Semiclassics of Andreev billiards

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Abstract

By coupling a superconductor to a quantum dot one may decide whether its classical counterpart has integrable or chaotic dynamics by looking at the density of states. Random matrix theory (RMT) predicts a true gap of the order of the Thouless energy $E_T$ in the classical motion is chaotic and an exponential damping in the case of integrable motion. We use semiclassical techniques developed in the recent years to reproduce the RMT predictions.

Using the same techniques we show that the existence of a superconductor in proximity to a quantum dot has crucial effects on the conductance.

Andreev reflection and Andreev billiards

In 1964 Andreev found a new type of scattering process at the interface between a normal and a superconducting region [1]. An electron (hole) in the normal region hitting the superconductor is reflected as a hole (electron) which additionally accumulates a phase $e^{i2\eta}$ where $\eta$ is the superconducting phase. This scattering mechanism gives rise to a new type of billiards: Andreev billiards consist of a ballistic region included by a boundary partially Andreev reflecting. Experimentally this can be realized by ballistic quantum dots coupled to superconductors.

Quantum mechanically Andreev billiards are described by the Bohr-Sommerfeld equations [2]

$$\left[ \hat{H}^S - A^S - \Delta \right] |\psi\rangle = E |\psi\rangle$$

Within the so-called scattering approach the average density of states of Andreev billiards with two superconductors is normalized to the density of states of the isolated billiard via the mean dwell time $N \sim 1/K$.

$$S_G(E) = \frac{1}{\pi} \sum \lambda^{\phi}(E) \left( \frac{N}{S} \right)$$

where $S_G$ is the Andreev conductance, $E$ is the energy, and $\lambda^{\phi}$ is the stability amplitude for the $\phi$th channel.

Density of states

The density of states is related to the correlation functions $G_{\alpha,\beta}(r, \tau) = \langle \delta \xi_{\alpha,\beta}(r) \delta \xi_{\alpha,\beta}(r) \rangle$ of $n$ scattering matrices. In leading order in the inverse channel number $n$, the correlation functions can be calculated by identifying the set of classical trajectories by nested plane waves, where an Emitter is represented by a node of degree 2 and the paths are straight lines. Each tree is then characterized by a vector $\theta$ whose $n$th row $\theta_i$ is the number of encounters and satisfies $\sum_i 1 - \theta_i + n + 1 = 0$. By cutting each tree at the top node such that it decomposes into several minors, one finds a recursion relation for each tree. If $N_1 \geq N_2$, this yields a fourth order equation for the generating function of the correlation function $G_{\alpha,\beta}(r, \tau) = \sum_{n=0}^{\infty} C_{\alpha,\beta} \phi_n \phi_n^{\ast}$ in general and a second order equation for $G_{\alpha,\beta}(r, \tau) = \sum_{n=0}^{\infty} C_{\alpha,\beta} \phi_n$ in special cases. Solving these equations and integrating over one variable yields the density of states $N \sim 1/K$.

Within the so-called scattering approach the average density of states of Andreev billiards with two superconductors is normalized to the density of states of the isolated billiard.

Conductance

The conductance for electrical transport between the leads 1 and 2 of an Andreev billiard with $n$ channels is given by [3]

$$G_{12} = \frac{1}{\pi} \sum \lambda^{\phi}(E)$$

where $S_G$ is the Andreev conductance, $E$ is the energy, and $\lambda^{\phi}$ is the stability amplitude for the $\phi$th channel.

Conclusion and Outlook

Semiclassics based on the diagonal contribution is not sufficient to explain Andreev billiards. In fact, correlation between several orbits is important to describe the effects of superconductors in proximity to a quantum dot.

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Further possible investigations would be the shot noise of Andreev billiards or the effect of tunnel barriers.

References