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Measuring the scattering mean free path in heterogeneous media

Anne Obermann, Éric Larose, Vincent Rossetto, Ludovic Margerin

CNRS, Universit®Joseph Fourier, Grenoble

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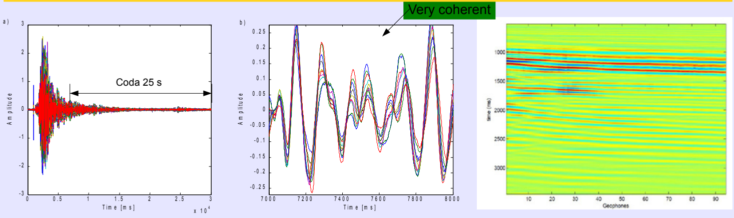
Goal:
Study the phase statistics in coda waves and determine the scattering mean free path ℓ from spatial phase decoherence.

1) Field experiment on a volcano in Auvergne

- 96 receivers in total
- 48 receivers with 0.6 m spacing
- 18 explosives + 17 hammer sledges

2) Raw data + Processing steps

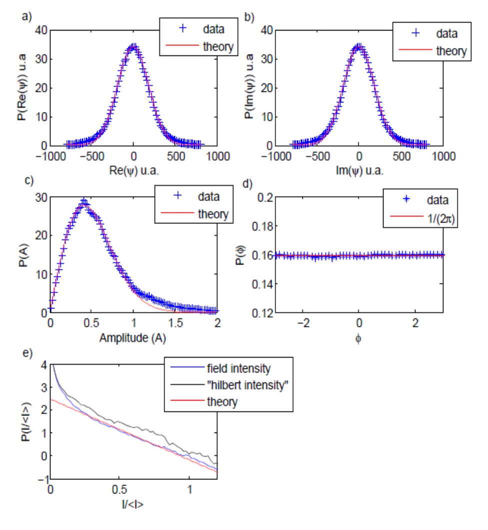


Processing steps

- Bandpassfilter: 30 Hz ± 5%
- Select time window
- Hilbert transformation
- Spatial phase unwrapping

$$\Psi(\vec{r},t) = A(\vec{r},t)\exp(i\Phi(\vec{r},t))$$

3) Gaussian statistics



Criteria for signals that obey circular gaussian statistics:

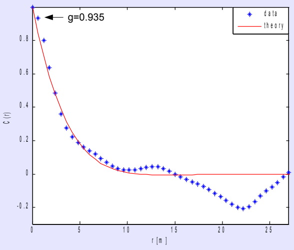
a) + b) The real and imaginary part of the field follow a Gaussian distribution

c) The amplitude distribution follows a Rayleigh distribution $P(A) = \frac{2A}{\langle I\rangle}\exp\left(-\frac{A^2}{\langle I\rangle}\right)$

d) The probability distribution of the phase is uniform $P(\Phi) = \frac{1}{2\pi}$

e) The amplitude decays exponentially $P(I) = \frac{1}{\langle I\rangle}exp\left(\frac{-I}{\langle I\rangle}\right)$

4) Field correlation



$$C(r) = \xcancel{J_0(kr)}\exp(-\tfrac{r}{2l})$$

g=C(Δr)=0.935 ℓ= (3 ±1) m

Correlation coefficient between adjacent receivers

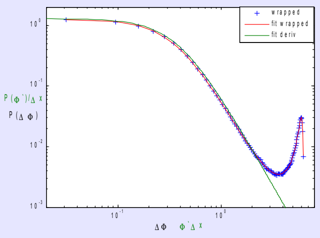
Scattering mean free path

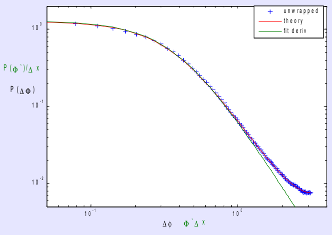
5) Phase difference distributions

First phase derivative distribution

Wrapped phase Φ

Unwrapped phase Φ_u





$$P(\Delta\Phi) = \frac{2\pi - |\Delta\Phi|}{4\pi^2}\left[\frac{1-g^2}{1-F^2}\right]\left[1+\frac{F\cos^{-1}(-F)}{\sqrt{1-F^2}}\right]$$

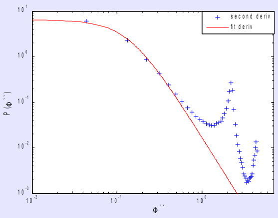
$$F = g_1\cos(\Delta\Phi)$$

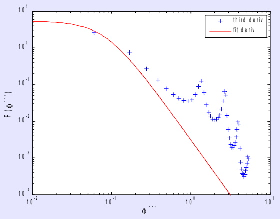
$$P(\Delta\Phi_u) = \frac{1}{2\pi}\left[\frac{1-g^2}{1-F^2}\right]\left[1+\frac{F\cos^{-1}(-F)}{\sqrt{1-F^2}}\right]$$

$$F = g_1\cos(\Delta\Phi_u)$$

$$P(\Phi') = \frac{1}{2}\frac{Q}{[Q+\Phi'^2]^{3/2}} \Rightarrow Q = -C''(0) \simeq \frac{1}{2}k^2$$

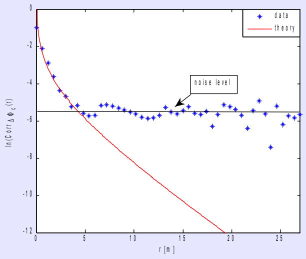
Higher order phase derivative distributions

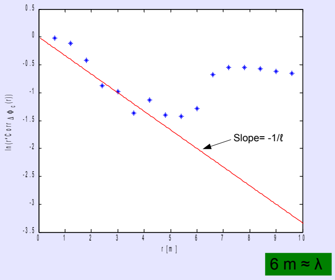




Q = 0.155 ⇨ λ =(6 ± 2) m g = 0.935

6) Phase difference correlation





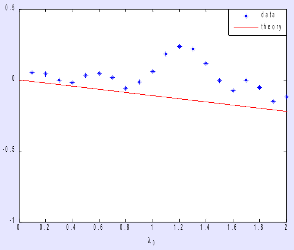
6 m ≈ λ

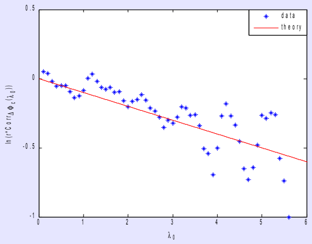
$$C_{\Phi'}(r > \lambda) \longrightarrow \tfrac{1}{2}\left[(C')^2 - C''C\right] = (k/\pi r)\exp(-r/l)$$

Comparison to numerical simulation:

48 receivers

267 receivers





ℓ = (4 ±1) m traceability of ℓ: 5 m ≈3/4 λ