QED & Quantum Vaccum, Low Energy Frontier, 02004 (2012)
DOI: 10.1051/iesc/2012qed02004

Lecture on the Theory of Casimir phenomenae

Bart van Tiggelen

Laboratoire de Physique et Modélisation des Milieux Condensés -
Grenoble - France

Lecture on the Theory of Casimir phenomenae

QED CNRS School 2012, IESC Cargèse , April 2012

laboratoire de physique et de modélisation des milieux condensés

Bart van Tiggelen

Università di Corsica Pasquale Paoli

Université Nice

Institut d'Études Scientifiques de Cargèse

The institute started in the 60's as anindependent non profit association in « villa Menasina » under the direction of Maurice Levy, an eminent theoretical physicist. At that time two or three schools of theoretical physics were held in the venue during summer. In 1975 the association was granted by governmental institutions with new facilities allowing a progressive increase in the activity of the institute. During the last 10 years, the IESC opened up for emerging disciplines such as biophysics of membranes, environmental sciences, social, and economic sciences and humanities.

Since 1996 the institute is affiliated to Centre Nationale de la Recherche Scientifique , University of Corsica and University of Nice Sophia Antipolis. The IESC is open from February to November. The typical format is one week for a workshop and two weeks for a School. Applications are welcome. See IESC website

Summary of course

1. Important events
2. Lorentz invariance of Casimir energy
   - Accelerated observer & Unruh effect
   - the UV catastrophe (Einstein equation, sonoluminescence, Casimir momentum)
3. Fluctuation-dissipation theorem
4. Casimir force between ideal plates
   - ..and on a metallic shell
   - Proximity force approximation, dispersion, finite T)
4. Casimir –Polder attraction
5. Lifshitz formula
6. Quantum friction
7. Connection with QED:
   - Casimir mass and Casimir momentum of H.
8. Bibliography

1.1 Casimir energy

1. Black body radiation (Planck, 1912 [1], Einstein & Stern, 1913 [2])

$$\frac{h\nu}{\exp(h\nu/kT)-1}+\frac{1}{2}h\nu=\left(kT-\frac{1}{2}h\nu\right)+\frac{1}{2}h\nu=kT+O(1/T)$$

2. Isotropic radiation with power spectrum ω^3 is Lorentz-invariant (Einstein, 1917 [3]);
3. Van der Waals force $1/r^6$ as a dispersion force due to quantum fluctuations (London, 1930 [4])
3. Relation to Cosmological constant (Pauli, 1934, Davies, 1984 [5])
4. Casimir Polder Force $1/r^7$ (1948 [6])
5. Lamb shift (Lamb & Retherford[7], 1947, Bethe,[8] 1947); anomalous magnetic moment of electron (Schwinger [19] 1948)
7. Attraction between metallic plates (Casimir, 1948 [10]), refuted by Pauli as « absolute nonsense »
8. Lifshitz formula for dielectric bodies (Lifshitz, 1956 [11])
9. "The general theory of Van der Waals forces" (Lifshitz, Dzyalonishiniskii, Pitaevskii 1961 [12])

1.2 Casimir energy

10. Observation of Casimir effect (Sparnaay [13] (100 %), 1958, Lamoureux [14] (5%), 1997), Mohideen & Roy [15], 1998, Ederth [16] (1%), 2000)
    (the third for plane sphere-on cantilever geometry, the latter for crossed cilinders)
11. Stability of the electron (Casimir, 1956 [17], Boyer, 1968 [18])
12. Unruh effect & Hawking radiation (Hawking 1974 [19], Unruh 1976 [20])
13. Dynamical Casimir effect (« moving mirror radiation »), Fulling and Davies, 1976 [21]
14. Bag model for hadrons (Jaffe etal, 1974 [22])
15. Cosmological constant problem field theory(Weinberg, 1989 [23])
16. Confined Casimir energy has inertial mass (Jeakel &Reynaud, 1993 [24])

1.3 Casimir energy

17. Sonoluminescence as dynamical Casimir effect (Schwinger, 19932[25], Eberlein, 1996[26])
18. Quantum friction and shearing the quantum vacuum (Levitov, 1989[27]; Pendry, 1997[28])
19. Casimir dies at age of 90 (May 4, 2000)
20. Casimir momentum in magneto-electric media (Feigel, 2004[29])
21. An attractive Casimir force theorem for dielectrics (Kenneth, Klich, 2006[30])
22. Repulsive Van der Waals force in colloids (Feiler etal, 2008[31]) ; quantum Casimir levitation of silicon sphere (Capasso etal 2009[32])
23. Casimir energy has gravitational mass (Milton, Fulling etal, 2007[33])
24. Scattering theory for Casimir energy: finite temperatures ,beyond Proximity Force Approximation, corrugated surfaces (Lambrecht & Reynaud , Dalvit etal, Bordag etal[34-37]).
25. Observation of dynamical Casimir effect with SQUID (Wilson etal 2011)[38]
26. Observation of thermal Casimir force between plates, favoring Drude model (Sushkov, Dalvit, Lamoreaux, 2011)[39]

1.4 Casimir energy before Casimir

"At this point it should be noted that it is more consistent here, in contrast to the material oscillator, not to introduce a zero-point energy of $1/2\hbar\omega$ *per degree of freedom. For, on the one hand, the latter would give rise to an infinitely large energy per unit volume due to the infinite number of degrees of freedom, on the other hand, it would be principally unobservable since nor can it be emitted, absorbed or scattered and hence, cannot be contained within walls and, as is evident from experience, neither does it produce any gravitational field."*

Pauli, Die Allgemeinen Prinzipien der Wellenmechanik,
in *Handbuch der Physik* 24 1 (1933)

2.1 Casimir energy is Lorentz invariant (to be continued)

$$dI(\theta,\omega) = \rho(\omega,\theta)\,d\omega\,d\cos\theta = d\left(\tfrac{1}{2}\mathbf{E}^2 + \tfrac{1}{2}\mathbf{B}^2\right)$$

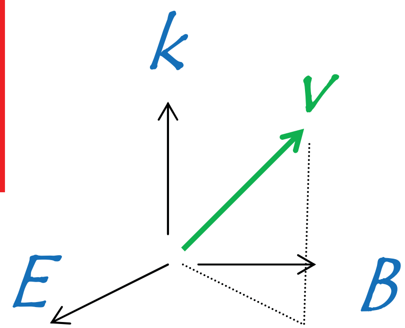
$$\boldsymbol{\beta} = \frac{\mathrm{v}}{c_0}\left(c\hat{\mathbf{k}} + s\cos\phi\,\hat{\mathbf{x}} + s\sin\phi\,\hat{\mathbf{y}}\right)$$

$$\omega' = \gamma\omega(1-\beta c)$$

$$c' = \frac{c-\beta}{1-\beta c}$$

$$\mathbf{E}' = \gamma\left(\mathbf{E} + \boldsymbol{\beta}\times\mathbf{B} - \frac{\gamma^2}{\gamma+1}\boldsymbol{\beta}\boldsymbol{\beta}\cdot\mathbf{E}\right)$$

$$\mathbf{B}' = \gamma\left(\mathbf{B} - \boldsymbol{\beta}\times\mathbf{E} - \frac{\gamma^2}{\gamma+1}\boldsymbol{\beta}\boldsymbol{\beta}\cdot\mathbf{B}\right)$$



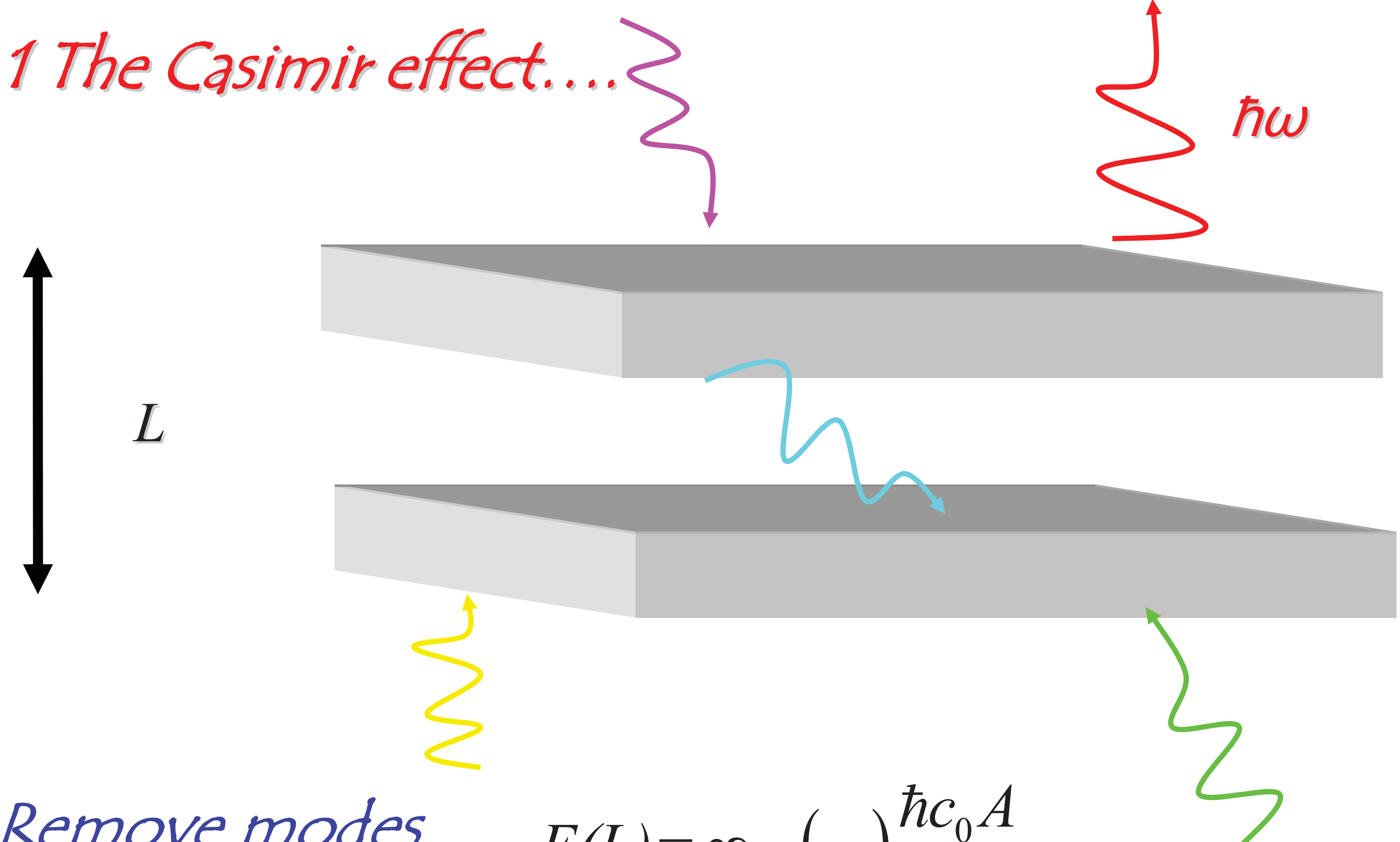
$$\frac{d(\omega',c')}{d(\omega,c)} = \frac{1}{\gamma(1-\beta c)}$$

$$dI' = \gamma^2(1-\beta c)^2\,dI$$

$$\rho'(\omega',c') = \rho\left(\frac{\omega'}{\gamma(1-\beta c)}, c\right)\gamma^3(1-\beta c)^3$$

$\rho(\omega,c) = \omega^3$ Lorentz invariant (Einstein, 1917 [3])

3.1 The Casimir effect….



Remove modes

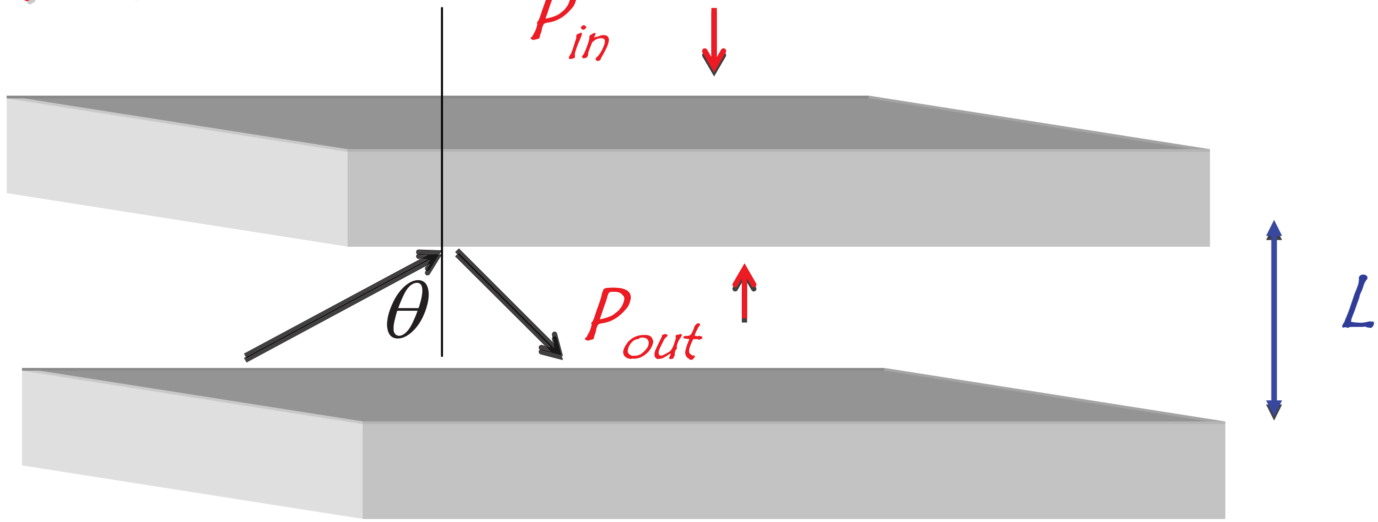
$$E(L) = \infty - (\cdots)\frac{\hbar c_0 A}{L^3}$$

$$F(L) = -\frac{\partial E}{\partial L} = -3(\cdots)\frac{\hbar c_0 A}{L^4}$$

Negative pressure

No momentum exchange between matter and radiation

3.2 The Casimir effect….



$$P_{\text{out}}(\theta) = \frac{F\left(\Delta t = 2L/\cos\theta\, c_0\right)}{A} = \frac{2 \times \frac{1}{2}\hbar k \times \cos\theta}{A} \times \frac{c_0 \cos\theta}{2L} \times 2 \Rightarrow P_{\text{out}} = \frac{\hbar c_0}{V} \sum_{\substack{\text{modes} \\ k_z > 0}} \frac{k_z^2}{k}$$

$$P = P_{in} - P_{out} = \frac{\hbar c_0}{L} \sum_{n=1}^{\infty} \int \frac{d^2\mathbf{k}}{(2\pi)^2} \frac{(n\pi/L)^2}{\sqrt{k^2 + (n\pi/L)^2}}$$

$$- \frac{\hbar c_0}{\pi} \int_0^{\infty} dk_z \int \frac{d^2\mathbf{k}}{(2\pi)^2} \frac{k_z{}^2}{\sqrt{k^2 + k_z{}^2}}$$

3.3 The Casimir effect….

$$F(n) = n^2 \int_0^\infty dx \frac{1}{\sqrt{x+n^2}} = n^2 \int_{n^2}^\infty \frac{dy}{\sqrt{y}}$$

$$P = \frac{\hbar c_0}{4\pi L}\left(\frac{\pi}{L}\right)^3 \left[\sum_{n=1}^{\infty} F(n) - \int_0^\infty dn\, F(n)\right]$$

$$= \frac{\pi^2 \hbar c_0}{4L^4}\left[-\tfrac{1}{2}F(0) - \tfrac{1}{12}F'(0) + \tfrac{1}{720}F'''(0) + \ldots\right]$$

0 0 -12 0

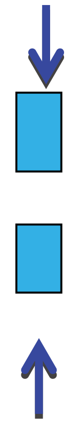
$$= -\frac{\pi^2}{240}\frac{\hbar c_0}{L^4}$$

130 nN/ cm² / L⁴(μm)

Casimir energy diverges in UV but…

THE Casimir effect is a low energy phenomenon

3.4 The Casimir effect of scalar bosons in 1D



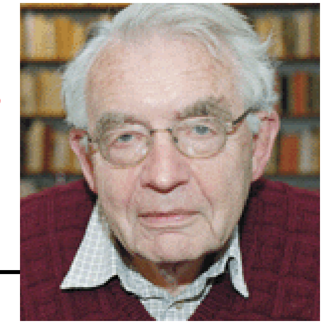
$$F = F_{in} - F_{out} = \frac{\hbar c_0}{2L}\sum_{n=1}^{\infty} 2\times\frac{1}{2}\frac{n\pi}{L} - \frac{\hbar c_0}{2\pi}\int_0^{\infty} dk\, k$$

$$= \frac{\hbar c_0 \pi}{2L^2}\lim_{\kappa\downarrow 0}\left(\sum_{n=1}^{\infty} n\exp(-\kappa n) - \int_0^{\infty} dn\, n\exp(-\kappa n)\right)$$

$$= \frac{-\pi}{24}\frac{\hbar c_0}{L^2}$$

Hendrik Casimir,
Proc. Koninklijke Nederlandse Academie voor Wetenschappen
51 (1948), 79

(thank you Astrid Lambrecht for providing)



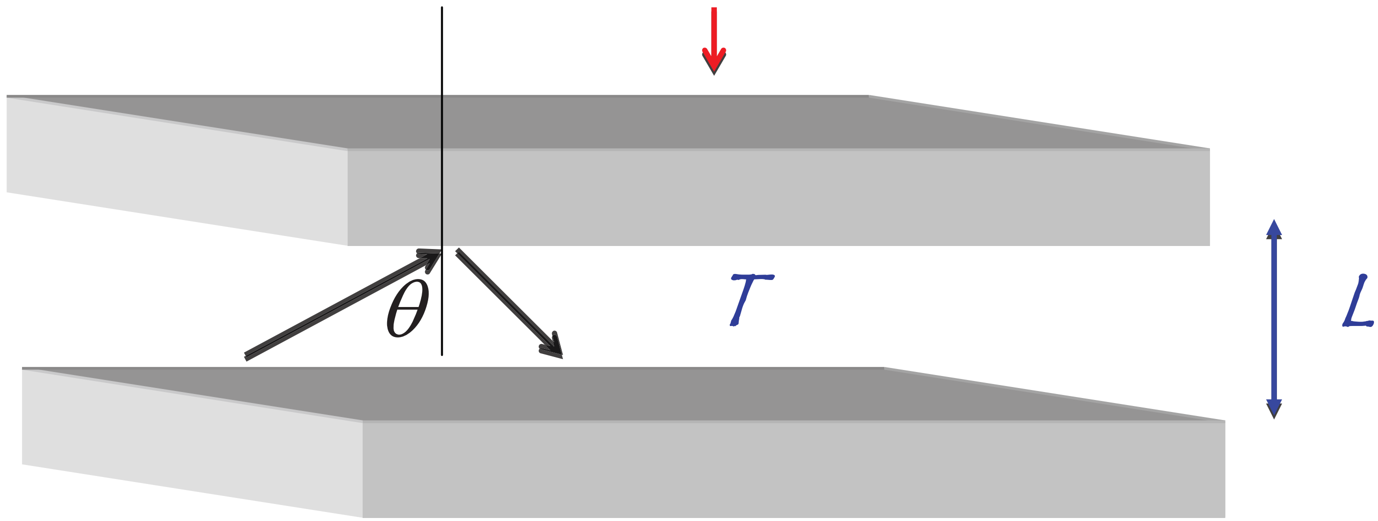
In order to obtain a finite result it is necessary to multiply the integrands by a function $f(k/k_m)$ which is unity for $k \ll k_m$ but tends to zero sufficiently rapidly for $(k/k_m) \rightarrow \infty$, where k_m may be defined by $f(1) = \frac{1}{2}$. The physical meaning is obvious: for very short waves (X-rays e.g.) our plate is hardly an obstacle at all and therefore the zero point energy of these waves will not be influenced by the position of this plate.

