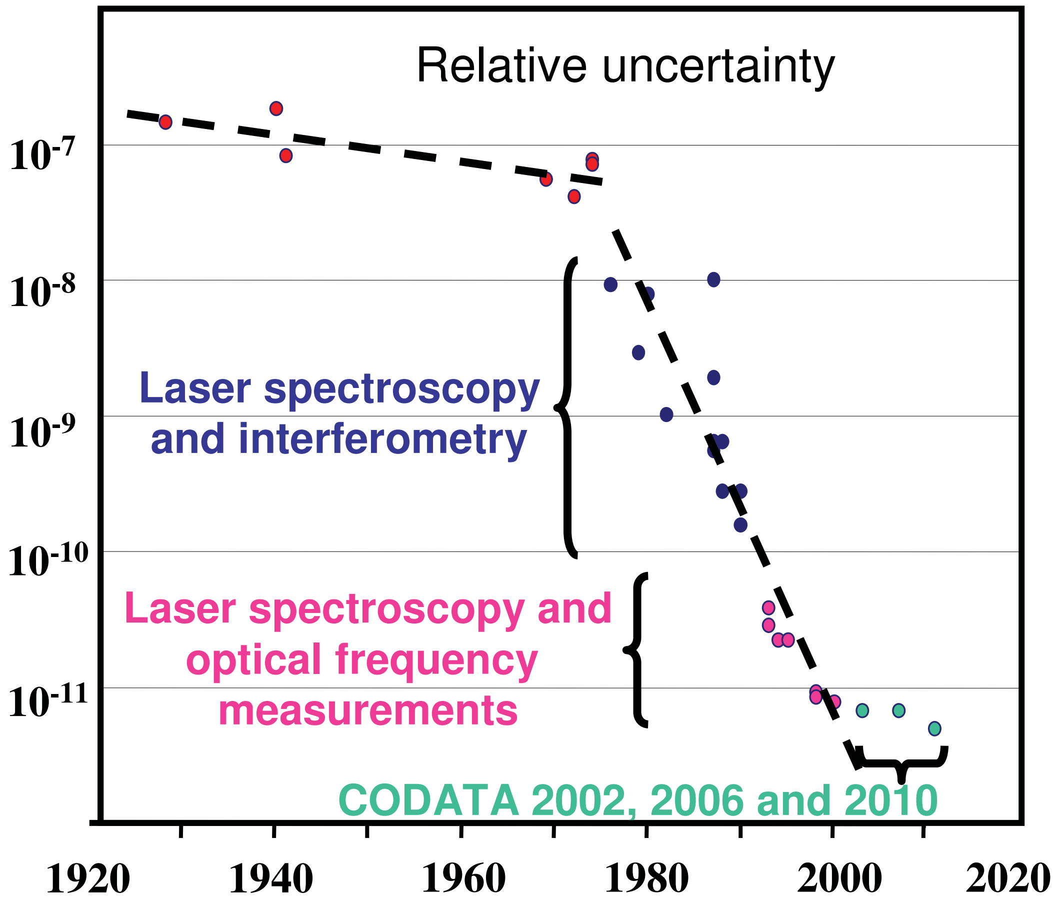
P.J. Mohr, B.N. Taylor and D.B. Newell, arXiv:1203.5425 (2012)

The Rydberg constant : conclusion

Although the 1S-2S frequency has been measured with an accuracy better than 1 part in 10^{14}, the best determination of the Rydberg constant is now limited to ~ 5 x 10^{-12}

- because of the knowledge of the proton charge radius

- because of the limitations of the 2S - nS/nD measurements



It is why experiments have been undertaken
- to measure the proton radius in another atomic system
- to study other optical transitions in hydrogen :
  improved 2S-nS/nD and 1S-3S experiments and 1S-4S prospect

The Lamb shifts

In the same manner as for the Rydberg constant, they can be deduced from the 1S-2S and 2S-nD intervals, using the scaling law of the Lamb shift

The result in hydrogen are :

$$L_{2S_{1/2}} - L_{2P_{1/2}} = 1\,057.8440\,(30)\ \text{MHz}$$

This value, which uses the theoretical value of the 2P Lamb shift, is more precise than the direct determination by microwave spectroscopy

and : $$L_{1S_{1/2}} = 8\,172.838\,(24)\ \text{MHz}$$

From this value and taking into account the QED theoretical predictions, one can deduce a value of the rms charge radius of the proton

$$r_{\text{p}} = 0.8764\,(89)\ \text{fm}$$

The 1S Lamb shift : discussion

The table below gives the various contributions to the Lamb shift

The uncertainties of the one-loop corrections are mainly due to α

(2006 values)

Term of the Lamb shift	Value for the 1S level	Uncertainties
Self-energy (one-loop)	8 383 339.466 kHz	0.083 kHz
Vacuum polarization (one-loop)	−214 816.607 kHz	0.005 kHz
Recoil corrections	2 401.782 kHz	0.010 kHz
Proton size	1 253.000 kHz	50 kHz
Two-loop corrections	731.000 kHz	3.300 kHz
Radiative recoil corrections	−12.321 kHz	0.740 kHz
Vacuum polarization (muon)	−5.068 kHz	$<$0.001 kHz
Vacuum polarization (hadron)	−3.401 kHz	0.076 kHz
Proton self-energy	4.618 kHz	0.160 kHz
Three-loop corrections	1.800 kHz	1.000 kHz
Nuclear size corrections to SE and VP	−0.149 kHz	0.011 kHz
Proton polarization	−0.070 kHz	0.013 kHz
1S Lamb shift	8 172 894(51) kHz	

The experimental uncertainty is ~ 24 kHz

Optical spectroscopy of hydrogen gives a test of QED until the two-loop corrections if the proton radius is known