Mesoscopic Physics in Complex Media, 03001 (2010)
DOI:10.1051/iesc/2010mpcm03001

Locating a small change in a multiple scattering environment

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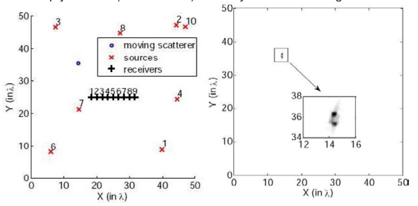
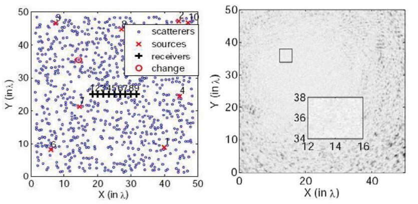
1) Imaging in multiple scattering media

Numerical simulations in 2 dimensions

In an empty medium (no scatterers) it is easy to locate a change.



In a scattering medium, applying the same technique fails.



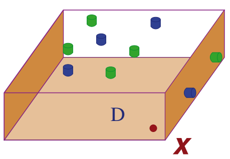
4) Numerical model

A numerical model of the system takes into account geometry and boundary conditions. The diffusivity D describes the heterogeneity of the system.

$$\mathrm{D} = \frac{c\ell^2}{3}$$

ℓ is the mean free path of a wave. Typically, ℓ is of the same order of magnitude as the heterogeneities.

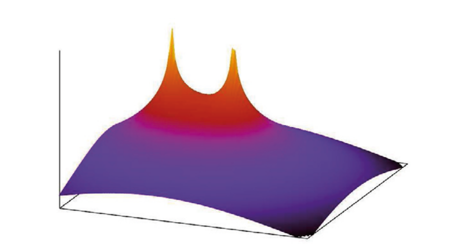
The wave intensity evolves according to a diffusion equation (diffuse waves).



- Sources and receivers located at the same positions as in the experimental setup.
- In the model, a change occurs at an arbitrary position $\boldsymbol{x}$.
- In a 3-D, infinite medium, there is an exact formula for the correlation :

$$K_{ij}^{\mathrm{num}}(\sigma, \boldsymbol{x}) = 1 - \frac{c\sigma}{2}\frac{1}{4\pi\mathrm{D}}\frac{\|\boldsymbol{x} - \boldsymbol{S_i}\| + \|\boldsymbol{x} - \boldsymbol{R_j}\|}{\|\boldsymbol{x} - \boldsymbol{S_i}\| \times \|\boldsymbol{x} - \boldsymbol{R_j}\|} \times \exp\left[\frac{\|\boldsymbol{S_i} - \boldsymbol{R_j}\|^2 - \left(\|\boldsymbol{x} - \boldsymbol{S_i}\| + \|\boldsymbol{x} - \boldsymbol{R_j}\|\right)^2}{4\mathrm{D}t}\right]$$

8) Sensitivity kernel



The sensitivity kernel is maximum along the line between the source and the receiver.

The model is valid only at a distance larger than ℓ from sources and receivers. This assumption is actually necessary to compute the correlation.

If the change is located close to a source or a receiver, the informations from this device are not usable and have to be removed from the inversion.

Notations

The typical time after which a system of size L has been fully explored by the waves is called the *Thouless time*

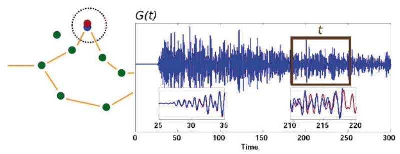
$$\tau_{\mathrm{D}} = \frac{L^2}{\mathrm{D}}.$$

The width of the probability density around the located position is called *resolution* and is noted δ.

$$\delta^2 = \frac{1}{\mathcal{C}}\int (\boldsymbol{x} - \mathrm{x}_0)^2 \exp\left[-N\chi_n^2(\sigma, \boldsymbol{x})\right] \mathrm{d}\boldsymbol{x}$$

2) The multiply scattered signal

In a multiple scattering environment, a pulse is received at several times, corresponding to paths of different lengths.



Measurements of the impulse response (Green's function)
Long lasting signal (*coda*)
The *coda* is sensitive to small changes

The *coda* is an excellent observable for the **detection** of small changes

→ can we use it to **locate** a small change ?

5) The inversion technique

We use a χ^2 method to compare the correlations K^{exp} and K^{num}.

$$\chi_n^2(\sigma, \boldsymbol{x}) = \frac{1}{N\varepsilon^2}\sum_{i,j}\left(K_{ij}^{\mathrm{exp}} - K_{ij}^{\mathrm{num}}(\sigma, \boldsymbol{x})\right)^2$$

- ε is the accuracy of the measurements
- N is a number of source-receiver pairs

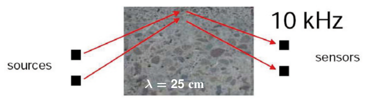
We look for the the *minimum* of $\chi_n^2(\sigma, \boldsymbol{x})$.