Introducing

We are thus led to the following conclusions. There exists an attractive force between two metal plates which is independent of the material of the plates as long as the distance is so large that for wave lengths comparable with that distance the penetration depth is small compared with the distance. This force may be interpreted as a zero point pressure of electromagnetic waves.

Although the effect is small, an experimental confirmation seems not unfeasable and might be of a certain interest.

3.5 The Casimir effect at finite temperatures….



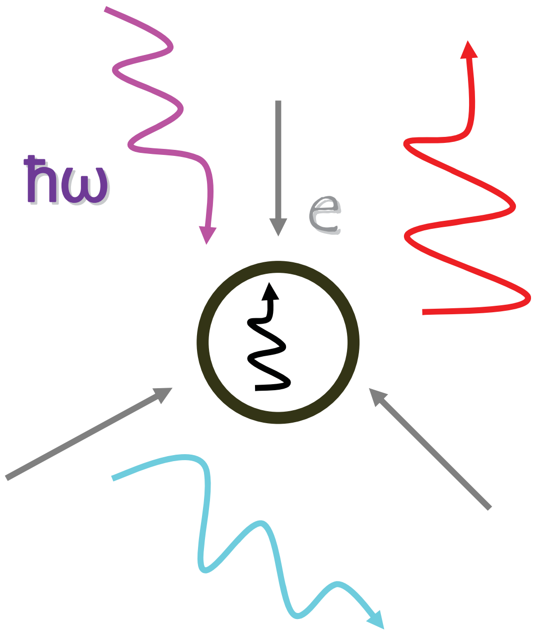
$$P = -\frac{\pi^2}{240}\frac{\hbar c_0}{L^4}\left(1+\frac{16}{3}\left[\frac{kTL}{\hbar c_0}\right]^4+\ldots\right)$$ K.A. Milton, ch. 2

$\underbrace{\qquad}_{L=10\,\mu m,\,T=300\,K\,:\ \text{correction}=19}$

$$\frac{kTL}{\hbar c_0} >> 1 : P = -2.4\frac{kT}{4\pi L^3}$$

3.6 The Casimir effect for a shell (Casimir 1956 [17])

Does Casimir force stabilize the electron ?



$$F_{\text{Casimir}} = -(\ldots)\frac{\hbar c_0 A}{r_e^4} = -\alpha \frac{\hbar c_0}{2r_e^2}$$

?

$$F_{\text{Coulomb}} = +\frac{e^2}{8\pi\varepsilon_0 r_e^2}$$

$$\alpha = \frac{e^2}{4\pi\varepsilon_0 \hbar c_0} = 0.0073..$$

But force is repulsive!

Boyer (1968) [18]

$$F_{\text{Casimir}} = +0.094\frac{\hbar c_0}{2r_e^2}$$

4.1 Fluctuation dissipation theorem at finite temperature

$$\rho = \frac{1}{Z}\exp(-\beta H) \quad H = \sum_k \hbar\omega_k (a^*_{kg}a_{kg} + \tfrac{1}{2})$$

$$f_k = \frac{1}{\exp(\beta\hbar\omega_k) - 1}$$

$$\left\{ \begin{array}{l} \mathrm{Tr}\,\rho\, a^*_{kg} a_{k'g'} = f_k \delta_{kk'}\delta_{gg'} \\ \mathrm{Tr}\,\rho\, a_{k'g'} a^*_{kg} = (f_k + 1)\delta_{kk'}\delta_{gg'} \end{array} \right.$$

vacuum

$$\mathbf{E}(\mathbf{r},t) = \sum_{kg} i\left(\frac{\hbar\omega_k}{2\varepsilon_0}\right)^{1/2} \left(a_{k'g'}\mathbf{E}_{kg}(\mathbf{r})\exp(-i\omega_k t) - a^*_{kg}\mathbf{E}^*_{kg}(\mathbf{r})\exp(i\omega_k t)\right)$$

Positive frequencies — negative frequencies

$$\mathbf{E}(\omega,\mathbf{r}) = \int_{-\infty}^{+\infty} dt\, \mathbf{E}(t,\mathbf{r})\exp(i\omega t) \qquad \mathbf{E}(\omega)^* = \mathbf{E}(-\omega)$$

4.2Fluctuation dissipation theorem at finite temperature

$$G^{\pm}(\omega,\mathbf{r},\mathbf{r}'))=\sum_{kg}\frac{\mathbf{E}_{kg}(\mathbf{r})\mathbf{E}^{*}_{kg}(\mathbf{r}')}{(\omega\pm i0)^2-\omega_k^2}$$

Classical retarded/advanced
Green function
for electric field

$$\omega>0:\quad \langle \mathrm{E}_{\mathrm{n}}(\mathbf{r},\omega)\mathrm{E}_{\mathrm{m}}(\mathbf{r}',\omega')\rangle=2\pi\delta_{\omega\omega'}\frac{\hbar\omega^2}{2\pi\varepsilon_0}i\left[G^{+}_{nm}(\omega,\mathbf{r},\mathbf{r}')-G^{-}_{nm}(\omega,\mathbf{r},\mathbf{r}')\right](f_{\omega}+1)$$

$$\langle \mathrm{E}_{\mathrm{n}}(\mathbf{r},-\omega)\mathrm{E}_{\mathrm{m}}(\mathbf{r}',-\omega')\rangle=2\pi\delta_{\omega\omega'}\frac{\hbar\omega^2}{2\pi\varepsilon_0}i\left[G^{+}_{nm}(\omega,\mathbf{r},\mathbf{r}')-G^{-}_{nm}(\omega,\mathbf{r},\mathbf{r}')\right]f_{\omega}$$

No negative frequencies at
T=0

$$\langle \mathrm{E}_{\mathrm{n}}(\mathbf{r},\omega)\mathrm{E}_{\mathrm{m}}(\mathbf{r}',\omega')+\mathrm{E}_{\mathrm{n}}(\mathbf{r},-\omega)\mathrm{E}_{\mathrm{m}}(\mathbf{r}',-\omega')\rangle=2\pi\delta_{\omega\omega'}\frac{-\hbar\omega^2}{\pi\varepsilon_0}\mathrm{Im}G^{+}_{nm}(\omega,\mathbf{r},\mathbf{r}')(2f_{\omega}+1)$$

$$G^{-}(\omega)=G^{+}(-\omega)\ ;\ \mathbf{E}_{kg}(\mathbf{r})\in\Re$$

$\mathrm{cotanh}(\beta\hbar\omega/2)$

4.3 Fluctuation dissipation theorem at finite temperature

What if medium is dissipative (finite conductivity)?

$$\left[\varepsilon_0 + \frac{4\pi i\sigma(\mathbf{r})}{\omega}\right]\omega^2\mathbf{E}(\mathbf{r},\omega) + \nabla\times\nabla\times\mathbf{E}(\mathbf{r},\omega) = \mathbf{j}(\mathbf{r},\omega)$$

dissipation

$$\sigma > 0$$

Current fluctuations

$$\left\langle j_i(\mathbf{r},\omega)\, j^*{}_j(\mathbf{r}',\omega')\right\rangle = K\times 4\pi\omega\sigma(\mathbf{r})\times 2\pi\delta_{\omega\omega'}\delta_{ij}\delta_{\mathbf{rr}'}$$

$$\mathbf{E}(\mathbf{r},\omega) = \int d\mathbf{x}\; G^+(\mathbf{r},\mathbf{x},\omega)\cdot\mathbf{j}(\mathbf{r},\omega)$$

$$\Rightarrow \left\langle E_n(\mathbf{r},\omega)E^*_m(\mathbf{r}',\omega')\right\rangle = \int d\mathbf{x}\int d\mathbf{x}' G^+_{ni}(\mathbf{r},\mathbf{x},\omega)\left\langle j_i(\mathbf{r},\omega)\, j_j(\mathbf{r}',\omega')\right\rangle\cdot G^-_{jm}(\mathbf{x},\mathbf{r},\omega)$$

$$= 2\pi\delta_{\omega\omega'}K\left\langle \mathbf{r}\middle| G^+(\omega)\,4\pi\omega\sigma(\mathbf{x})G^-(\omega)\middle|\mathbf{r}'\right\rangle$$

$$= 2\pi\delta_{\omega\omega'}\frac{K}{-2i}\left\langle \mathbf{r}\middle| G^+(\omega) - G^-(\omega)\middle|\mathbf{r}'\right\rangle$$

$$K = \frac{\hbar\omega^2}{\pi\varepsilon_0}\operatorname{cotanh}(\beta\hbar\omega/2)$$

$$\left\langle \mathrm{E}_n(\mathbf{r},\omega)\,\mathrm{E}^*_m(\mathbf{r}',\omega')\right\rangle = 2\pi\delta_{\omega\omega'}\frac{-\hbar\omega^2}{\pi\varepsilon_0}\operatorname{Im}G^+_{nm}(\omega,\mathbf{r},\mathbf{r}')\left(2f_\omega + 1\right)$$

Positive and negative frequencies added

4.4 Fluctuation dissipation theorem at finite temperature

What if medium is dispersive and dissipative ?

$$\left[\varepsilon(\mathbf{r})+\frac{4\pi i\sigma(\mathbf{r})}{\omega}\right]\omega^2\mathbf{E}(\mathbf{r},\omega)+\nabla\times\nabla\times\mathbf{E}(\mathbf{r},\omega)=\mathbf{j}(\mathbf{r},\omega)$$

dispersion dissipation current fluctuations

$\sigma>0$

$$\left\langle j_i(\mathbf{r},\omega)\,j^*{}_j(\mathbf{r}',\omega')\right\rangle=K\times4\pi\omega\sigma(\mathbf{r})\times2\pi\delta_{\omega\omega'}\delta_{ij}\delta_{\mathbf{rr}'}$$

Substitute $\boldsymbol{\psi}(\mathbf{r},\omega)=\sqrt{\varepsilon(\mathbf{r})}\mathbf{E}(\mathbf{r},\omega)$ and show that

$$\left\langle \mathrm{E}_n(\mathbf{r},\omega)\mathrm{E}_m^*(\mathbf{r}',\omega')\right\rangle=2\pi\delta_{\omega\omega'}\frac{-\hbar\omega^2}{\pi\sqrt{\varepsilon(\mathbf{r})\varepsilon(\mathbf{r}')}}\mathrm{Im}G_{nm}^{+}(\omega,\mathbf{r},\mathbf{r}')\left(2f_\omega+1\right)$$

Dangerous to quantize macroscopic media!

2. 2 Continued: Casimir energy is Lorentz invariant

$$\partial_t \frac{1}{2}\left(\varepsilon_0 E^2 + \frac{1}{\mu_0} B^2\right) + c_0^2 \nabla \cdot \varepsilon_0 \mathbf{E}\times\mathbf{B} = 0$$

$$\partial_t\left(\varepsilon_0 \mathbf{E}\times\mathbf{B}\right) + \nabla \cdot \left\{ \frac{1}{2}\left(\varepsilon_0 E^2 + \frac{1}{\mu_0} B^2\right)\delta_{ij} - \left(\varepsilon_0 E_i E_j + \frac{1}{\mu_0} B_i B_j\right) \right\} = 0$$

$$\partial_\mu T^\mu{}_\nu = 0$$

$$T^\mu{}_\mu = 0$$

$$T_{\mu\nu} = T_{\nu\mu}$$

$$\frac{\omega^2}{c_0^2} G_{ij}(\omega,\mathbf{k}) = \left(\frac{\omega^2}{c_0^2}\delta_{ij} - k_i k_j\right)\frac{1}{(\omega+i0)^2/c_0^2 - k^2}$$

$$\left\langle T_{\mu\nu}\right\rangle \propto -2\hbar\,\mathrm{Im}\int \frac{d\omega}{2\pi}\int \frac{d^3\mathbf{k}}{(2\pi)^3}\frac{1}{\frac{(\omega+i0)^2}{c_0^2} - k^2}\begin{pmatrix} \omega^2/c_0^2 & \omega k_j/c_0 \\ \omega k_i/c_0 & k_i k_j \end{pmatrix}$$

Lorentz invariant measure

Lorentz scalar

Lorentz covariant tensor

2.3 continued : Casimir energy is Lorentz invariant

$$\mathrm{Im}\int_0^\infty \frac{d\omega}{2\pi}\frac{\omega^2/c_0^2}{\frac{(\omega+i0)^2}{c_0^2}-k^2}$$

doesn't allow Wick rotation (Milton, chapter 10; 1997 [40])

$$\mathrm{Im}\int_0^\infty \frac{d\omega}{2\pi}\left[\frac{\omega^2/c_0^2}{\frac{(\omega+i0)^2}{c_0^2}-k^2}-1\right]=\mathrm{Im}\int_0^\infty \frac{d\omega}{2\pi}\frac{k^2}{\frac{(\omega+i0)^2}{c_0^2}-k^2}=-\int_0^\infty d\zeta\,\frac{k^2}{\frac{\zeta^2}{c_0^2}+k^2}$$

$$\mathrm{Im}\int_0^\infty \frac{d\omega}{2\pi}\frac{\langle k_i k_j\rangle}{\frac{(\omega+i0)^2}{c_0^2}-k^2}=-\int_0^\infty \frac{\frac{1}{3}k^2}{\frac{\zeta^2}{c_0^2}+k^2}$$

$$\omega\rightarrow\langle T_{\mu\nu}\rangle=E_{\mathrm{casi}}\begin{pmatrix}1 & & 0 & 0\\ 0 & \frac{1}{3} & 0 & 0\\ 0 & 0 & \frac{1}{3} & 0\\ 0 & 0 & 0 & \frac{1}{3}\end{pmatrix}\qquad T^{\mu}{}_{\mu}=0$$

$$\neq K g_{\mu\nu}$$

Not cosmological constant but ultrarelativistic dust E=3P (L & L, Theory of Fields)

Divergence of Lorentz invariant theory is not Lorentz invariant?

