Locating a small change in a multiple scattering environment

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1) Imaging in multiple scattering media

Numerical simulations in 2 dimensions

In an empty medium (no scatterers) it is easy to locate a change.

In a scattering medium, applying the same technique fails.

2) The multiply scattered signal

In a multiple scattering environment, a pulse is received at several times, corresponding to paths of different lengths.

Measurements of the impulse response (Green's function)

Long lasting signal (coda)
The coda is sensitive to small changes

The coda is an excellent observable for the detection of small changes

→ can we use it to locate a small change?

3) Correlations

Sources (in green) and receivers (in blue) are used to measure the impulse response in a medium. The set of all records is a footprint of the medium.

To compare the footprints before and after the change, we compute the correlation between the signals:

\[ C_{\text{ref}, \text{mod}}(t) = \langle G_{\text{ref}}(s', t') G_{\text{mod}}(s, t) \rangle \]

The set of \( C(t) \) contains information concerning the position of the change.

4) Numerical model

A numerical model of the system takes into account geometry and boundary conditions. The diffusivity \( D \) describes the heterogeneity of the system.

\[ D = \varepsilon \sigma \]

If \( \sigma \) is the mean free path of a wave, typically, \( \sigma \) is of the same order of magnitude as the heterogeneities.

The wave intensity evolves according to a diffusion equation (diffuse waves).

5) The inversion technique

We use a \( L^2 \) method to compare the correlations \( K^{\text{ref}} \) and \( K^{\text{mod}} \):

\[ x \text{ is the accuracy of the measurements} \]

\[ N \text{ is a number of source-receiver pairs} \]

We look for the minimum of \( \mathcal{E} \):

\[ \mathcal{E} = \sum_{i,j} |K^{\text{mod}}(x_i, x_j) - K^{\text{ref}}(x_i, x_j)|^2 \]

This inversion procedure:

\[-\text{the effective scattering cross-section} \nu \text{ of the change} \]

\[-\text{the location} x \text{ of the change} \]

6) Laboratory experiments using concrete

In concrete \( \varepsilon = 2000 \) cm-1, \( \sigma = 4 \) cm.

A hole drilled in the side where the transducers stand. The hole is located correctly.

7) Results

Numerical simulations in 2 dimensions

A hole drilled in the opposite side is also correctly located.

8) Sensitivity kernel

The sensitivity kernel is maximum along the line between the source and the receiver.

The model is valid only at a distance larger than \( r \) from sources and receivers.

This assumption is actually necessary to compute the correlation.

If the change is located close to a source or a receiver, the informations from this device are not usable and have to be removed from the inversion.

9) Accuracy of the technique

Resolution as a function of the cross-section \( \nu \),

Resolution as a function of time for the ideal setup with \( N \) receivers.

The correlations are computed with \( D \)

Correlation due to the time window integration \( [\tau - T, \tau + T] \)

Applications in seismology

Exploratory work: Parkfield

- Green's functions retrieved using noise correlation.
- Correlation of Green's functions before and after 2004 earthquake in Parkfield.
- Noise correlations: how late in the coda can we go?
- Can we improve correlation computation specifically for the use of noise?
- Imaging of small heterogeneous areas like volcanics.
- Deep imaging possible?

Outlook

- Perform real-size experiments
- Locate extended changes and or several changes at the same time
- Extend to weakly heterogeneous media
- Improve the inversion technique
- How to use the early coda?
- Locate fault movements in cracks

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Notations

The typical time after which a system of size \( L \) has been fully explored by the waves is called the Thouless time

\[ t^* = \frac{L^2}{D} \]

The width of the probability density around the located position is called resolution and is noted \( \delta \)

\[ \delta^2 = \frac{1}{2} \int (\sigma - \sigma)^2 \exp(-\|\mathbf{x} - \mathbf{x}_0\|^2) \]

References