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Density of States in a Strongly Scattering 'Mesoglass'

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Introduction

A reliable method of measuring the density of states in strongly scattering random media is important for obtaining a complete picture of wave transport. We present such a robust procedure, which was used to detect the vibrational modes of elastic ultrasonic waves in a highly porous sintered glass bead network. Individual modes may be resolved and counted in the Fourier transform of a transmitted pulse. A statistical approach was used to account for the possibility of missing or overlapping modes, enabling the density of states to be determined. In the intermediate frequency regime, the density of states was found to be independent of frequency. Level spacing statistics was also measured; level repulsion is seen, consistent with random matrix theory predictions.

Mesoglass Samples

A 1:1 mixture of polydisperse glass and iron beads was sintered to create a 'mesoglass' material. The iron was then removed by etching, leaving a highly porous, disordered structure.

The sample material was cut into sufficiently small pieces (typically < 1 mm^3) so that individual vibrational modes may be resolved and counted.

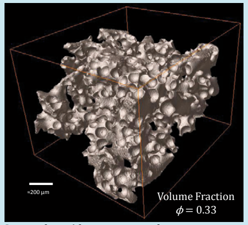


Image taken with x-ray tomography

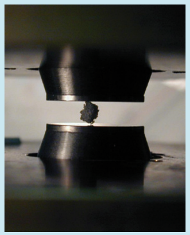
Three frequency regimes were identified for this material [1]:

1. **Effective Homogenous Medium Regime** – Length scales larger than the disorder of the structure; the material behaves as a uniform medium with properties influenced by porosity of the structure.
2. **Intermediate (Strong Scattering) Regime** – Length scales on the order of the pore sizes of the material; fractal behaviour and very strong scattering occur.
3. **Bulk Material Regime** – Length scales smaller than the constituent parts of our network; properties of the bulk medium (glass) dominate.

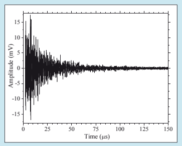
These length scales define the important frequency regimes in our medium.

Measuring Vibrational Modes

To measure the density of states, samples were placed between two transducers as shown below, and vibrational states were excited by a short pulse.



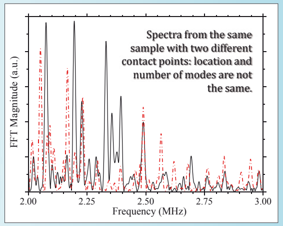
- Measurements were performed under vacuum to reduce absorption.
- Detected pulses were averaged to improve resolution.
- Multiply-scattered signals persist over long times (right).
- Modes are seen as peaks in the Fourier transform of detected pulses.



Mode Counting

The effect of varying contact points between sample and transducers was studied (right). Some of the same modes may be identified in both spectra, but some are shifted or missing altogether.

The observed mode frequencies also vary with the pressure between sample and transducers, as well as the vacuum pressure.



For a single sample orientation, one can not be confident that all the modes are detected within the frequency range of interest. This limitation was overcome using the following statistical method.

Statistical Approach

By making many measurements, the density of states was determined using statistical analysis. The process may be outlined as follows [for more details see ref. 2]:

1. Assume that for any one data set for a particular sample, there exists a probability p of detecting the true number of modes, n.
2. Since a mode is either detected or not detected, and assuming that mode detection is a random process, the Binomial distribution should be used as a parent distribution for the number of modes observed.
3. The detection probability and total number of modes can then be written in terms of the measured mean $\mu = np$ and standard deviation $\sigma = \sqrt{np(1-p)}$:

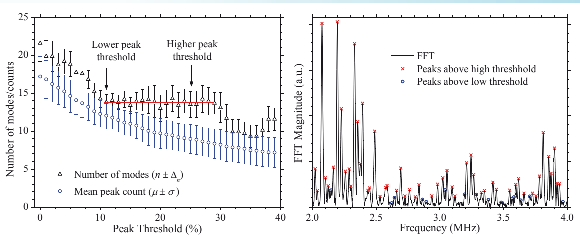
$$\boxed{p = \frac{\mu - \sigma^2}{\mu}} \qquad \boxed{n = \frac{\mu^2}{\mu - \sigma^2}}$$

with corresponding uncertainties.