2.4 continued Casimir energy is Lorentz invariant

Wick rotation when regularized invariantly:

Analytic function bounded in C

$$\omega \to i\zeta c_0 : \langle T_{\mu\nu} \rangle = \frac{\hbar c_0}{8\pi^4} \int_0^\infty d\zeta \int d^3\mathbf{k} \frac{1}{\zeta^2 + k^2} \begin{pmatrix} -\zeta^2 & i\zeta k_i \\ i\zeta k_j & k_i k_j \end{pmatrix} f(\zeta^2 + k^2)$$

$$= \frac{\hbar c_0}{2\pi^3} \int_0^\infty ds\, s^3 f(s) \int_0^{2\pi} d\phi \begin{pmatrix} -\sin^2\phi \cos^2\phi & 0 \\ 0 & \tfrac{1}{3}\delta_{ij} \sin^4\phi \end{pmatrix}$$

$$= -g_{\mu\nu} \frac{\hbar c_0}{8\pi^2} \int_0^\infty ds\, s^3 f(s) = -E_{\text{casi}}\, g_{\mu\nu}$$

Regularized quantum vacuum is formal candidate for cosmological constant

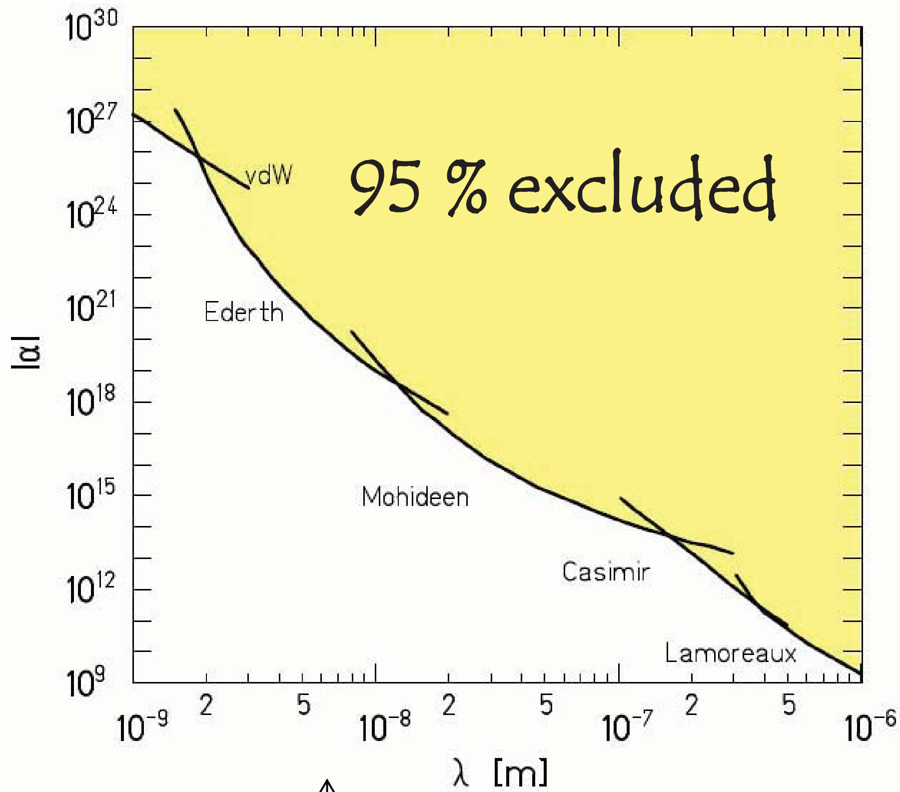
$$\boxed{R_{\mu\nu} - R g_{\mu\nu} + \Lambda g_{\mu\nu} = -\frac{8\pi G}{c_0^4} T_{\mu\nu}} \qquad \Rightarrow \Lambda = \frac{8\pi G}{c_0^4} E_{\text{casi}} \quad ?$$

$$\frac{\Lambda c_0^4}{8\pi G} \approx \rho_c = 10^{-5}\,\text{GeV/cm}^3 := \frac{\hbar c}{8\pi^2} \int_0^{1/a} ds\, s^3 \Rightarrow a \approx 13\,\mu\text{m}$$

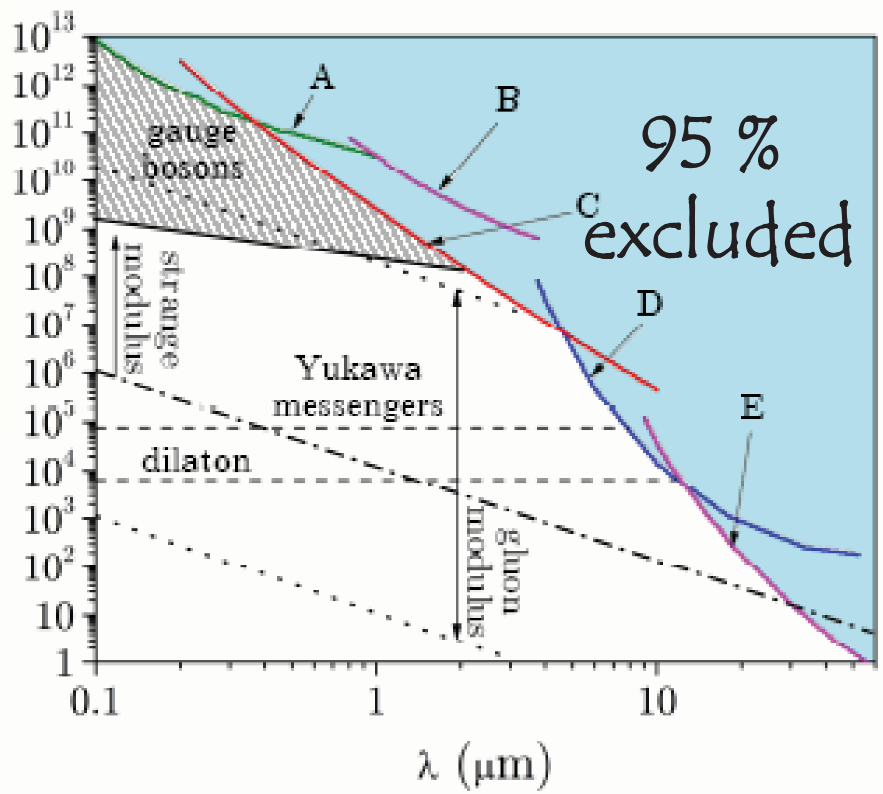
Is regularization result of stabilized extra , large « enrolled » dimensions d › 4 of this size?

2.5 Non Newtonian gravity deduced from Casimir experiments

1 mm thick plate (10 g/cm³) $F_{casi}=F_z \rightarrow L=13\ \mu m$



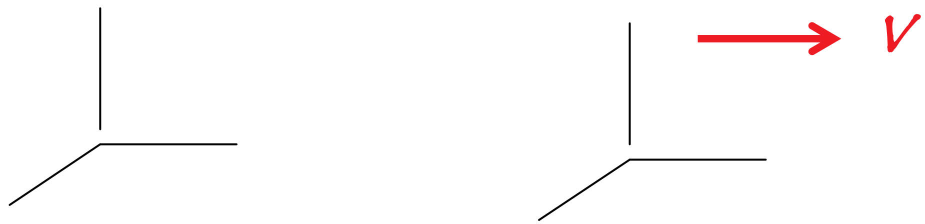
$$\frac{V}{V_{\text{Newton}}} = \alpha \exp(-r/\lambda)$$



E. Adelberger et al (2003) [41]
Kapner etal, 2007[42]
Sushkov, Kim, Dalvit , Lamoreaux [43] 2011

2.6 Accelerated Casimir energy becomes Planck law

Constant acceleration a in comoving frame with speed v (Milonni, 1994[44])



$$\frac{du_{//}'}{dt'} = \frac{du_{//}}{dt}\frac{1}{\gamma}\frac{1}{\left(1-\mathbf{v}\cdot\mathbf{u}/c_0^2\right)^2} = a \quad \text{for} \quad \mathbf{u}=\mathbf{v} \Rightarrow \frac{dv}{dt} = a\left(1-\frac{v^2}{c_0^2}\right)^{3/2}$$

$$v(t) = \frac{at}{\sqrt{1+a^2t^2/c_0^2}} \longrightarrow d\tau = \sqrt{1-v^2/c_0^2}\,dt \Rightarrow t(\tau) = \frac{c_0}{a}\sinh\frac{a\tau}{c_0}$$

$$x(t) = \frac{c_0^2}{a}\left[\sqrt{1+a^2t^2/c_0^2}-1\right] \longrightarrow x(\tau) = \frac{c_0^2}{a}\left(\cosh\frac{a\tau}{c_0}-1\right)$$

$$x^2(\tau) - t^2(\tau) = -\frac{c_0^4}{a^2}\sinh^2\frac{a\tau}{2c_0}$$

2.7 Accelerated Casimir energy becomes Planck law

$$\mathbf{E} = \partial_t \mathbf{A}$$

$$\left\langle \mathrm{A_n}(\mathbf{r},\omega)\,\mathrm{A_m^*}(\mathbf{r}',\omega)\right\rangle = 2\pi\delta_{\omega\omega'}\frac{-\hbar}{\pi\varepsilon_0}\,\mathrm{Im}\,G^{+}_{nm}(\omega,\mathbf{r},\mathbf{r}')\left(2f_\omega+1\right)$$

$$\underline{T=0; a\neq 0}:\left\langle \mathrm{A_n}(0,t=0)\,\mathrm{A_m^*}(\mathbf{r}(t),t)\right\rangle = \frac{\hbar}{\varepsilon_0}\int_0^\infty d\omega \int \frac{d^3\mathbf{k}}{(2\pi)^3}\,\delta\!\left(\frac{\omega^2}{c_0^2}-k^2\right)\delta_{nm}\exp(i\mathbf{k}\mathbf{r}(t)-i\omega t)$$

$$= \frac{\hbar}{(2\pi)^3 c_0}\frac{\delta_{nm}}{r^2-c_0^2t^2} \propto -\frac{\hbar a^2/c_0^5}{\sinh^2\frac{a\tau}{2c_0}}$$

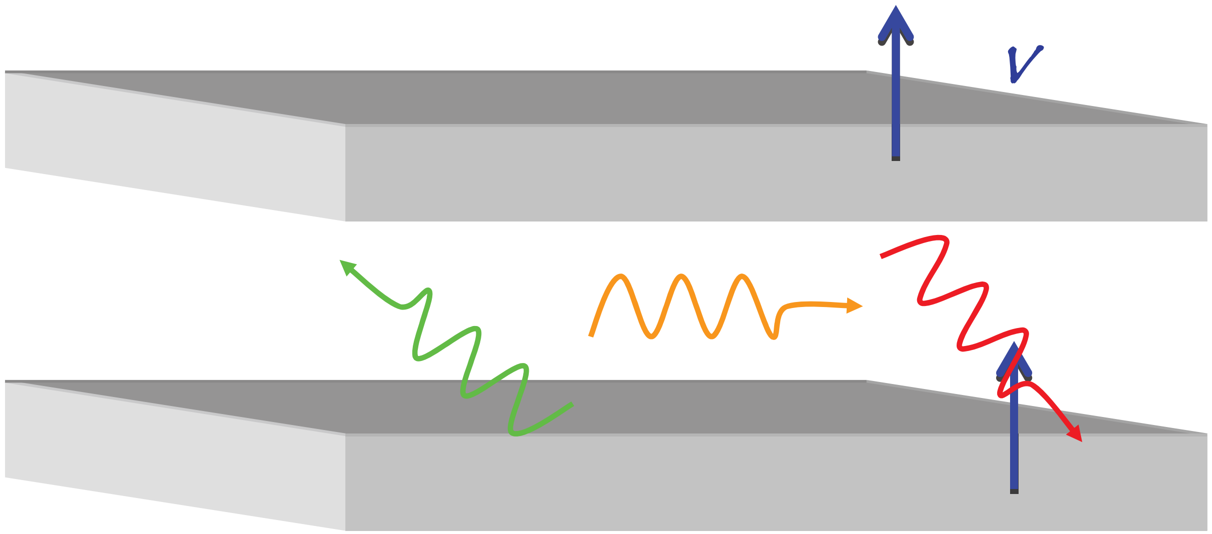
$$\underline{T\neq 0; a=0}:\left\langle \mathrm{A_n}(0,t=0)\,\mathrm{A_m^*}(\mathbf{r}=0,t)\right\rangle = \frac{\hbar}{\varepsilon_0}\int_0^\infty d\omega \int \frac{d^3\mathbf{k}}{(2\pi)^3}\,\delta\!\left(\frac{\omega^2}{c_0^2}-k^2\right)\delta_{nm}\exp(-i\omega t)(2f_\omega+1)$$

$$= \ldots. - \frac{\hbar\left(\pi kT/\hbar\right)^2/c_0^3}{\sinh^2\left(\frac{\pi kT}{\hbar}\tau\right)}$$

$$\boxed{T = \frac{\hbar a}{2\pi k c_0}}$$

Unruh/Hawking temperature

4.1 Inertial Casimir mass?

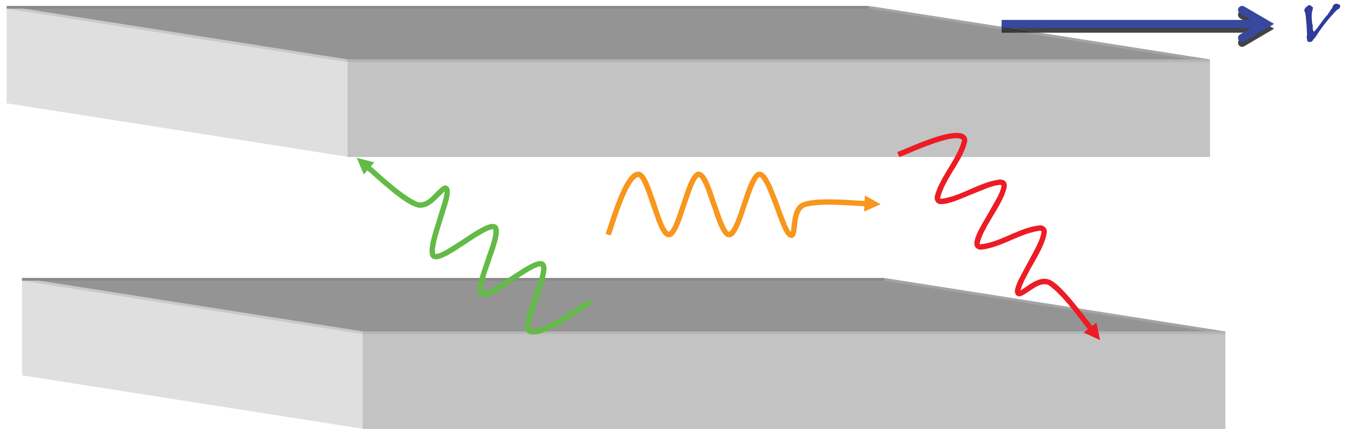


question

?

$$\mathbf{P}/A = m\mathbf{v}/A = \frac{M + E_{casi}/A}{c_0^2}\mathbf{v} = \frac{M/A - {}^{\pi^2}\!/_{720}\hbar c_0/L^2}{c_0^2}\mathbf{v}$$

4.2 Casimir friction?



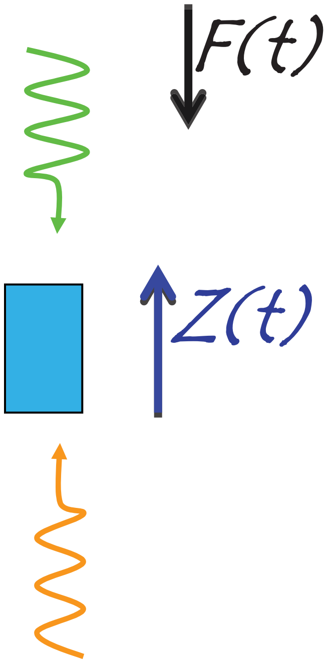
question

$$\mathbf{F}/A = -\gamma\,\mathbf{v} \quad ?$$

4.3 vacuum force on moving object

Fulling and Davies (1976[21])

$$F(t) = \frac{\hbar}{6\pi c_0^2} z'''(t)$$



F=0 for uniform speed (by Lorentz invariance)
or acceleration (Unruh effect)

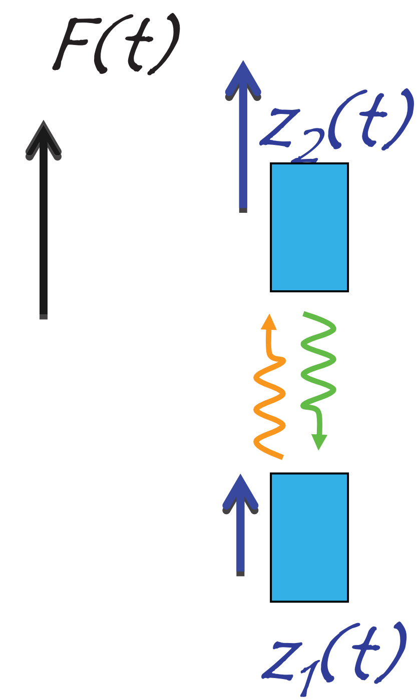
4.4 vacuum force on moving mirrors: Casimir mass

Jaekel and Reynaud(1993 [24])

Retarded reaction force from mirror 2 via vacuum on mirror 1

$$\delta F_1(t) = \frac{\hbar}{6\pi c_0^2}\left(z_1'''(t) - z_2'''(t-\tau) + z_1'''(t-2\tau) \mp \ldots\right)$$
$$- \frac{\hbar c_0 \pi \tau}{12L^3}\left(z_1'(t) - z_2'(t-\tau) + z_1'(t-2\tau) \mp \ldots\right)$$

Retarded variaton in Casimir force $\delta F_1(t) = -F'(z_2 - z_1)\times z_1'(t)\times\tau$



Keep z_1–z_2 constant: $z' = z_1' = z_2'$

$$\delta F_{1+2}(\omega) = 2\left(\frac{i\hbar\omega^3}{6\pi c_0^2} + \frac{i\omega\hbar c_0 \pi \tau}{12L^3}\right)\left(1 - \exp(i\omega\tau) + \exp(2i\omega\tau) \mp \ldots\right) z(\omega)$$
$$= \frac{\frac{i\hbar\omega}{6\pi c_0^2}\left(\omega^2 + \frac{\pi^2}{2\tau^2}\right)}{\cos\frac{1}{2}\omega\tau}\exp\left(-\tfrac{1}{2}i\omega\tau\right)z(\omega) = -\frac{\hbar\pi}{12L^2}(-i\omega)z(\omega) - \frac{\hbar}{24\pi c_0 L}(-i\omega)^2 z(\omega)$$

Quantum friction (not found by ref 24)? $\gamma = \frac{\hbar\pi}{L^2}$ $m = \frac{E_{casi}}{c_0^2}$ Casimir mass

5.1 Van der Waals and Casimir Polder interaction

$$\left\langle \mathrm{E}_{\mathrm{n}}(\mathbf{r},\omega)\,\mathrm{E}^{*}_{\mathrm{m}}(\mathbf{r}',\omega')\right\rangle = 2\pi\delta_{\omega\omega'}\frac{-\hbar\omega^{2}}{\pi\sqrt{\varepsilon(\mathbf{r})\varepsilon(\mathbf{r}')}}\,\mathrm{Im}\,G^{+}_{nm}(\omega,\mathbf{r},\mathbf{r}')\left(2f_{\omega}+1\right)$$

$$\left\langle \int d\mathbf{r}\left[\frac{1}{2}\varepsilon(\mathbf{r})\mathbf{E}^{2}(\mathbf{r},t)+\frac{1}{2\mu_{0}}\mathbf{B}^{2}(\mathbf{r},t)\right]\right\rangle = -2\hbar\,\mathrm{Im}\int_{0}^{\infty}\frac{d\omega}{2\pi}\int\frac{d\mathbf{k}}{(2\pi)^{3}}\left(k^{2}\delta_{nm}-k_{n}k_{m}\right)G^{+}_{nm}(\omega,\mathbf{k},\mathbf{k})\left(2f_{\omega}+1\right)$$

