Mixed Phase Space and Tunneling

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Coupled Standard Maps: Phase-Space Section

Designing a Model with Clean Phase Space

- Hamiltonian:

$$H = T(\vec{p}) + \sum_{n \in \mathbb{Z}} \delta(t-n) V(\vec{q})$$

with $T(\vec{p}) = \frac{p_1^2}{2} + \frac{p_2^2}{2}$

- Map:

$$\begin{pmatrix} p_1' \\ p_2' \\ q_1' \\ q_2' \end{pmatrix} = \begin{pmatrix} p_1 - \frac{\partial V}{\partial q_1}(q_1') \\ p_2 - \frac{\partial V}{\partial q_2}(q_2') \\ q_1 + p_1 \\ q_2 + p_2 \end{pmatrix}$$

with periodic boundaries

- Coupled standard maps:

[Froeschlé, 1971]

$$V(\vec{q}) = \frac{K_1}{4\pi^2} \cos(2\pi q_1) + \frac{K_2}{4\pi^2} \cos(2\pi q_2) + \frac{\xi_{12}}{4\pi^2} \cos(2\pi(q_1 + q_2))$$

- Designed map:

Potential

Phase space

Quantum Mechanics $U|\psi\rangle = e^{2\pi i \phi} |\psi\rangle$

Time evolution operator
 $U = e^{-iV/h_{\text{eff}}} e^{-iT/h_{\text{eff}}}$
 with $h_{\text{eff}} = 1/40$

Open System

Designed Map:
Direct Regular-to-Chaotic Tunneling

Fictitious integrable system approach
 [Bäcker, Ketzerick, Lück, Schilling, PRL 2008]

$$\gamma_{m,n}^d = \|(1 - P_{\text{reg}})U|\psi_{m,n}^{\text{reg}}\|^2$$

approach applicable for higher dimensional systems

Designed Map:
Nonlinear Resonances

$$V(q_1, q_2) = \frac{r_1}{2} q_1^2 + \frac{r_2}{2} q_2^2 + \frac{R_1}{3} q_1^3 + \frac{R_2}{3} q_2^3 + R_{12} q_1^2 q_2$$

Frequency Map

[Laskar, Icarus 1990, Laskar et al., Physica D 1992]

important resonances

$$m_1 \omega_1 + m_2 \omega_2 = n:$$

$$5\omega_1 + 0\omega_2 = 1$$

$$-\omega_1 + \omega_2 = 0$$

Designed Map:
Tunneling Rates & Nonlinear Resonances

For 2D maps:

- Resonance-assisted tunneling

[Broder, Schlagheck, Ulms PRL 2001]

[Wintberger, Schlagheck, Eltschka, Beichtolder PRL 2006]

- Combination with Direct-regular-to-chaotic tunneling

[Lück, Bäcker, Ketzerick, Schlagheck, PRL 2010]

$$\gamma_m = \|(1 - P_{\text{reg}})U|\psi_{m,n}^{\text{reg}}\|^2$$

$$|\psi_{m,n}^{\text{reg}}\rangle = |\psi_{m,n}^{\text{reg}}\rangle + \frac{V_{rs}}{E_m - E_{m+r}} |\psi_{m,n}^{\text{reg}}\rangle + \frac{V_{rs}}{E_m - E_{m+r}} |\psi_{m,n}^{\text{reg}}\rangle + \dots$$

$$\gamma_m = \gamma_m^d + \left| \frac{V_{rs}}{E_m - E_{m+r}} \right|^2 \gamma_{m+r}^d + \left| \frac{V_{rs}}{E_m - E_{m+r}} \right|^2 \gamma_{m+2r}^d + \dots$$

Classical Description of Nonlinear Resonances

$$H_\varepsilon(\vec{I}, \vec{\theta}, t) = H_{\text{reg}}(\vec{I}) + \varepsilon V(\vec{I}, \vec{\theta}, t)$$

$$\sum_{m_1, m_2, n \in \mathbb{Z} \setminus 0} \tilde{V}^{m_1, m_2, n} \cos(\vec{m} \vec{\theta} + 2\pi n t)$$

most important structure: period 5 orbit

→ $5:0:1$ and $-1:1:0$ resonance

→ also $5 \cdot \alpha - \beta : \beta : \alpha$ resonances for all $\alpha, \beta \in \mathbb{Z} \setminus 0$

after averaging over fast degrees of freedom:

$$H_\varepsilon(\vec{J}, \vec{\theta}) = \tilde{H}_{\text{reg}}(\vec{J}) + \varepsilon \sum_{\alpha, \beta \in \mathbb{Z} \setminus 0} \tilde{V}^{5\alpha - \beta, \beta, \alpha} \cos((5 \cdot \alpha - \beta)\theta_1 + \beta\theta_2)$$

aim: read from classical phase space

Perturbation Theory

What influences quasi modes $|\psi_{m,n}^{\text{reg}}\rangle$?

$$|\tilde{\psi}_{m,n}^{\text{reg}}\rangle = |\psi_{m,n}^{\text{reg}}\rangle + \sum_{k,l \neq m,n} \frac{\langle \psi_{k,l}^{\text{reg}} | V | \psi_{m,n}^{\text{reg}} \rangle}{E_{m,n} - E_{k,l}} |\psi_{k,l}^{\text{reg}}\rangle + \dots$$

$$\langle \psi_{k,l}^{\text{reg}} | V | \psi_{m,n}^{\text{reg}} \rangle \sim$$

$$\sum_{\alpha, \beta \in \mathbb{Z} \setminus 0} \delta_{k, m+5\alpha-\beta} \delta_{l, n+\beta}$$

Understanding Peak Positions

Outlook

- Direct regular-to-chaotic tunneling:

— apply fictitious-integrable-system approach to generic systems — $P_{\text{reg}}, |\psi^{\text{reg}}\rangle$?

- Resonance-assisted tunneling:

— Quantitative understanding ($\tilde{V}^{5\alpha-\beta, \beta, \alpha}$ from classical phase space)
 — Presence of multiple resonances separated in phase space

- Influences of non-barrieriness of tori (Arnol'd-diffusion-like behaviour)?

- Role of partial barriers?

- Influences of different types of fixed points?