By making several measurements to find μ for each sample, and combining the results for all samples in quadrature, the true number of modes, n, and the probability p, were found.

Thresholding

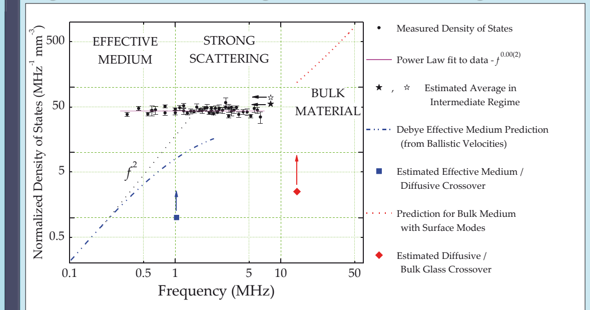
When identifying peaks in the Fourier transform, a threshold was set to discern between genuine modes and noise. To find the appropriate threshold level, an interactive method was employed. For each sample, the threshold was varied relative to the average, and the resulting μ and n plotted versus threshold. The correct threshold is in the region where n is independent of μ, enabling n to be found unambiguously.



Left: The red line denotes the range where the mode counts are independent of threshold level.
Right: Peaks found using the interactive thresholding method.

Density of States Results

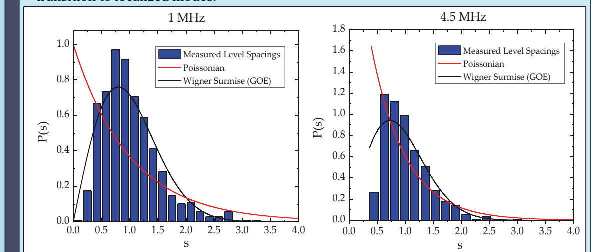
The measured density of states is independent of frequency in the strong scattering regime, with this behaviour extending even into the effective medium regime.



The power law fit can be interpreted using a fractal model [3,4], which predicts a density of states proportional to $\omega^{d_f - 1}$, where d_f is the fracton dimension. We measure d_f =1.0, in between the values for bond-bending (d_f = 0.89, [3]) and scalar elasticity (d_f = 4/3, [4]) models. Our results are in good agreement with the value d_f = 1.05 inferred for sintered glass networks from low frequency velocity measurements [5].

Level Spacing Statistics

The level spacing statistics was measured and compared with predictions from random matrix theory. For a disordered system, we expect to see fluctuations consistent with the Gaussian Orthogonal Ensemble (well approximated below by the Wigner surmise) showing level repulsion [6,7]. In a localized system, we expect a transition to a Poisson distribution near the mobility edge [8]. Our results are close to the predictions of Gaussian statistics at low frequencies. At higher frequencies, the system behaviour appears to be between Gaussian and Poissonian statistics suggesting the approach to a transition to localized modes.



Conclusion

The density of states in a highly porous sintered glass bead network was measured. In the intermediate frequency regime, the density of states was found to be independent of frequency. Level spacing statistics show level repulsion, consistent with random matrix theory predictions. These results suggest that Anderson localization may occur at higher frequencies near the upper crossover.

[1] J. H. Page *et al.*, Phys. Stat. Sol. C **1**, 2925 (2004): [2] W.K. Hildebrand *et al.*, J. Acoust. Soc. Am. **127**, 2819 (2010): [3] I. Webman, G.S. Grest, Phys. Rev. B **31**, 1689 (1985): [4] A. Alexander, R. Orbach, J. Physique **43**, 625 (1982): [5] H.P. Schriemer, *et al.*, Waves Random Media **6**, 361 (1996); [6] R.L. Weaver, J. Acoust. Soc. Am **85**, 1005 (1989): [7] M. Mehta , *Random Matrices* (Academic Press, Boston, 1991), 2nd ed.: [8] M. Sade *et al.* , Phys. Rev. E **72**, 066123 (2005).