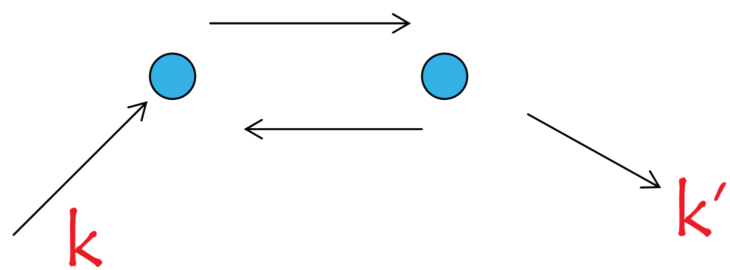
$$\mathbf{G}^{+}(\omega,\mathbf{k},\mathbf{k}') = \mathbf{G}^{+}_{0}(\omega,\mathbf{k})\delta_{\mathbf{k}\mathbf{k}'} + \mathbf{G}^{+}_{0}(\omega,\mathbf{k})\cdot\mathbf{T}_{\mathbf{k}\mathbf{k}'}(\omega)\cdot\mathbf{G}^{+}_{0}(\omega,\mathbf{k}')$$

T-matrix of 2 point dipoles:

$$\mathbf{T}_{\mathbf{k}\mathbf{k}'}(\omega) = \frac{t(\omega)e^{i(\mathbf{k}-\mathbf{k}')\mathbf{r}_{1}} + t(\omega)e^{i(\mathbf{k}-\mathbf{k}')\mathbf{r}_{2}} + t^{2}\mathbf{G}^{+}_{0}(\omega,\mathbf{r}_{12})\left[e^{i\mathbf{k}\mathbf{r}_{1}-i\mathbf{k}'\mathbf{r}_{2}}+e^{i\mathbf{k}\mathbf{r}_{2}-i\mathbf{k}'\mathbf{r}_{1}}\right]}{1-t^{2}\mathbf{G}^{2}_{0}(\omega,\mathbf{r}_{12})}$$

$$\mathbf{G}^{+}(\omega,\mathbf{k}) = \frac{1-c_{0}^{2}\mathbf{k}\mathbf{k}/\omega^{2}}{(\omega+i0)^{2}/c_{0}^{2}-k^{2}}$$

$$t(\omega) = -\frac{\alpha(0)\omega^{2}\omega_{0}^{2}/c_{0}^{2}}{\omega_{0}^{2}-\omega^{2}-\tfrac{2}{3}i\gamma\omega/c_{0}}$$



5.2 Van der Waals interaction

Van Tiggelen, 1999 [45])

$$\langle \delta E \rangle = -\hbar \,\mathrm{Im}\,\mathrm{Tr}\int_0^\infty \frac{d\omega}{2\pi}\,\omega \frac{d}{d\omega}\left\{2\log t(\omega) + \log\left(1 - t^2(\omega)\mathbf{G}^2(\omega,\mathbf{r}_{12})\right)\right\}$$

$$= \hbar \,\mathrm{Im}\,\mathrm{Tr}\int_0^\infty \frac{d\omega}{2\pi} 2\log t(\omega) + \hbar\,\mathrm{Im}\,\mathrm{Tr}\int_0^\infty \frac{d\omega}{2\pi}\log\left(1 - t^2(\omega)\mathbf{G}^2(\omega,\mathbf{r}_{12})\right)$$

$$r_{12} < \omega_0 / c_0$$

$$\langle \delta E \rangle = 2 \times {}^{3}\!/_{2}\,\hbar\omega_0$$

Ground state energy

=

Casimir energy

(this anticipates Lamb shift as a « Casimir effect »)

$$\langle \delta E \rangle = \hbar\,\mathrm{Im}\,\mathrm{Tr}\int_0^\infty \frac{d\omega}{2\pi}\left\{ -t^2(\omega)\left(\frac{1 - 3\hat{\mathbf{r}}\hat{\mathbf{r}}}{4\pi\omega^2 r^3 / c_0^2}\right)^2 \right\}$$

$$= -6\hbar\int_0^\infty \frac{ds}{2\pi}\left(\frac{\alpha(is)}{4\pi}\right)^2 \frac{1}{r^6} = -\frac{3}{4}\hbar\omega_0 \frac{\alpha(0)^2}{(4\pi)^2 r^6}$$

Van de Waals energy = Casimir energy

5.3 Casimir Polder interaction

$$\langle \delta E \rangle = -\hbar \, \mathrm{Im\,Tr} \int_0^\infty \frac{d\omega}{2\pi} \omega \frac{d}{d\omega} \left\{ 2 \log t(\omega) + \log(1 - t^2(\omega) \mathbf{G}^2(\omega, \mathbf{r}_{12})) \right\}$$

$$= \hbar \, \mathrm{Im\,Tr} \int_0^\infty \frac{d\omega}{2\pi} 2 \log t(\omega) + \hbar \, \mathrm{Im\,Tr} \int_0^\infty \frac{d\omega}{2\pi} \log(1 - t^2(\omega) \mathbf{G}^2(\omega, \mathbf{r}_{12}))$$

$$\langle \delta E \rangle = 2 \times \tfrac{3}{2} \hbar \omega_0$$

Ground state energy

=

Casimir energy

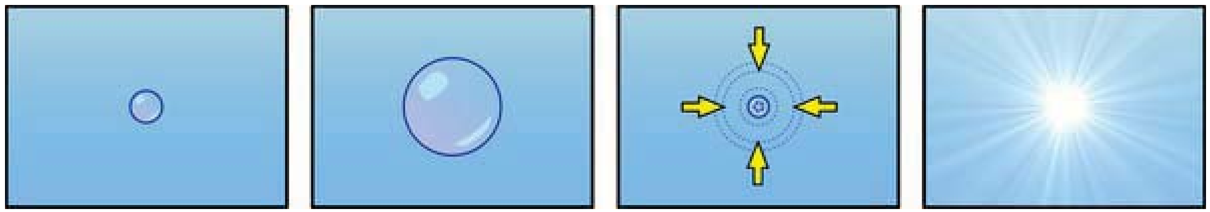
$$r_{12} > \omega_0 / c_0$$

$$\langle \delta E \rangle = -\frac{\hbar \alpha(0)^2}{c_0^4} \mathrm{Im\,Tr} \int_0^\infty \frac{d\omega}{2\pi} \omega^4 \mathbf{G}^2(\omega, \mathbf{r})$$

$$= \frac{\hbar \alpha(0)^2}{c_0^4} \int_0^\infty \frac{ds}{2\pi} s^4 \frac{f(isr/c_0)^2}{(4\pi r)^2} = -\frac{23}{4\pi} \frac{\hbar c_0 \alpha(0)^2}{(4\pi)^2 r^7}$$

Casimir Polder interaction energy

5.4 UV catastrophe in sonoluminescence (> 1934)



$(a = 40\,\mu m)$

Schwinger (1993) [7]

$$\Delta E(\text{bubble}) = \int d^3 r \left\{ \int d^3 k \,\frac{1}{2}\hbar\omega_k(\text{bubble in water}) - \int d^3 k\,\frac{1}{2}\hbar\omega_k(\text{water no bubble}) \right\}$$

$$\approx \frac{\hbar a^3\ \omega_c^4}{c_0^3}\left(1 - \frac{1}{\sqrt{\varepsilon}}\right) \approx 10\,\text{MeV}$$

← cut-off in the UV ?

Dimensional regularisation [46]? →

$$\langle \delta E \rangle = \int d^{d>8}\mathbf{r}\, \frac{-23\hbar c_0 \alpha(0)^2}{(4\pi)^3 r^7}$$

$$= \frac{+23}{1536\pi}\,\frac{\hbar c_0(\varepsilon(0)-1)^2}{L}$$

$$= 0.001\, eV$$

Identity of the van der Waals Force and the Casimir Effect and the Irrelevance of These Phenomena to Sonoluminescence

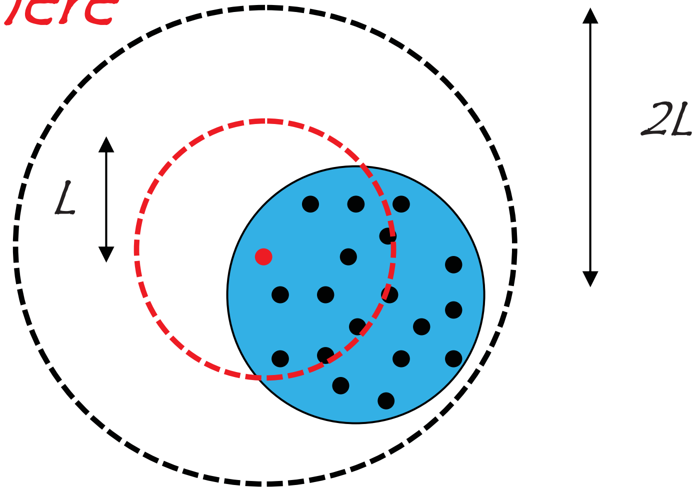
Iver Brevik*
Division of Applied Mechanics, Norwegian University of Science and Technology, N-7491 Trondheim, Norway

Valery N. Marachevsky[†]
Department of Theoretical Physics, St. Petersburg University, 198 904 St. Petersburg, Russia

Kimball A. Milton[‡]
Department of Physics and Astronomy, The University of Oklahoma, Norman, Oklahoma 73019

(Received 12 October 1998)

5.5 Casimir energy of dielectric sphere



$$\langle \delta E \rangle = N \times \tfrac{3}{2}\hbar\omega_0 - \frac{1}{2}\sum_{r_i, r_j < L} \frac{23\hbar c_0 \alpha(0)^2}{(4\pi)^3 |\mathbf{r}_i - \mathbf{r}_j|^7}$$

$$= N \times \tfrac{3}{2}\hbar\omega_0 - \frac{1}{2}\sum_{r_i < L, r_j} \frac{23\hbar c_0 \alpha(0)^2}{(4\pi)^3 |\mathbf{r}_i - \mathbf{r}_j|^7} + \frac{1}{2}\sum_{r_i < L, L < r_j < 2L} \frac{23\hbar c_0 \alpha(0)^2}{(4\pi)^3 |\mathbf{r}_i - \mathbf{r}_j|^7} + \frac{1}{2}\sum_{r_i < L, r_{ij} > 2L} \frac{23\hbar c_0 \alpha(0)^2}{(4\pi)^3 |\mathbf{r}_i - \mathbf{r}_j|^7}$$

$$= N \times \tfrac{3}{2}\hbar\omega_0 + \frac{\hbar c_0 \alpha(0)^2}{(4\pi)^2 a^7}\left(-0.11\frac{L^3}{a^3} + 0.4\frac{L^2}{a^2} - 0.006\frac{L}{a}\right) + \frac{1}{2} n^2 \frac{4}{3}\pi L^3 \int_{r>2L} d^3\mathbf{r} \frac{23\hbar c_0 \alpha(0)^2}{(4\pi)^3 r^7}$$

Contribution to latent heat[4,44]

$$q[J/kg] = \frac{0.11}{(4\pi)^2}\frac{\hbar c_0 \alpha(0)^2}{a^7 \times \tfrac{4}{3}\pi\rho a^3}$$

$$+\frac{23}{1536\pi}\frac{\hbar c_0 (\varepsilon(0)-1)^2}{L}$$

Regularized Casimir energy stems from missing atoms far outside (Kawka, 2010)[47]

Liquid helium

$$\alpha(0) = 2\ A^3 \quad a = 3\,A$$

$$\rho = 0.15\,g/cm^3 \Rightarrow q = 14\,J/g$$

5.7 The magical mystery world of regularization

$$\int \frac{d^d\mathbf{k}}{(2\pi)^d}(x^2+k^2)^{-p/2} = \frac{x^{d-p}\Gamma\left(\frac{p-d}{2}\right)}{(4\pi)^{d-1}\Gamma\left(\frac{p}{2}\right)} \quad (p>d)$$

$$\sum_{n=1}^{\infty}\frac{1}{n^x} = \zeta(x) \quad (\operatorname{Re} x > 1)$$

$$\Rightarrow P_{\text{casi}} = \frac{\hbar c_0}{L}\sum_{n=1}^{\infty}\int \frac{d^2\mathbf{k}}{(2\pi)^d}\frac{(n\pi/L)^2}{\sqrt{(n\pi/L)^2+k^2}} = \frac{\hbar c_0}{4\pi L}\sum_{n=1}^{\infty}\frac{\Gamma(-\frac{1}{2})}{\Gamma(\frac{1}{2})}\left(\frac{n\pi}{L}\right)^2\frac{n\pi}{L}$$

$$= \frac{\pi^2}{4}\frac{\hbar c_0}{L^4}\frac{\Gamma(-\frac{1}{2})}{\Gamma(\frac{1}{2})}\zeta(-3) = \frac{\hbar c_0}{L^4}\frac{\pi^2}{4}\frac{-2\sqrt{\pi}}{\sqrt{\pi}}\frac{1}{120} = -\frac{\pi^2}{420}\frac{\hbar c_0}{L^4}$$

« Regularized » Casimir pressure

$$\int_{x<1} d^d\mathbf{x}\int_{y<1} d^d\mathbf{y}\frac{1}{|\mathbf{x}-\mathbf{y}|^{\gamma}} = \frac{\pi^{d-\frac{1}{2}}2^{d-\gamma}\Gamma\left(\frac{d-\gamma+1}{2}\right)}{(d-\gamma)\Gamma(d/2)\Gamma(d+1-\gamma/2)} \quad (d>\gamma/2)$$

$$\Rightarrow \int_{x<a} d^3\mathbf{x}\int_{y<a} d^3\mathbf{y}\frac{-23\alpha^2N^2}{4\pi|\mathbf{x}-\mathbf{y}|^7} \xrightarrow[\gamma=7]{d=3} +\frac{23(\varepsilon-1)^2}{1536\,a}$$

« Regularized » Casimir energy of N dipoles distributed in a sphere

Brevik etal, 1998

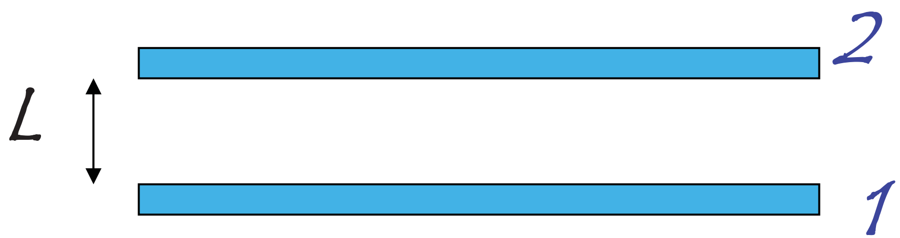
5.8 Lifshitz formula

Two weakly polarizable bodies 1 and 2

$$\langle \delta E \rangle = -\hbar \,\mathrm{Im}\,\mathrm{Tr}\int_0^\infty \frac{d\omega}{2\pi}\,\delta t_1(\omega)\delta t_2(\omega)\mathbf{G}^2(\omega,\mathbf{r}_{12})$$

$$= -\hbar\,\mathrm{Im}\,\mathrm{Tr}\int_0^\infty \frac{d\omega}{2\pi}\, n_1\alpha_1(\omega)d\mathbf{r}_1 \times n_2\alpha_2(\omega)d\mathbf{r}_2\,\frac{\omega^4}{c_0^4}\mathbf{G}^2(\omega,\mathbf{r}_{12})$$

$$\langle E_{casi} \rangle = -\frac{\hbar}{2\pi c_0^4}\int d\mathbf{r}_1 \int d\mathbf{r}_2 \int_0^\infty ds\, s^4\left[\varepsilon_1(is)-1\right]\left[\varepsilon_2(is)-1\right]\mathrm{Tr}\,\mathbf{G}^2(is,\mathbf{r}_{12})$$



$$\langle E_{casi} \rangle = -\frac{\hbar}{2\pi c_0^4}\int d\mathbf{r}_1 \int d\mathbf{r}_2 \int_0^\infty ds\, s^4\left[\varepsilon_1(is)-1\right]\left[\varepsilon_2(is)-1\right]\mathrm{Tr}\,\mathbf{G}^2(is,\mathbf{r}_{12})$$

$$= -\frac{\hbar}{2\pi c_0^4} A d^2 \int d^2\mathbf{x} \int_0^\infty ds\, s^4\left[\varepsilon_1(is)-1\right]\left[\varepsilon_2(is)-1\right]\mathrm{Tr}\left(\frac{1 - 3\hat{\mathbf{r}}\hat{\mathbf{r}}}{4\pi\left(x^2+L^2\right)^{3/2}\left(s/c_0\right)^2}\right)^2$$

