Exotic Solids Made by Elastic Metamaterials

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What are metamaterials?

Metamaterials are **artificially structured** materials that can have **profoundly unique** electromagnetic or optical properties.

<http://people.ee.duke.edu/~drsmith/metamaterials.html>

e.g.

negative refraction

R. A. Shelby et al., **Science** 292, 77 -79 (2001)

cloaking

D. Schurig et al., **Science** 314, 977 -980 (2006)₂

Metamaterials and Effective Medium Theory

Microstructures

Material parameters, (ϵ, μ) , that can describe the macroscopic properties

$$n = \sqrt{\epsilon} \sqrt{\mu} \begin{cases} \text{both } \epsilon \text{ and } \mu \text{ are negative} \longrightarrow n < 0 : \text{negative bands} \\ \text{only one negative in } \epsilon \text{ and } \mu \longrightarrow n \text{ imaginary: gap} \end{cases}$$

What are elastic waves?

Wave equation: $\rho \frac{\partial^2 u_i}{\partial t^2} = \nabla \cdot (\mu \nabla u_i) + \nabla \cdot \left(\mu \frac{\partial \vec{u}}{\partial x_i} \right) + \frac{\partial}{\partial x_i} (\lambda \nabla \cdot \vec{u})$

ρ mass density
 μ shear modulus
 $\mu = 0$ for liquids
 λ lamé constant
 κ bulk modulus $\kappa = \lambda + \frac{2}{d} \mu$

Two types of waves & three material parameters.

longitudinal : $n_l = \sqrt{(\lambda + 2\mu)/\rho}$

transverse : $n_t = \sqrt{\mu/\rho}$

	$\kappa_e + \mu_e > 0$	$\kappa_e + \mu_e < 0$	$\kappa_e + \mu_e > 0$	$\kappa_e + \mu_e < 0$
	$\mu_e > 0$	$\mu_e > 0$	$\mu_e < 0$	$\mu_e < 0$
$\rho_e > 0$	$n_{le} > 0, n_{te} > 0$	$l : \text{gap}, n_{te} > 0$	$n_{le} > 0, t : \text{gap}$	$l, t : \text{gap}$
$\rho_e < 0$	$l, t : \text{gap}$	$n_{le} < 0, t : \text{gap}$	$l : \text{gap}, n_{te} < 0$	$n_{le} < 0, n_{te} < 0$

Previously proposed elastic metamaterials: (a) $\rho < 0$, $\kappa > 0$ and $\mu > 0$ (gap)

dipolar resonance

Z. Liu et al., *Science* 289, 1734 (2000)

(b) $\rho < 0$, $\kappa < 0$, $\mu > 0$ ($n_l < 0$)

Y. Q. Ding *et al.*, *Phys. Rev. Lett.* **99**, 093904 (2007)

Motivation

How about $\mu < 0$?

$\kappa_e + \mu_e > 0$		$\kappa_e + \mu_e < 0$	
$\mu_e > 0$		$\mu_e < 0$	
$\rho_e > 0$	$n_{le} > 0, n_{te} > 0$	$n_{le} > 0, t: \text{gap}$	$l, t: \text{gap}$
$\rho_e < 0$	$l, t: \text{gap}$	$l: \text{gap}, n_{te} < 0$	$n_{le} < 0, n_{te} < 0$

We will design “exotic solids” with $\mu < 0$, also with $\rho < 0$, $\kappa < 0$:

$\langle \text{---} \rangle$ both n_l and $n_t < 0$

$\langle \text{---} \rangle$ “Incompressible-solid-like”: only support transverse wave

$\langle \text{---} \rangle$ “Fluid-like” : only support longitudinal waves

Methods

- Effective medium analysis : guiding the design
- Numerical calculations
 - Multiple-scattering calculation
 - COMSOL Multiphysics

Effective medium theory for elastic metamaterials:

$$\frac{(\kappa_0 - \kappa_e)}{(\mu_0 + \kappa_e)} = \frac{4\tilde{D}_0^{II}(s)}{i\pi r_0^2 k_{l0}^2}$$

$$\frac{(\rho_0 - \rho_e)}{\rho_0} = -\frac{8\tilde{D}_1^{II}(s)}{i\pi r_0^2 k_{l0}^2}$$

$$\frac{\mu_0(\mu_0 - \mu_e)}{(\kappa_0\mu_0 + (\kappa_0 + 2\mu_0)\mu_e)} = \frac{4\tilde{D}_2^{II}(s)}{i\pi r_0^2 k_{l0}^2}$$

monopolar

dipolar

quadrupolar

Y. Wu, Y. Lai and Z.Q. Zhang, *Phys. Rev. B* **76**, 205313 (2007).

Exotic solids I

$$\mu < 0, \rho < 0$$

Microstructure

$$r_s = 0.32a \quad r_w = 0.24a$$

Effective medium parameters

Band structure and transmission

longitudinal in.

transverse in.

transverse
wave filter
“incompressi-
ble-solid-
like”

negative refraction

Snell's law: $k_t \sin i = k_{l0} \sin \theta_{l0} = k_{t0} \sin \theta_{t0}$

Plug in k_t $\phi_{t0} = 58^\circ$ $\phi_{l0} = 99^\circ$

Exotic solids II

From single mass LRM

Hard core enhances the **dipolar resonances** which can induce negative mass density

to...

Can we use many hard objects to enhance **quadrupolar resonances**?

Multi-mass locally resonant materials

System design

model

microstructures

can enhance both **monopolar** and **quadrupolar** resonances

Band structure and states

“fluid-like” solid

“Super-anisotropic” solid:
“incompressible-solid-like” in ΓM direction and “fluid-like” in ΓX direction

Near μ resonance, $\mu \ll 0$, $\kappa + \mu \ll 0$

If $\rho < 0$, we get $c_l < 0$ and $c_t < 0$, i.e. two degenerate states at $\mu = -\infty$.

Each state is longitudinal and transverse in different directions.

Symmetry breaking chooses one of them $\Rightarrow$ “super-anisotropic”.

Conclusions

- Design elastic metamaterials in two dimensions
 - guided by the effective medium theory
 - enhance quadrupolar resonance to produce negative μ
- Exotic solids (two examples):
 - 1) Rubber-coated-water cylinders in a foam host would produce negative bands for shear wave
 - 2) Multi-mass locally resonant structures would generate complex bands

The End