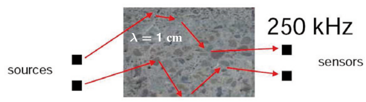
This inversion provides

- the effective scattering cross-section σ of the change
- the location $\boldsymbol{x}$ of the change

6) Laboratory experiments using concrete

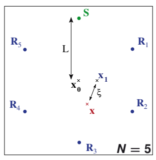
In concrete $c = 2500\ \mathrm{m.s^{-1}}$.



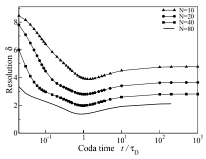


We use high frequencies such that the wavelength is of the order of the heterogeneities' size.

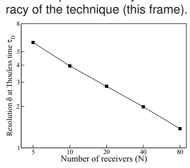
9) Accuracy of the technique



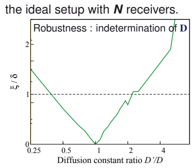
Ideal setup for the study of the accuracy of the technique (this frame).



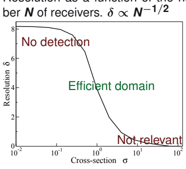
Resolution as a function of time for the ideal setup with N receivers.



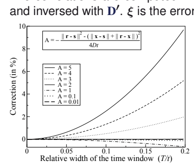
Resolution as a function of the number N of receivers. $\delta \propto N^{-1/2}$



The correlations are computed with D and inversed with D'. ξ is the error.



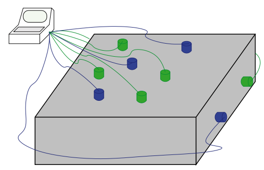
Resolution as a function of the cross-section σ.



Correction due to the time window integration $[t - T, t + T]$.

3) Correlations

Sources (in green) and receivers (in blue) are used to measure the impulse response in a medium. The set of all records is a footprint of the medium.



To compare the footprints *before* and *after* the change, we compute the correlation between the signals:

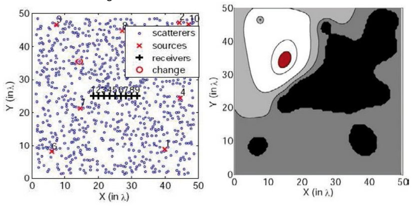
$$K_{ij}^{\mathrm{exp}} = \left\langle G^{\mathrm{before}}(\boldsymbol{S_i}, \boldsymbol{R_j}, \tau)^* \, G^{\mathrm{after}}(\boldsymbol{S_i}, \boldsymbol{R_j}, \tau)\right\rangle_{t-T\leqslant\tau\leqslant t+T}$$

The set of K_{ij} contains informations concerning the position of the change.

7) Results

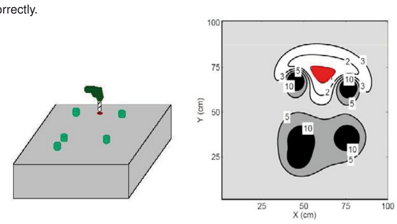
Numerical simulations in 2 dimensions

The inversion technique is able to locate the change. The accuracy is lower than in the non scattering medium : $\delta \simeq \ell$.

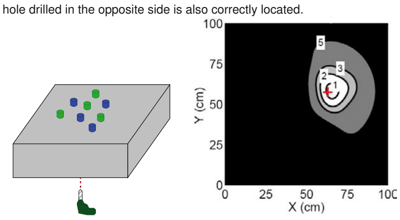


Experiment in a block of concrete

A hole is drilled in the side where the transducers stand. The hole is located correctly.



A hole drilled in the opposite side is also correctly located.



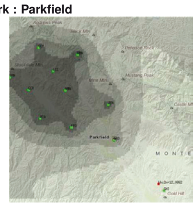
Applications in seismology

Exploratory work : Parkfield

- Green's functions retrieved using noise correlation.

$$G_{AB}(t) = \langle \Psi_A(\tau)\Psi_B(\tau + t)\rangle_\tau$$

- Correlation of Green's functions *before* and *after* 2004 earthquake in Parkfield.



- Noise correlations : how late in the *coda* can we go ?
- Can we improve correlation computation specifically for the use of noise ?
- Imaging of small heterogeneous areas like volcanoes.
- Deep imaging possible ?

References

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Outlook

- Perform real-size experiments
- Locate extended changes and or several changes at the same time
- Extend to weakly heterogeneous media
- Include complex boundary conditions
- How to use the early *coda* ?
- Locate fluid movements in cracks

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