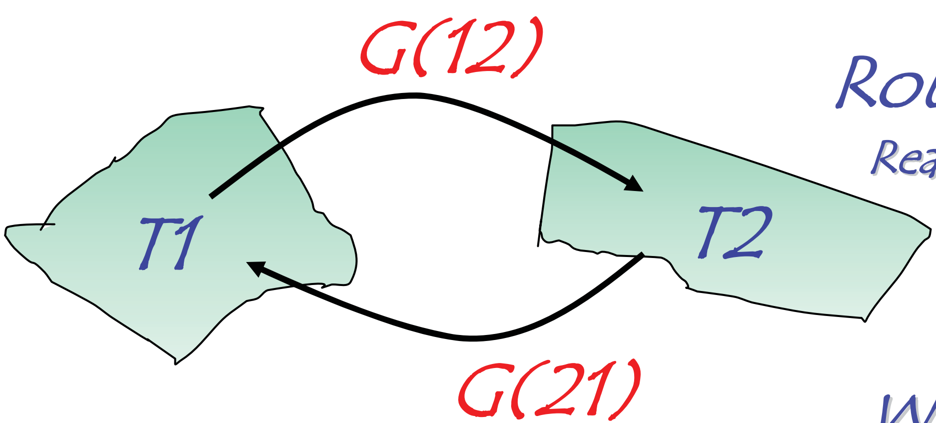
$$= -\frac{3\hbar}{2(4\pi)^2}\frac{Ad^2}{L^4}\int_0^\infty ds\left[\varepsilon_1(is)-1\right]\left[\varepsilon_2(is)-1\right]$$

5.9 Rigorous Lifshitz formula

$$\langle \delta E \rangle = \frac{\hbar}{2\pi} \operatorname{Im} \operatorname{Tr} \int_0^{i\infty} d(i\zeta) \log\left[1 - \mathbf{T}_1(i\zeta)\cdot\mathbf{G}_{12}(i\zeta)\cdot\mathbf{T}_2(i\zeta)\cdot\mathbf{G}_{21}(i\zeta)\right]$$

$$= \frac{\hbar}{2\pi} \operatorname{Tr} \int_0^{\infty} d\zeta \log\left[1 - \mathbf{T}_1(i\zeta)\cdot\mathbf{G}_{12}(i\zeta)\cdot\mathbf{T}_2(i\zeta)\cdot\mathbf{G}_{21}(i\zeta)\right]$$

$$= \frac{\hbar}{2\pi} \int_0^{\infty} d\zeta \log\det\left[1 - \mathbf{T}_1(i\zeta)\cdot\mathbf{G}_{12}(i\zeta)\cdot\mathbf{T}_2(i\zeta)\cdot\mathbf{G}_{21}(i\zeta)\right]$$



Roundtrip operator

Real-valued for imaginary frequencies

Controversy:

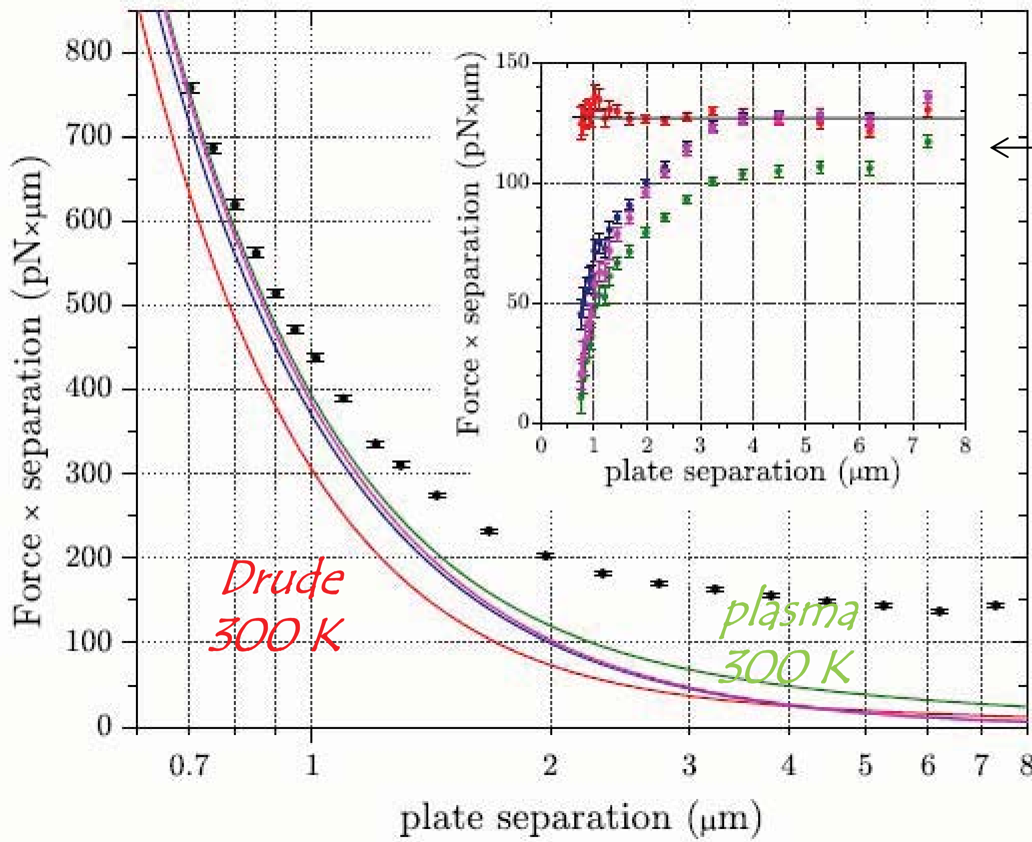
What model for ε(ω) ?

5.10 Thermal Casimir force: Drude or plasma model?

$$\varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2 + i\gamma\omega}$$

$$\varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2}$$

Drude or Plasma ?

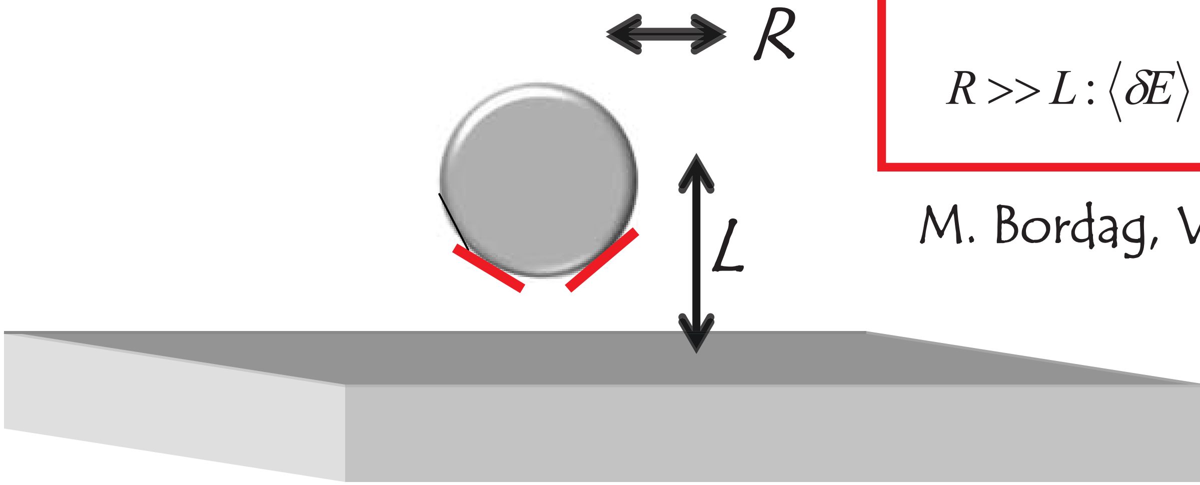


Exp-theo: Drude wins ?

electrostatic

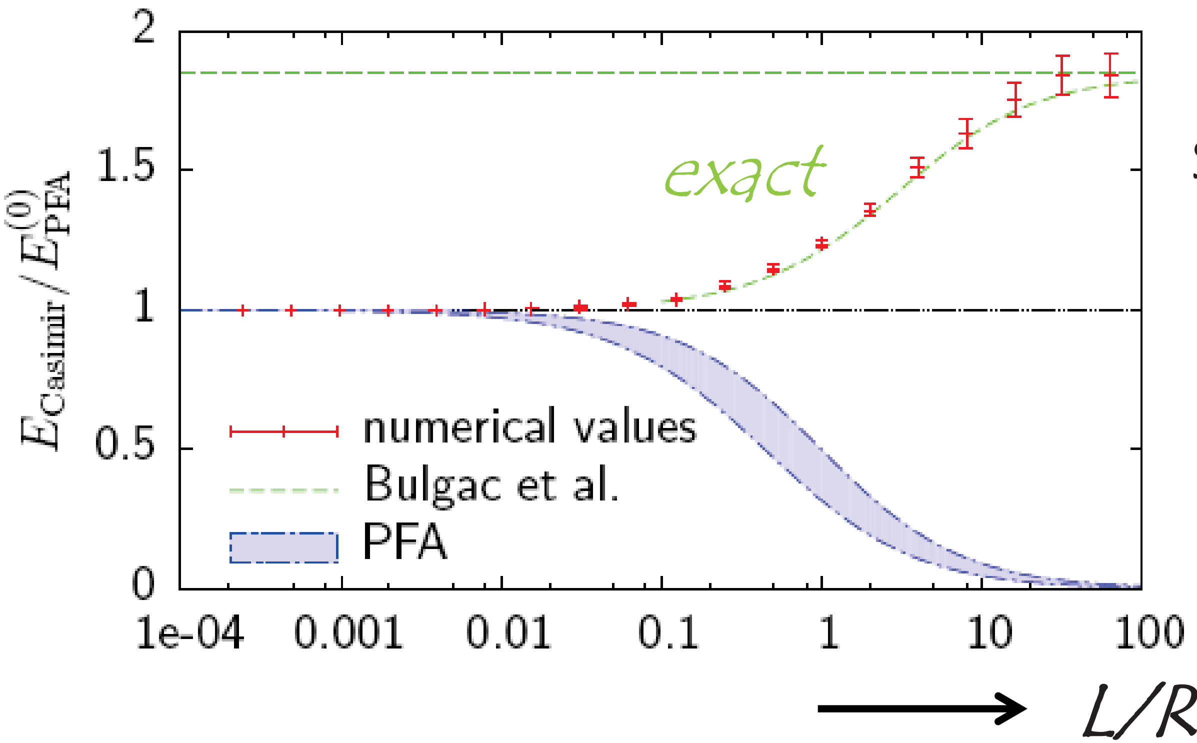
Sushkov, Kim, Dalvit, Lamoreaux, 2011

5.11 Proximity Force approximation



$$R >> L : \langle \delta E \rangle = -\frac{\pi^3 \hbar c_0}{720} \frac{R}{L^2}\left(1 - \frac{L}{R} + \ldots\right)$$

M. Bordag, V. Nikolaev, 2008 [37]

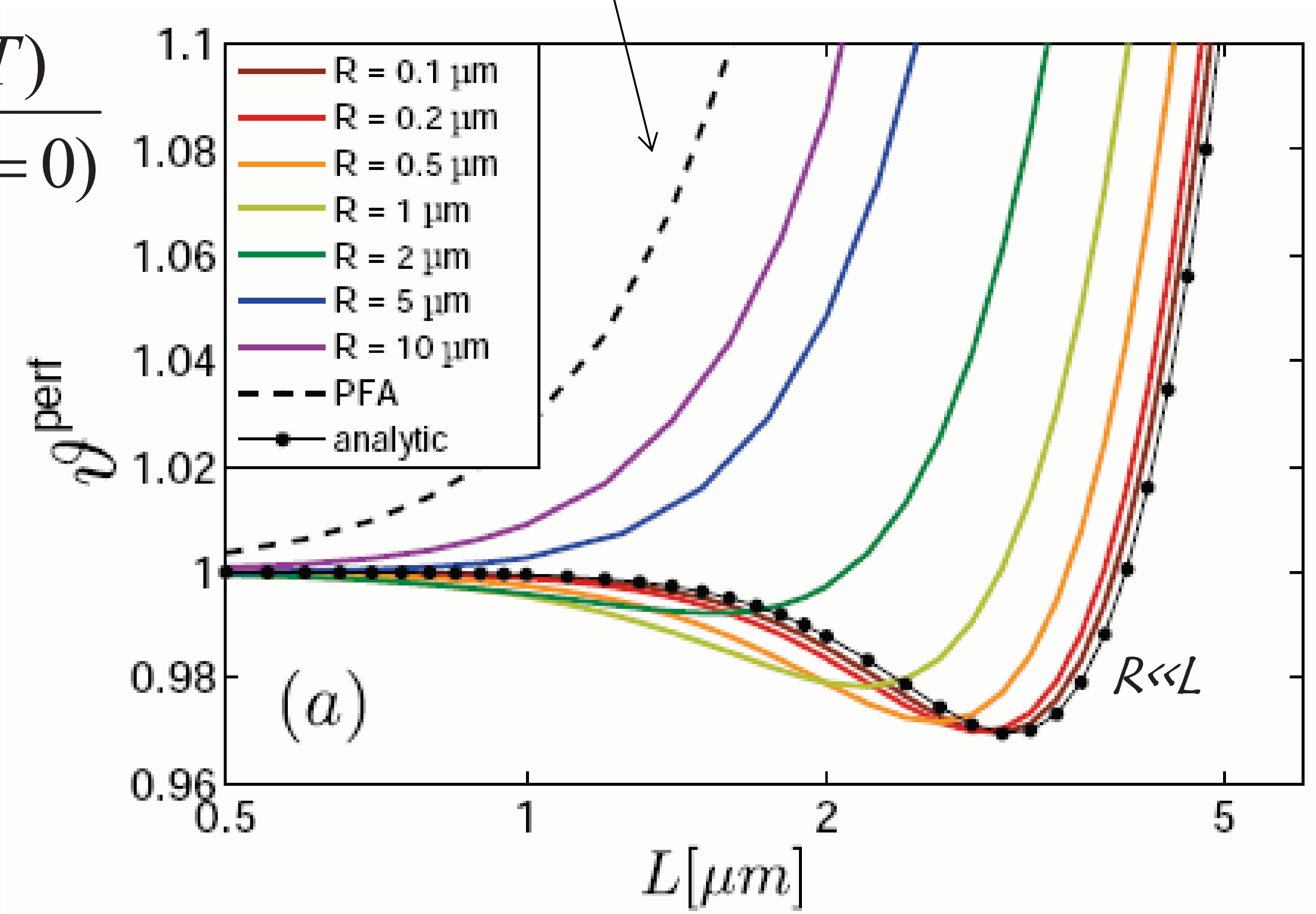


Scalar field with Dirichlet BC's
Gies & Klingmüller,
2006 [48]

5.12 Casimir force sphere-plane at ambient temperature

Proximity force approximation

$$\frac{F(T)}{F(T=0)}$$



Repulsive thermal photons

A. Canaguier-Durand, P. A. Maia Neto, A. Lambrecht, S. Reynaud, *Phys. Rev. Lett.* **104**, 040403 (2010). *Ref. 36*

6. Quantum vacuum friction: friction or fiction?

6.2 Quantum vacuum friction: friction or fiction?

A little (nonexhaustive) history:

Einstein 1917: friction of moving atom in thermal field (and zero at T=0) [3]

L Levitov: Van der Waals friction between moving dielectric bodies, Europhys Lett. 1989 [27]

J. Pendry 1997 Quantum vacuum shearing J. Phys. Cond Matt. 1997 [28]: disagrees with Levitov

Dupays, Rizzo etal: quantum friction from vacuum induced magnetic moment by rotating neutron stars, EPL 2008 [49]→ *paper seems unnoticed in Casimir community*

Philbin etal: macroscopic G-function diagonal: no poor man's friction à la Pendry, 2008 [50]

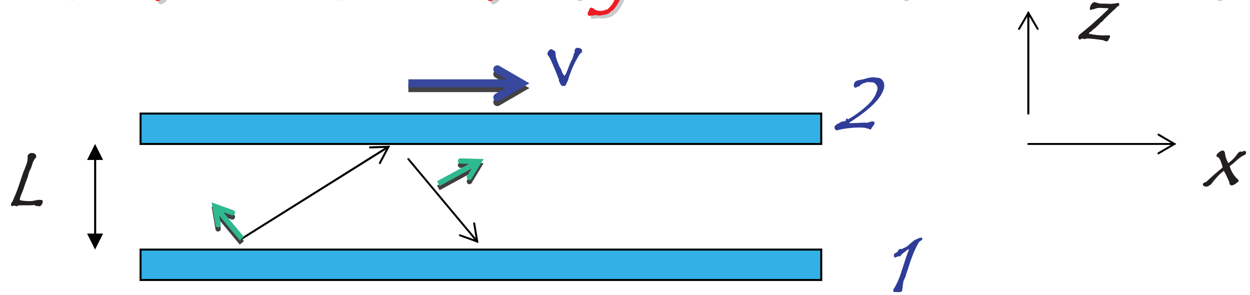
J. Pendry returns: poles of G Doppler shift in complex plane: friction = fact no friction, 2010 [51] (but in fact Ref 50 claims diagonaility throughout the complex plane)

Hoye and Brevik, dissipative quantum friction for T > 0 between moving oscillators, EPL 2010 [52], claims agreement with Barton

Volokitin and Persson (2011) [53] contest Ref 50 since no excitations. Contested by Ref. 50 [54] .

G. Barton, Van der Waals friction between atoms and between halfspaces (claims agreement with Pendry, contests Levitov, contests Hoye & Brevik) 2010, 2011 [55]

6.3 Quantum vacuum shearing : friction or fiction?



John Pendry,[28] : friction caused by evanescent low frequency modes

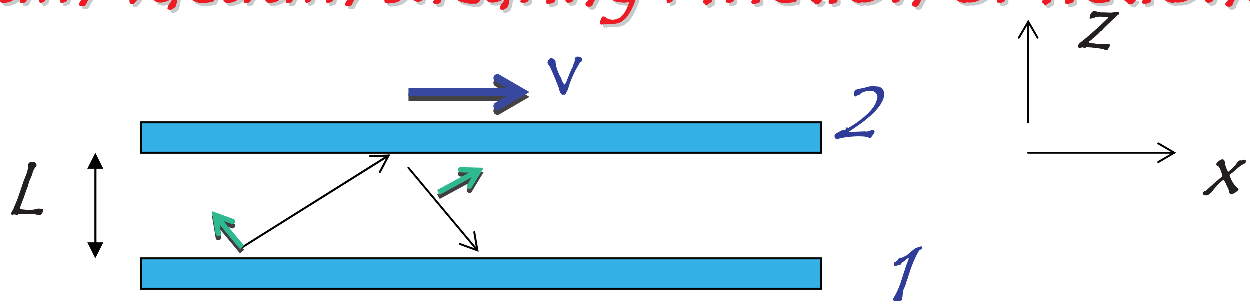
$$\langle T_{xz} \rangle \propto \langle E_x(z,\mathbf{k}_{//}) E_z^*(z,\mathbf{k}_{//}) \rangle \text{ for } v \neq 0$$

$$F_x = \frac{T_{xz}}{A} = \frac{\hbar}{2} \frac{\int d^2\mathbf{k}_{//}}{(2\pi)^2} \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} \operatorname{sign}(\omega) \exp(-k_{//}L) k_x \operatorname{Im} R_2(\omega + k_x v) \operatorname{Im} R_1(\omega)$$

$$= \frac{\hbar}{2(2\pi)^2} \int_0^{\infty} dk_x \int_{-\infty}^{+\infty} dk_y \int_0^{k_x \mathrm{v}} \frac{d\omega}{2\pi} \exp(-k_{//}L) k_x \operatorname{Im} R_2(k_x \mathrm{v} - \omega) \operatorname{Im} R_1(\omega)$$

No dissipation → F=0

6.4 Quantum vacuum shearing : friction or fiction?



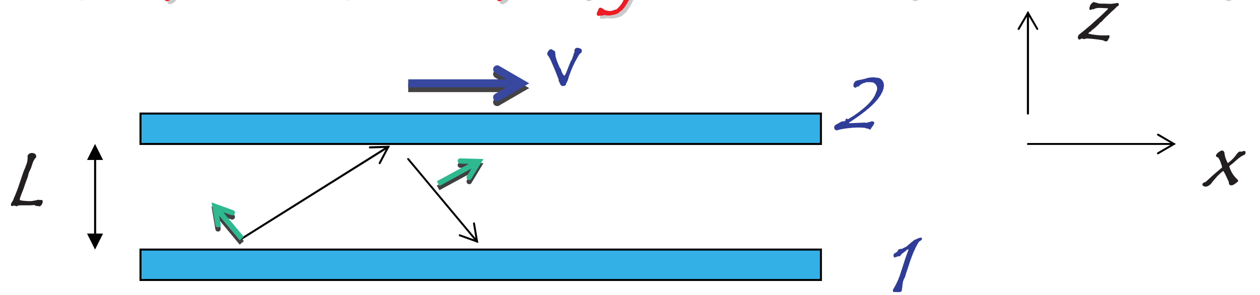
John Pendry,[28]: friction caused by evanescent low frequency modes

$$\varepsilon(\omega) = 1 + \frac{\sigma}{-i\omega\varepsilon_0} \Rightarrow \frac{F}{A} = \frac{5\hbar\varepsilon_0^2}{2^8\pi^2\sigma^2}\frac{\mathrm{v}^3}{\mathrm{L}^6}$$

$$\mathrm{v} = 1\ \mathrm{m/s}, \sigma = 0.1/\Omega\mathrm{m},\ L = 1\mathrm{nm} \Rightarrow \mathrm{F/A} = 3\cdot 10^{-7}\ \mathrm{N/cm^2}$$

Large!, but......

6.5 Quantum vacuum shearing : friction or fiction?



T. Philbin, U. Leonardt, 2009[50]

$$\left\langle T_{xy}(\mathbf{r}_{//},z)\right\rangle = \int \frac{d^2\mathbf{k}_{//}}{(2\pi)^2}\int_0^\infty \frac{d\omega}{2\pi}\left\langle \varepsilon_0 E_x(z,\mathbf{k}_{//})E_z^*(z,\mathbf{k}_{//})\right\rangle$$

$$= -2\hbar\int \frac{d^2\mathbf{k}_{//}}{(2\pi)^2}\int_0^\infty \frac{d\omega}{2\pi}\frac{\omega^2}{c_0^2}\,\mathrm{Im}\,G_{xz}\left(z,z,\omega,\mathbf{k}_{//}\right)$$

Green function is diagonal!

No friction?

Quantizing dissipative media

neglects exitations ?

Real microscopic theory?

7. Casimir Momentum

Work done in collaboration with

Geert Rikken (LNCMI)
James Babington (LPMMC)
Sébastien Kawka (LPMMC)

Support ANR Photonimpuls

7.2 « Momentum from Nothing »

VOLUME 92, NUMBER 2 PHYSICAL REVIEW LETTERS week ending 16 JANUARY 2004

Quantum Vacuum Contribution to the Momentum of Dielectric Media

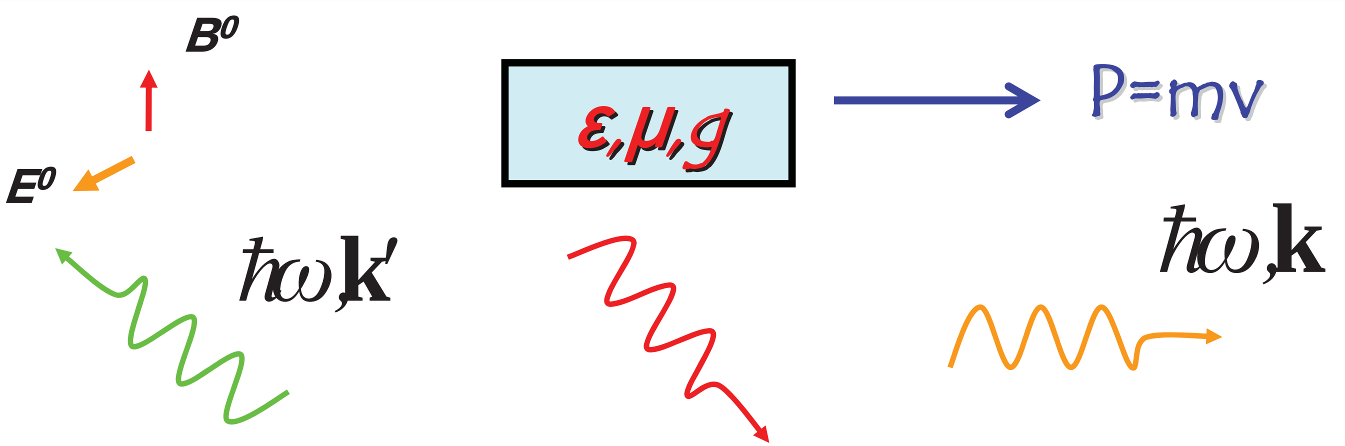
A. Feigel*

Department of Complex Systems, Weizmann Institute of Science, Rehovot 76100, Israel

(Received 3 February 2003; published 16 January 2004)

Momentum transfer between matter and electromagnetic field is analyzed. The related equations of motion and conservation laws are derived using relativistic formalism. Their correspondence to various, at first sight self-contradicting, experimental data (the so-called Abraham-Minkowski controversy) is demonstrated. A new, Casimir-like, quantum phenomenon is predicted: contribution of vacuum fluctuations to the motion of dielectric liquids in crossed electric and magnetic fields. Velocities of about 50 nm/s can be expected due to the contribution of high frequency vacuum modes. The proposed phenomenon could be used in the future as an investigating tool for zero fluctuations. Other possible applications lie in fields of microfluidics or precise positioning of micro-objects, e.g., cold atoms or molecules.

DOI: 10.1103/PhysRevLett.92.020404 PACS numbers: 03.50.De, 42.50.Nn, 42.50.Vk



Ref 29

7.3 Bi-anisotropic Media

$$\mathbf{D}(\omega) = \varepsilon(\omega)\mathbf{E}(\omega) + \mathbf{g}(\omega)\cdot\mathbf{B}(\omega)$$

$$\mathbf{H}(\omega) = \mathbf{g}^T(\omega)\cdot\mathbf{E}(\omega) + \mu(\omega)^{-1}\mathbf{B}(\omega)$$

Fresnel dispersion law

$$\det\left(\varepsilon\frac{\omega^2}{c_0^2} - k^2 + \mathbf{kk} - \frac{\omega}{c_0}\mathbf{g}\cdot(\boldsymbol{\varepsilon}\cdot\mathbf{k}) + \frac{\omega}{c_0}(\boldsymbol{\varepsilon}\cdot\mathbf{k})\cdot\mathbf{g}^*\right) = 0$$

$$g_{ij}(\omega) = i\omega\, g\delta_{ij}$$

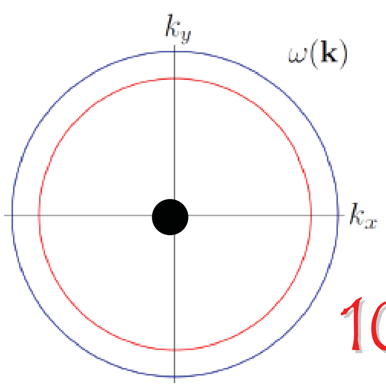
Rotatory power

$$g_{ij}(\omega) = (1-\varepsilon)\,\varepsilon_{ijl}\frac{v_l}{c_0}$$

Fizeau effect

$$g_{ij}(\omega) = g\left(E_i^0 B_j^0 - B_i^0 E_j^0\right)$$

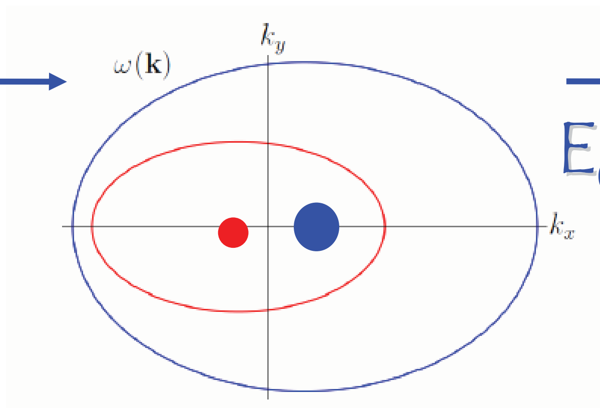
Magneto-electric birefringence



10^{-2}

V

10^{-8}



$E_0 \times B_0$

10^{-15}

7.4 phenomenological continuum theory

$$\partial_t\left(\rho\,\mathbf{v}+\varepsilon_0\mathbf{E}\times\mathbf{B}\right)=-\nabla\cdot\mathbf{T}^0$$

$$T_{ij}^0=\frac{1}{8\pi}\left(\varepsilon_0E^2+\frac{1}{\mu_0}B^2\right)\delta_{ij}-\frac{1}{4\pi}\left(\varepsilon_0E_iE_j+\frac{1}{\mu_0}B_iB_j\right)$$

Observed in X-ray

$$\left\langle 0\left|\frac{\mathbf{E}\times\mathbf{B}}{4\pi c_0}\right|0\right\rangle\propto\begin{cases}\dfrac{1}{c_0}\displaystyle\int d^3\mathbf{k}\,\frac{1}{2}\hbar\omega_k\;\times g(\omega)\mathbf{E}_0\times\mathbf{B}_0=\frac{2}{3}\frac{\hbar\omega_c^4}{\pi^3\,c_0^4}\,g\,\mathbf{E}_0\times\mathbf{B}_0\\[2ex]\dfrac{1}{c_0}\displaystyle\int d^3\mathbf{k}\,\frac{1}{2}\hbar\omega_k\;\times\left[\varepsilon(\omega)-1\right]\frac{\mathbf{v}}{c_0}=\rho_{casi}\mathbf{v}\end{cases}$$

Photonic momentum in dielectric media?

→ classical « Abraham » contribution already controversial

UV catastrophe of vacuum energy ?

Lorentz invariance of quantum vacuum?

Inertia of quantum vacuum?

7.5 Casimir momentum not excluded by Lorentz invariance

$$L(\mathbf{E},\mathbf{B}) = \frac{1}{2}(\mathbf{E}^2 - \mathbf{B}^2) + 2\nu(\mathbf{E}^2 - \mathbf{B}^2)^2 + \frac{\nu}{2}(\mathbf{E}\cdot\mathbf{B})^2$$

$$\mathbf{E} = \mathbf{E}_0 + \mathbf{E}(\omega)$$
$$\mathbf{B} = \mathbf{B}_0 + \mathbf{B}(\omega)$$

← Bi-anisotropic Lorentz-invariant vacuum

Fluctuation-Dissipation

$$\langle 0|E_i(\mathbf{r},\omega)E^*_j(\mathbf{r}',\omega')|0\rangle = -2\hbar\omega^2\,\mathrm{Im}G_{ij}(\mathbf{r},\mathbf{r}',\omega)\times 2\pi\delta(\omega-\omega')$$

$$\langle 0|\frac{c_0}{4\pi}\mathbf{E}^*\times\mathbf{H}|0\rangle = 0$$

Zero energy flow

$$\langle 0|\frac{\mathbf{E}^*\times\mathbf{B}}{4\pi c_0}|0\rangle = -\frac{4}{3}\nu K\ \mathbf{E}_0\times\mathbf{B}_0$$

infinite momentum density

$$K = \frac{1}{(2\pi)^3}\,\tfrac{1}{2}\hbar\int_0^\infty d\omega\int_{4\pi}d\Omega\,\rho_0(\omega,\Omega)$$

Infinite Lorentz scalar

Rikken & Rizzo (2003) [56]
Van Tiggelen, Rikken, 2009 [57]

7.6 ……..but does it satisfy general relativity?

$$R_{\mu\nu} - R g_{\mu\nu} + \Lambda g_{\mu\nu} = -\frac{8\pi G}{c_0^4} T_{\mu\nu} \Rightarrow T_{\mu\nu} = T_{\nu\mu}$$

$$T_{\nu\mu} = \begin{pmatrix} E_{casi} & (\mathbf{E}\times\mathbf{H})_{casi} = 0 \\ (\mathbf{E}\times\mathbf{B})_{casi} \\ = -\tfrac{4}{3}\nu E_{casi}(\mathbf{E}_0 \times \mathbf{B}_0) & [T_{ij}]_{casi} \\ \neq 0 & \end{pmatrix}$$

?

7.7 The Abraham Force

Macroscopic Maxwell → $\partial_t \mathbf{G}_M + \nabla \cdot \mathbf{T} = -\mathbf{f}$

$$\mathbf{G}_M = \mathbf{D} \times \mathbf{B}$$

$$\mathbf{f} = -E^2 \nabla \varepsilon - H^2 \nabla \mu$$

Minkowski

7.8 The Abraham Force (see Brevik [58])

$\mathbf{G}_A = \varepsilon_0 \mu_0 \mathbf{E} \times \mathbf{H}$

$= \frac{1}{c_0} \mathbf{S}$

$\mathbf{G}_N = \varepsilon_0 \mathbf{E} \times \mathbf{B}$

$= \mathbf{G}_0$

$$\partial_t \mathbf{G}_A + \nabla \cdot \mathbf{T} = -\mathbf{f} - \varepsilon_0 (\varepsilon_r - 1/\mu_r) \partial_t (\mathbf{E} \times \mathbf{B})$$

Abraham [58]

$$\partial_t \mathbf{G}_N + \nabla \cdot \mathbf{T} = -\mathbf{f} - \varepsilon_0 (\varepsilon_r - 1) \partial_t (\mathbf{E} \times \mathbf{B})$$

Nelson [59]

$$\ldots = -\mathbf{f} - \frac{2}{5} \left[\partial_t \mathbf{E}_0(t) \right] \times \mathbf{B}_0(t) - \mathbf{E}_0(t) \times \left[\partial_t \mathbf{B}_0(t) \right]$$

Peierls [60]

exp

$$\mathbf{F} = (\varepsilon - 1) V \partial_t \mathbf{E}_0(t) \times \mathbf{B}_0(t)$$

Walker & Walker, 1976 [61]

theo

Abraham momentum = kinetic momentum,
Minkowski momentum = conjugate momentum

Barnett (2010) [62]

Nelson momentum = pseudo momentum

Nelson (1991) [59]

7.9 The Abraham Force (Nelson version)

Macroscopic Maxwell →

$$\partial_t(\mathbf{D}\times\mathbf{B})-\partial_t(\mathbf{P}\times\mathbf{B})+\nabla\cdot\mathbf{T}=-\mathbf{f}-\partial_t(\mathbf{P}\times\mathbf{B})$$

$$\mathbf{f}=-E^2\nabla\varepsilon-H^2\nabla\mu$$

Maxwell-Lorentz force on induced polarization and current →

$$\partial_t\rho\mathbf{v}+\nabla\cdot\mathbf{U}=\mathbf{f}+\partial_t(\mathbf{P}\times\mathbf{B})$$

\+

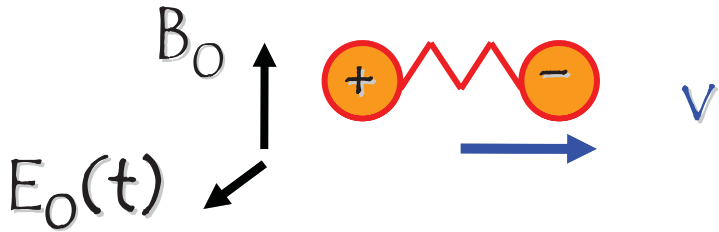
Microscopic Maxwell →

$$\partial_t\left(\rho\mathbf{v}+\varepsilon_0\mathbf{E}\times\mathbf{B}\right)+\nabla\cdot\mathbf{T}_0=0$$

$$\neq\mathbf{E}\times\mathbf{H}$$

symmetric

7.10 Classical Abraham momentum in crossed EM fields



$$\mathbf{r}_{1,2} = \mathbf{R} \pm \frac{1}{2}(\mathbf{x})$$

$$m\ddot{\mathbf{r}}_1 = +q\mathbf{E}(t) + q\dot{\mathbf{r}}_1 \times \mathbf{B} + \mathbf{f}(r_{12})$$

$$m\ddot{\mathbf{r}}_2 = -q\mathbf{E}(t) - q\dot{\mathbf{r}}_2 \times \mathbf{B} - \mathbf{f}(r_{12})$$

$$2m\dot{\mathbf{R}} + q\mathbf{x} \times \mathbf{B} = \text{constant} = 0$$

$$m\ddot{\mathbf{x}} = 2q\mathbf{E}(t) + 2q\,\dot{\mathbf{R}} \times \mathbf{B} - m\omega_0^2\,\mathbf{x} \approx 0$$

$$2m\dot{\mathbf{R}} = \frac{q^2/m}{\omega_0^2}\mathbf{E}_0(t) \times \mathbf{B}_0$$

No controversy exists in microscopic description
Consistent with Abrahams and Nelson version

$$\frac{q^2/m}{\varepsilon_0\omega_0^2} = \alpha(0) = \frac{\varepsilon - 1}{n}$$

7.11 The UV catastrophe is real in macroscopic description

Free electron (electric dipole)

$$\varepsilon(\omega)-1=-\frac{\omega_P^2}{\omega^2} \qquad \rho_{casi}=\frac{\hbar}{c_0^3}\int_0^\infty d\omega\,\omega^3\frac{\omega_p^2}{\omega^2}=\infty$$

magnetic dipole

$$g_{ME}(\omega)=-\frac{e^4 m_\Delta^2}{\omega_0^2 m^3 M^2}\Big[-\frac{\omega^2+\omega_0^2}{(\omega_0^2-\omega^2)^2}(\mathbf{E}_i^0\mathbf{B}_j^0-(\mathbf{E}^0\cdot\mathbf{B}^0)\delta_{ij})$$
$$+\frac{1}{\omega_0^2-\omega^2}(\mathbf{E}_i^0\mathbf{B}_j^0-\frac{1}{4}\mathbf{E}_j^0\mathbf{B}_i^0-\frac{1}{4}(\mathbf{E}^0\cdot\mathbf{B}^0)\delta_{ij})\Big].$$

Electric quadrupole

$$P_{casi}=\frac{\hbar}{c_0^3}\int dr\int_0^\infty d\omega\,\omega^3 g(\omega)E_0\times B_0=\infty$$

Rizzo etal, 2003, 2009 [63], Babington & BAvT, 2011, [64]

$$\mathbf{P}_{casi}=\frac{\hbar c_0 g(\varepsilon(0)-1)}{a}\mathbf{E}_0\times\mathbf{B}_0?$$

Dimensional regularization for object of size a?
BAvT 2009 [65]

7.11 Observation of the Abraham Force

Ex: Helium

E0=450 V/mm; B0=1 T

$\alpha(0)=0.22\ 10^{-40}\mathrm{Cm^2/V}\ \ (16.6a_0^3)$

$\rho=0.17\ \mathrm{kg/m^3}\ (\mathrm{room}\ T)$

$g=0.017\ 10^{-22}\mathrm{m/VT}$

(SI units)

$$v_{\mathrm{abr}} = \frac{\varepsilon_0 \alpha(0) EB}{2\mathrm{m}_p} \approx 0.3\ \mathrm{nm/sec}$$

Classical Abraham Force

$$F_{\mathrm{abr}} \approx 7 \cdot 10^{-32}\ N$$

$$N_{at} F_{\mathrm{abr}} \propto 10^{-13}\ N$$

$$v_{\mathrm{Feigel}} = \frac{\pi}{4} \frac{h}{\rho \lambda_c^4} gEB \approx 0.02\ \mathrm{nm/sec}$$

Semi-classical QED with cut-off 0.1 nm (Feigel [29])

$$\mathrm{v}_{QED} \propto \mathrm{v}_{\mathrm{abr}} \times (Z?\alpha)^2 \approx 0.001\mathrm{nm/sec}$$

Rigorous QED (Kawka, 2010 [66])

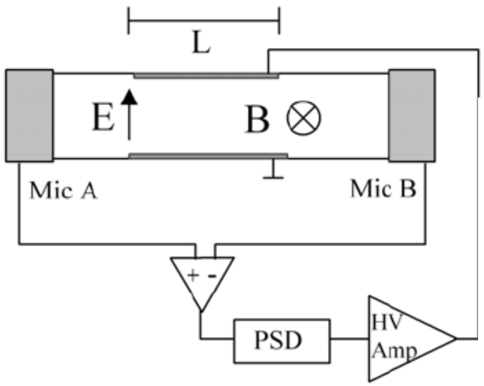
$$\frac{dp}{dt} = \alpha(0)\frac{dE}{dt} \times B$$

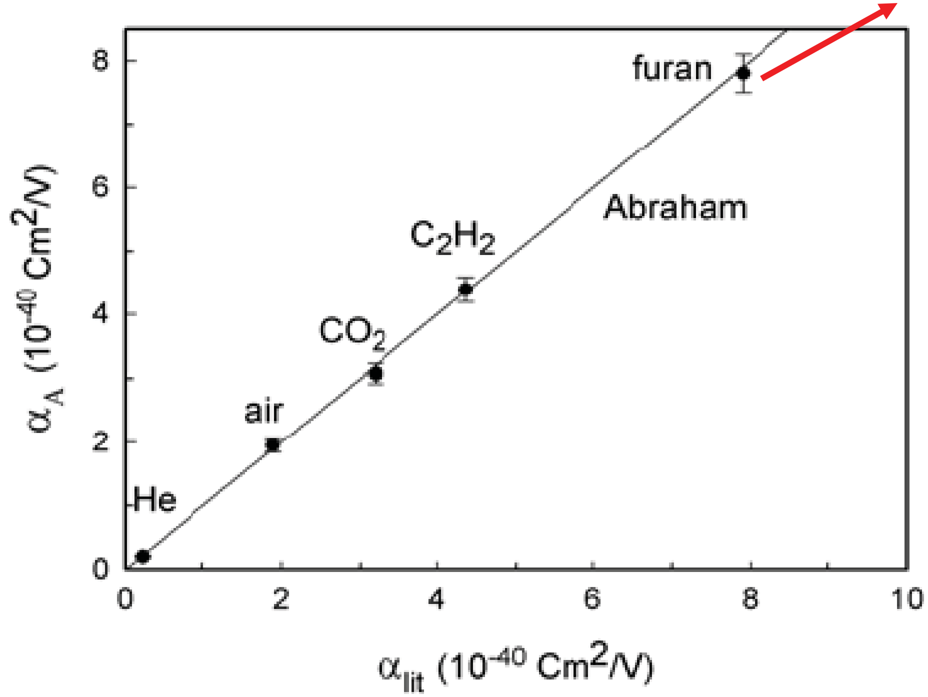
Abraham force

Acoustic pressure

$$P(\omega) = P_0 + \alpha(0) \times E \times B \times \omega \times \cos\omega t \times n \times L$$

V= 8 nm/sec+- 0.8
Feigel correction: 2 nm/sec
Excluded by errorbars





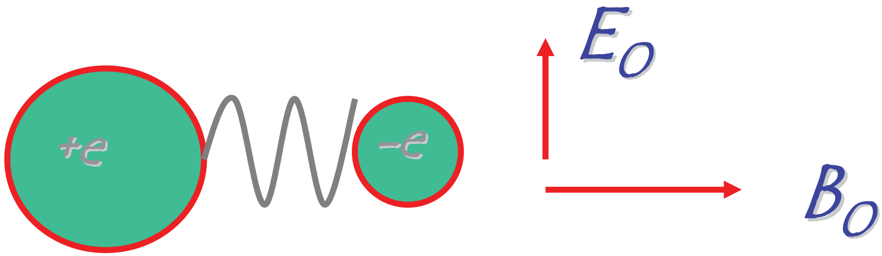
E=450 V/mm;
B=1 T;
f= 7.6 kHz

δP/(EB)

α(0)

Rikken / Van Tiggelen, 2011 [67]

Casimir momentum: 1/6
QED of atom in crossed fields



$$\mathbf{A}_0 = \frac{1}{2}\mathbf{B}_0 \times \mathbf{r} \qquad \phi = -\mathbf{E}_0 \cdot \mathbf{r}$$

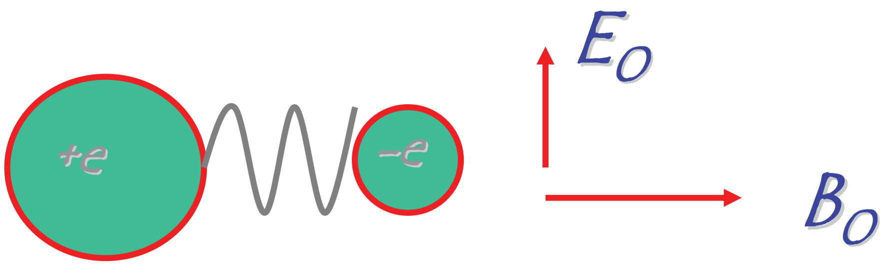
Coulomb Gauge

$$H = \frac{1}{2m_1}\left(\mathbf{p}_1 - e\mathbf{A}_0(\mathbf{r}_1) - e\mathbf{A}(\mathbf{r}_1)\right)^2 + \frac{1}{2m_2}\left(\mathbf{p}_2 + e\mathbf{A}_0(\mathbf{r}_2) + e\mathbf{A}(\mathbf{r}_2)\right)^2$$

$$+ e\mathbf{E}_0 \cdot \mathbf{r}_{21} + V(r_{12})$$

$$+ \sum_i \hbar\omega_i \left(a_i^* a_i + 1/2\right)$$

Casimir momentum: 2/6
QED of atom in crossed fields



Conjugate momenta
≠ kinetic momentum

$$\mathbf{p}_1 = m_1\mathbf{v}_1 + e\mathbf{A}_0(\mathbf{r}_1)$$
$$\mathbf{p}_2 = m_1\mathbf{v}_2 - e\mathbf{A}_0(\mathbf{r}_2)$$

Pseudo momentum

$$\hat{\mathbf{K}} = \mathbf{p}_1 + \mathbf{p}_2 + \frac{1}{2}e\mathbf{B}_0 \times \mathbf{r}_{21} = \mathbf{P}_{kin} + e\mathbf{B}_0 \times \mathbf{r}$$

Pseudo momentum is conserved

$$[\mathbf{K}, H] = 0$$

Coulomb Gauge

Ground state changes due to coupling with quantum vacuum

$$|\tilde{\Psi}_0\rangle = \left[1 - \frac{1}{2}\sum_{i\mathbf{Q}n}{}' \frac{|W_{i\mathbf{Q}n,0\mathbf{Q}_0 0}|^2}{(E_{0\mathbf{Q}_0 0} - E_{i\mathbf{Q}n})^2}\right] |0\mathbf{Q}_0\{0\}\rangle$$

$$+ \sum_{i\mathbf{Q}n}{}' \frac{W_{i\mathbf{Q}n,0\mathbf{Q}_0 0}}{E_{0\mathbf{Q}_0 0} - E_{i\mathbf{Q}n}} |i\mathbf{Q}n\rangle$$

$$+ \sum_{i\mathbf{Q}n}{}' \sum_{i'\mathbf{Q}'n'}{}' \frac{W_{i\mathbf{Q}n,i'\mathbf{Q}'n'} W_{i'\mathbf{Q}'n',0\mathbf{Q}_0 0}}{(E_{0\mathbf{Q}_0 0} - E_{i\mathbf{Q}n})(E_{0\mathbf{Q}_0 0} - E_{i'\mathbf{Q}'n'})} |i\mathbf{Q}n\rangle$$

$$- W_{0\mathbf{Q}_0 0,0\mathbf{Q}_0 0} \sum_{i\mathbf{Q}n}{}' \frac{W_{i\mathbf{Q}n,0\mathbf{Q}_0 0}}{(E_{0\mathbf{Q}_0 0} - E_{i\mathbf{Q}n})^2} \rangle |i\mathbf{Q}n\rangle \qquad (12)$$

$\delta M\mathbf{v}+$ $\langle\psi_0|e\mathbf{A}|\psi_0\rangle$

$0.84\alpha^3$

$\alpha(\omega=0, \mu+\delta\mu)$

$\langle 0|\mathbf{A}|0\rangle = 0$

Casimir momentum: 3/6
QED of hydrogen atom in crossed fields

$$\hat{\mathbf{K}} = m_1\mathbf{v}_1 + m_2\mathbf{v}_2 + e\mathbf{A}(\mathbf{r}_1) - e\mathbf{A}(\mathbf{r}_2) + e\mathbf{B}_0 \times \mathbf{r}_{21} + \sum_i \hbar\mathbf{k}_i\left(a_i^* a_i + 12\right)$$

No multipole approximation in $\mathbf{A}(\mathbf{r}) \propto \sum_{gk} \mathbf{g}_k \exp(i\mathbf{kr})\, a_{\mathbf{gk}} + c.c$

$$\langle \Psi_0 | \mathbf{K} | \Psi_0 \rangle = M\mathbf{v} + \delta M\, \mathbf{v} + \frac{8}{3}\frac{E_0}{c_0^2}\mathbf{v}$$

$$+ \varepsilon_0 \alpha(0)\mathbf{B}_0 \times \mathbf{E}_0 + \varepsilon_0\, \delta\mu\, \partial_\mu \alpha(0)\mathbf{B}_0 \times \mathbf{E}_0 + \mathbf{K}_1 + \mathbf{K}_2$$

$$\delta M = \delta\left(m_1 + m_2\right) \quad \delta\mu = \delta\left(\frac{m_1 m_2}{m_1 + m_2}\right) \quad \delta m_i = \frac{4}{3\pi}\alpha\hbar \int_0^\infty dk\, \frac{\hbar k}{\hbar^2 k^2/2\mathrm{m}_i + \hbar k c}$$

Casimir momentum: 4/6
QED of hydrogen atom in crossed fields

$$\hat{\mathbf{K}} = m_1\mathbf{v}_1 + m_2\mathbf{v}_2 + e\mathbf{A}(\mathbf{r}_1) - e\mathbf{A}(\mathbf{r}_2) + e\mathbf{B}_0 \times \mathbf{r}_{21} + \sum_i \hbar\mathbf{k}_i\left(a_i^* a_i + \frac{1}{2}\right)$$

$$-\frac{m_1^2\mathbf{v}_1^2}{2c_0^2}\mathbf{v}_1 + \text{idem } 2$$

Atomic binding energy dominated by Casimir energy

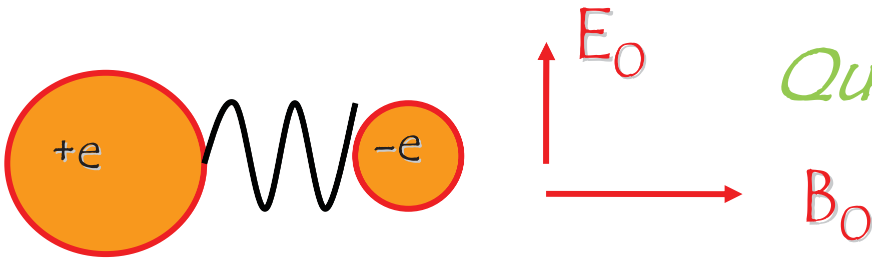
$$\langle\Psi_0|\mathbf{K}|\Psi_0\rangle = M\mathbf{v} + \delta M\,\mathbf{v} + \frac{8}{3}\frac{E_0}{c_0^2}\mathbf{v} - \frac{5}{3}\frac{E_0}{c_0^2}\mathbf{v}$$

$$+\varepsilon_0\,\alpha(0)\mathbf{B}_0\times\mathbf{E}_0 + \delta(\mu)\varepsilon_0\partial_\mu\,\alpha(0)\mathbf{B}_0\times\mathbf{E}_0$$

$$+\mathbf{K}_1 + \mathbf{K}_2 + \mathbf{K}_R$$

$$\delta M = \delta(m_1 + m_2) \quad \delta\mu = \delta\left(\frac{m_1 m_2}{m_1 + m_2}\right) \quad \delta m_i = \frac{4}{3\pi}\alpha\hbar\int_0^\infty dk\,\frac{\hbar k}{\hbar^2 k^2/2\mathrm{m}_i + \hbar kc}$$

Casimir momentum: 5/6
QED of hydrogen in crossed fields



Quantum vacuum contribution:

$$\mathbf{K}_1 = -\mathbf{B}_0 \times \mathbf{E}_0 \frac{1}{3}\frac{e^2\hbar^2}{a_0 c_0^2 \mu^2}\sum_n \langle 0 | \frac{e^2}{4\pi\varepsilon_0 r}\hat{\mathbf{r}} | n \rangle \cdot \frac{1}{(E_n - E_0)^2} \cdot \langle n | \mathbf{r} | 0 \rangle$$

$$= -\varepsilon_0 \alpha(0) \mathbf{B}_0 \times \mathbf{E}_0 \alpha^2 (\, 0.208 + 0.0045 \,) = -0.21\alpha^2 \, \mathbf{K}_A$$

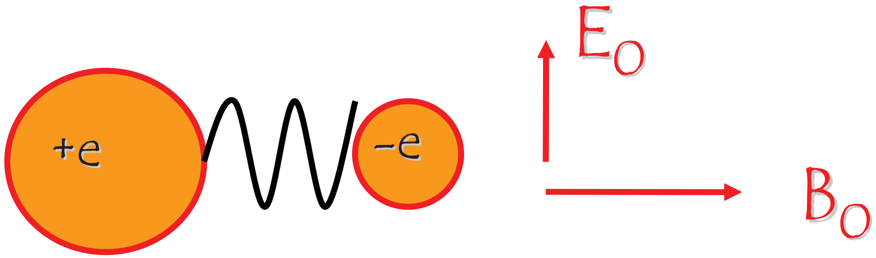
$$\mathbf{K}_2 = +\varepsilon_0 \alpha(0) \mathbf{B}_0 \times \mathbf{E}_0 \, \alpha^2 \frac{1}{27}\frac{e^2}{4\pi\varepsilon_0 a_0^2}\sum_n \langle 0 | \hat{\mathbf{r}} | n \rangle \cdot \frac{1}{E_n - E_0} \cdot \langle n | \mathbf{r} | 0 \rangle$$

$$= +\varepsilon_0 \alpha(0) \mathbf{B}_0 \times \mathbf{E}_0 \alpha^2 (\, 0.079 + 0.018 \,) = +0.1\alpha^2 \, \mathbf{K}_A$$

Discrete Rydberg states

Continuous spectrum
assuming
plane waves for electrons

Casimir momentum: 6/6
QED of hydrogen in crossed fields

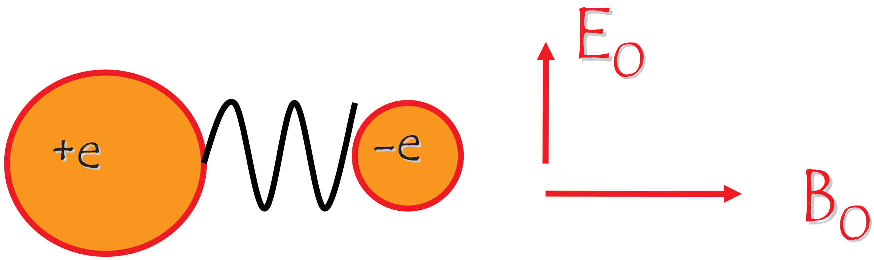


Relativistic contribution:

$$\mathbf{K}_R = -\frac{e^2}{2\mathrm{m}_e M c_0^2} \langle 0_E \,|\, p^2 (\mathbf{B}_0 \times \mathbf{x}) \,|\, 0_E \rangle$$

$$\propto \alpha^2 \frac{m_e}{M} \mathbf{K}_A$$

Casimir momentum: 6/6
QED of hydrogen in crossed fields



$$\langle \mathbf{K} \rangle = (m_e + m_p)\mathbf{v} + \frac{E_0}{c_0^2}\mathbf{v} + \mathbf{K}_A - 0.1\,\alpha^2 \mathbf{K}_A + O(\alpha^3, \alpha^2 \frac{m_e}{m_p})$$

$$\mathbf{K}_A = \varepsilon_0 \alpha(0) \mathbf{B}_0 \times \mathbf{E}_0$$

Casimir momentum of H atom exists
and slightly reduces the classical Abraham momentum

Casimir mass being equivalent to binding energy is same physics

BaVT, Kawka, Rikken, submitted to EPJD [68]

SUMMARY
Casimir momentum in crossed E,B

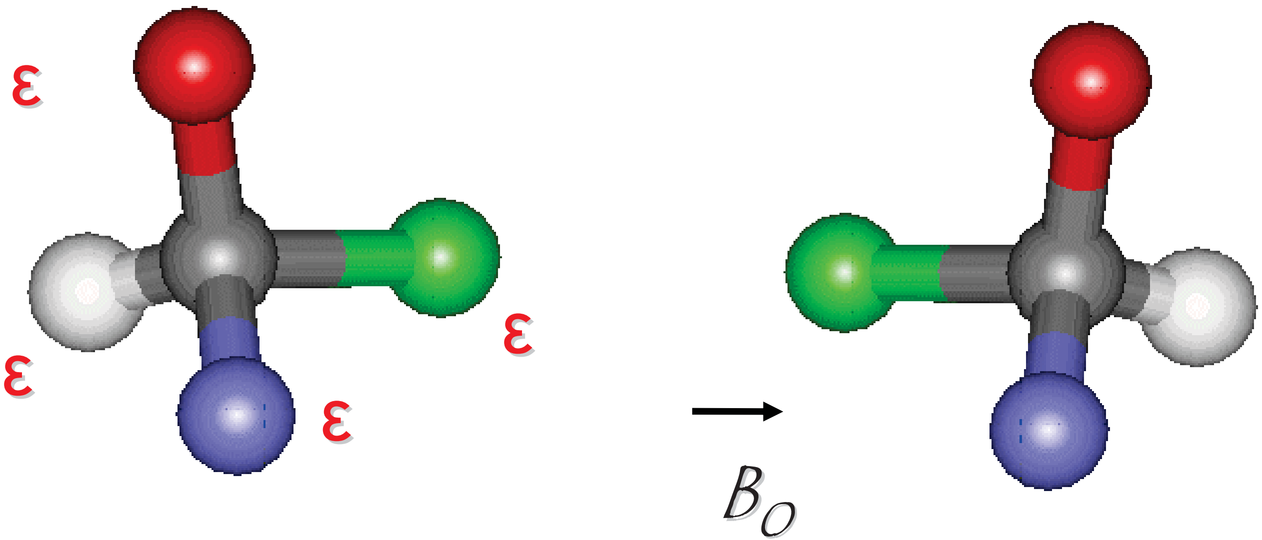
- Classical Abraham force , linear in E_0 and B_0 ,
  is observed for neutral atoms
  (and for strong dielectrics)
- QED contribution by Feigel is not observed
- UV divergencies disappear in mass renormalization .
  or cancel.
  Need to go beyond multipole approximation
- Quantum vacuum contributes to
  Abraham momentum in order $-(1/137)^2$
  Will this be $-(Z/137)^2$ for $Z > 1$??

Magneto-Chiral Casimir momentum?

$$\langle E \times B \rangle = g B_0 \,?$$

- Classically no equivalent Abraham version in charge neutral systems
- g must be a pseudo scalar
  → medium must be chiral (on nanoscale)
- Describe chirality microscopically,
  not phenomenologically using Lifshitz formula
  via « magneto-chiral « index of refraction ($\Delta n = g\, B_0.k$)
- Would separate enantiomers using magnetic fields
  = Pasteurs dream !
- Medium must have induced magnetic dipole since $\langle E \times H \rangle = 0$

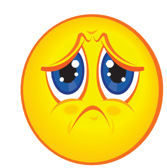
Pasteur's dream with a Casimir momentum $P = g\, B_O$?



Chiral geometry with electric polarizabilities with Zeeman splitting

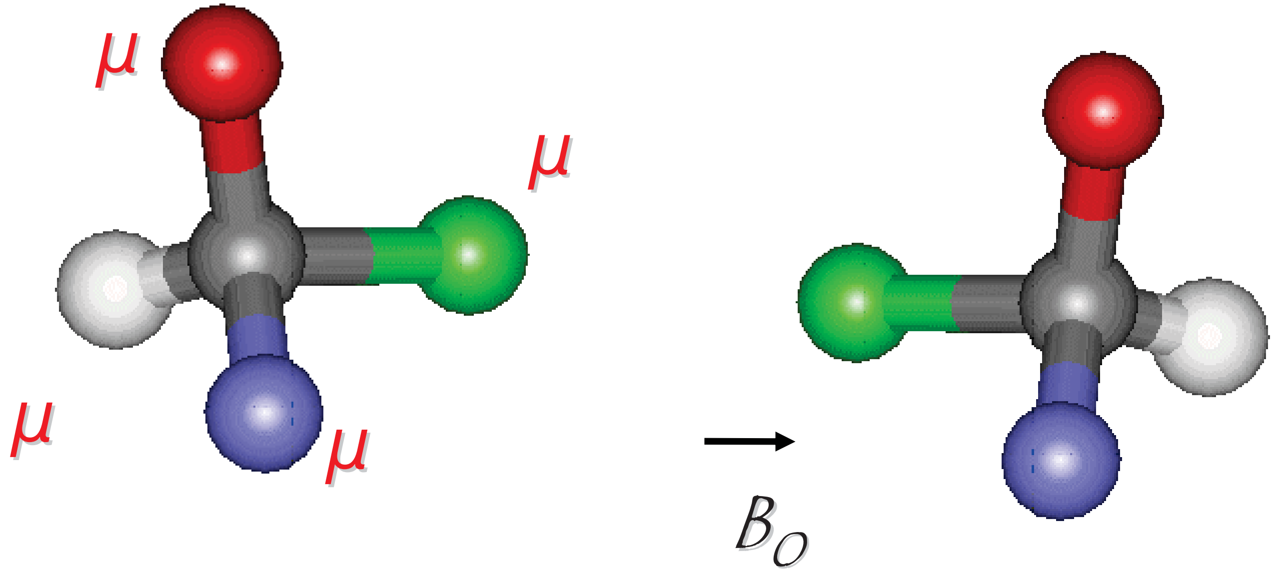
Pinheiro and BAvT [69]

$$\alpha(\omega,\sigma) = \frac{4\pi c_0^2}{\omega_0^2}\,\frac{\gamma}{\omega^2 - \omega_0^2 + i\sigma VB + i\gamma\omega_0}$$



$$\mathbf{B} = \mu_0 \mathbf{H} \Rightarrow \langle 0 | \int d\mathbf{r}\, \mathbf{E}\times\mathbf{B} | 0 \rangle \propto \langle 0 | \int d\mathbf{r}\, \mathbf{E}\times\mathbf{H} | 0 \rangle = 0$$

A Casimir momentum $P = g\, B_0$? Pasteur's dream!



Chiral geometry with magnetic polarizabilities with Zeeman splitting

$$\chi(\omega,\sigma) = \chi(0)\frac{\omega_0^2}{\omega^2 - \omega_0^2 + i\sigma VB + i\gamma\omega}$$

$$\langle 0 | \int d\mathbf{r}\, \mathbf{E}\times\mathbf{H} | 0 \rangle = 0$$

$$\langle 0 | \int d\mathbf{r}\, \mathbf{E}\times\mathbf{B} | 0 \rangle = g\mathbf{B}_0$$

$$g = \left(\frac{4\hbar c}{3e\mu_0}\right)\left(\iota\chi(0)\mu_0\right)^5\left(\frac{1.4\times 10^4}{L^{14}}\right)$$

Na Tetraeder L=10 nm → g/m = 1 nm/sec/T

Babington, BaVT, 2011[70]

Very nonexhaustive bibliography

See: James F. Babb (Harvard) : https://www.cfa.harvard.edu/~babb/casimir-bib.html

1. See M.L. Shih and P.W. Milonni , Am. J. Phys. 59, 684 (1991).
2. O. Stern and A. Einstein, Ann. Physik 40, 551 (1913).
3. For a simplified derivation see Appendix B of P.W. Milonni, The Quantum Vacuum, an introduction to Quantum Electrodynamics, Academic Press (San Diego 1994)
4. F. London, Z. Phys. 63, 245 (1930).
5. See e.g. N.D. Birrel and P.C.W. Davies, Quantum Fields in Curved Spaces (Cambridge, 1984)
6. H. B. G. Casimir, and D. Polder, Physical Review, Vol. 73 (4), 360 (1948).
7. W.E. Lamb and R.C Retherford, Phys. Rev.72 (3): 241 (1947).
8. H. Bethe, Phys. Rev. 72, 339 (1947).
9. J. Schwinger, Phys. Rev. 73, 416L (1948) .
10. H. B. G. Casimir, Proceedings of the Royal Netherlands Academy of Arts and Sciences, 51, 793–795 (1948).
11. E. M. Lifshitz, Soviet Phys. JETP, 2, 73 (1956).
12. I.E.Dzyaloshinskii, IE.M. Lifshitz, L.P. Pitaevskii,,Sov.Phys. Uspekhi 4 ;153 (1961).
13. M.J. Sparnaay, Nature 180 (4581): 334; Physica 24 (6–10): 751 (1958).
14. S.K. Lamoreaux, Phys.Rev.Lett.78 , 5 (1997).
15. U.Mohideen, A. Roy, Phys. Rev. Lett. 81 (21): 4549 (1998).

16. T. Ederth, Phys. A 62, 062104 (2000) .
17. H. G. B. Casimir, Physics 19, 846 (1956).
18. T. H. Boyer, Phys. Rev. 174, 1764 (1968).
19. S. W. Hawking, Nature 248 (5443): 30.
20. W.G. Unruh , Phys. Rev. D 14 (4): 870 (1976), . (1974).
21. S A Fulling and P C W Davies Proc. R. Soc. A 348 393 (1976).
22. A. Chodos, R.L. Jaffe, etal, Phys. Rev. D 9, 3471 (1974).
23. S. Weinberg, Rev. Mod. Phys. 61, 1–23 (1989) .
24. M.-T. Jaekel, S. Reynaud, J. Phys. I France 3 1093 (1993) .
25. J. Schwinger, Proc. Natl. Acad. Sci. USA 89, 4091 (1992).
26. C. Eberlein, Phys. Rev. Lett. 76, 3842–3845 (1996)
27. L. S. Levitov ,Europhys. Lett. 8 499 1989.
28. J. B. Pendry, J. Phys.:Cond. Matt. 9, 10301 (1997).
29. A. Feigel, Phys. Rev. Lett. 93, 268904 (2004).
30. O. Kenneth and I. Klich, Phys. Rev. Lett. 97 160401 (2006)
31. A.A. Feiler etal, Langmuir, 24 (6), 2274 (2008).
32. J.N. Munday, F. Capasso, V.A. Parsegian, Nature 457 , 7226 (2006).
33. K. A. Milton, S.A. Fulling etal, J.Phys.A41:164052 (2008).

34. A. Lambrecht, P.A. Maia-Neto, and S. Reynaud, NJP 8, 243 (2006);
35. A.Lambrecht,, A. Canaguier-Durand,, R. Guérout and S. Reynaud, Lecture Notes in Physics, 834 (2011).
36. A. Canaguier-Durand, P. A. Maia Neto, A. Lambrecht, and S. Reynaud Phys. Rev. A 82, 012511 (2010)
37. S Reynaud , P.A.M. Neto P. A. M., A. Lambrecht A , J. Phys. A: Math. Theor. 41 164004 (2008).
38. D. A. R. Dalvit, P. A. Maia Neto, A. Lambrecht, S. Reynaud, Phys. A: Math. Theor. 41, 164028 (2008) .
37. M. Bordag, V. Nikolaev, J. Phys. A. Math. Theor. 41, 164002 (2008).
38. C.M. Wilson etal, Nature 479, 376 (2011).
39. A. O. Sushkov, W. J. Kim, D. A. R. Dalvit & S. K. Lamoreaux, Nature Physics 7, 230 (2011)
40. K.A. Milton, *The Casimir effect*, World Scientific, Singapore, 2001
41. E.G. Adelberger, B.R. Heckel, A.E. Nelson, Ann.Rev. Nucl. Part. Science Vol. 53: 77-1217 (2003)
42. D.J. Kapner etal, Rev. Lett. 98, 021101 (2007).
43. A. O. Sushkov, W. J. Kim, D. A. R. Dalvit & S. K. Lamoreaux, Phys. Rev. Lett. 107, 171101 (2011).
44. P.W. Milonni, *The Quantum Vacuum, an introduction to Quantum Electrodynamics*, Academic Press (San Diego 1994).

45. B.A. van Tiggelen, in: *Atomic Matter Waves*, edited by F. David, Ch. Westbrook, and R. Kaiser (Kluwer, Dordrecht, 2000)
46. I. Brevik, V.N. Marachevsky, K.A. Milton, Phys. Rev. Lett 82, 3948 – 3951 (1999)
47. S. Kawka, Ph.D. thesis, Université de Grenoble 1 (2010).
48. H. Gies and K. Klingmüller, Phys. Rev. Lett. 96, 220401 (2006).
49. A. Dupays, C. Rizzo, D. Bakalov, G.F. Bignami, EPL 82 , 69002 (2008) .
50. T.G. Philbin and U. Leonhardt, New J. Phys. 11, 033035 (2009).
51. J. Pendry, NJP 12, 033028 (2010).
52. J.S. Hoye and I. Brevik, EPL 91, 60003 (2010).
53. A.I. Volokitin and B.N.J. Persson, New J. Phys. 13 068001(2011);
54. T.G. Philbin and U. Leonhardt , New J. Phys. 13 068002(2011) .
55. G. Barton, New J. Phys. 12 113044 (2010); New J. Phys. 12 113045 (2010).
56. G. L. J. A. Rikken and C. Rizzo, Phys. Rev. A 63, 012107 (2003).
57. B.A. van Tiggelen and G.L.J.A. Rikken Phys. Rev. Lett. 100, 248902 (2008).
58. I. Brevik, Phys. Rep. 52, 133 (1979).
59. D.F. Nelson, Phys. Rev. A 44, 3985 (1991).
60. R. Peierls, Proc. Roy. Soc. A347 475.(1976).
61. G.B. Walker and G. Walker, Nature 263, 401 (1976); Nature 265, 324 (1977).
62. S.M. Barnett, Phys. Rev. Let. 104, 070401 (2010),; S.M. Barnett and R. Loudon, Phil. Trans. R. Soc. A 368, 927 (2010)
63. A. Rizzo and S. Coriani, J. Chem. Phys. 119, 11064 (2003); A. Rizzo, D. Shcherbin and K. Ruud, Can. J. Chem. 87:1352-1361 (2009).

64. J. Babington and B.A. van Tiggelen, Eur. Phys. J. D 65, 367-372 (2011).
65. B.A. van Tiggelen, Eur. Phys. J. D 47, 261-269 (2008).
66. S. Kawka and B.A van Tiggelen, EPL 89, 11002 (2010) ;
erratum EPL 98, 29904 (2012).
67. G.L. J. A. Rikken, B. A. van Tiggelen ,Phys. Rev. Lett. 107, 170401 (2011) .
68. B.A. van Tiggelen, S. Kawka, G. L. J. A. Rikken , submitted to Eur. Phys. J. D
69. F. Pinheiro and B.A. van Tiggelen, Phys.Rev. E 66, 016607 (2002) .
70. J. Babington, B. A. van Tiggelen , EPL 93, 41002 (2011)

Thank you !