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Metamaterials for surface waves

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Introduction

Institut Fresnel

Photonics, Electromagnetism, Image and Signal Processing.
about 60 permanent researchers (universities, engineering school, and cnrs) and 140 people overall. www.fresnel.fr
In the city where the Fabry-Perot interferometer was invented

Planier lighthouse in Marseille

Construction supervised by A. Fresnel

Introduction

Collaborators
PhD students: M. Farhat, M. Kadic, G. Dupont,
CNRS researcher: S. Guenneau.

and also

A. B. Movchan, Liverpool University (water waves and thin plates)

R. Quidant's group, ICFO, Barcelona (plasmonics)

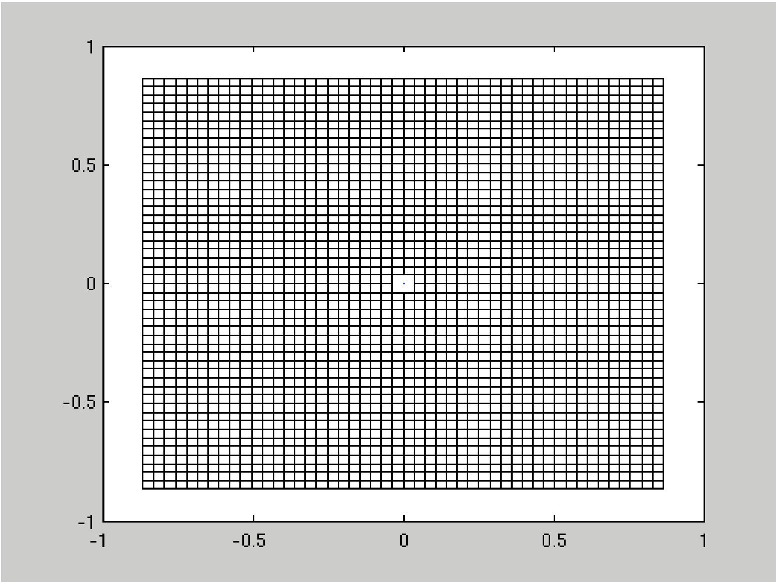
Cloaking: our motivations

- Looking for experiments for popularization of the metamamaterial’s concepts.
- Promote the metamaterials ideas for other type of waves (water waves, flexural waves, surface plasmons polaritons).

Cloaking by transformation (reminder)

J.B. Pendry, D. Schurig, & D. R. Smith, Science 312, 1780–1782 (2006).

Coordinates transform (2D case)



$$r' = R_1 + r\left(R_2 - R_1\right) / R_2$$

$$\theta' = \theta \quad \text{and} \quad x_3' = x_3$$

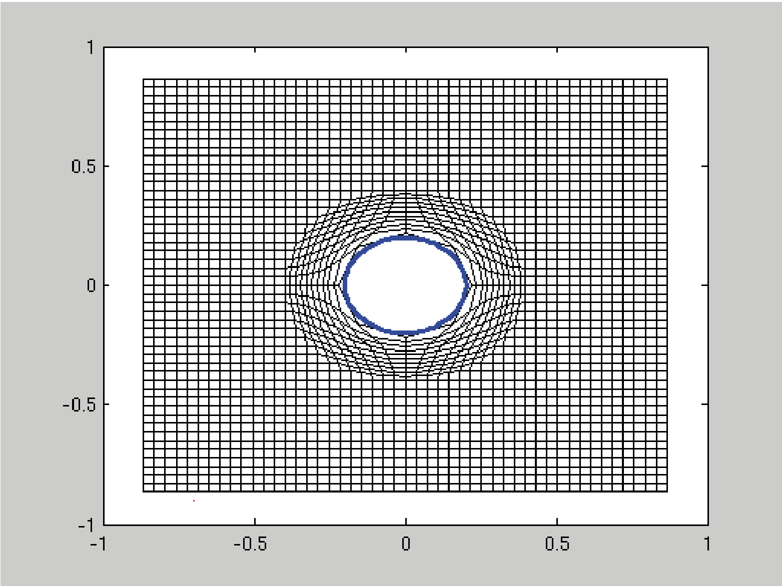
Permeability and permittivity

$$\varepsilon_r = \mu_r = \frac{r - R_1}{r} \qquad \varepsilon_\theta = \mu_\theta = \frac{r}{r - R_1} \qquad \varepsilon_3 = \mu_3 = \left(\frac{R_2}{R_2 - R_1}\right)^2 \frac{r - R_1}{r}$$

Cloaking by transformation

J.B. Pendry, D. Schurig, & D. R. Smith, Science 312, 1780–1782 (2006).

Coordinates transform



$$r' = R_1 + r\left(R_2 - R_1\right)/R_2$$

$$\theta' = \theta \quad \text{and} \quad x_3' = x_3$$

Permeability and permittivity

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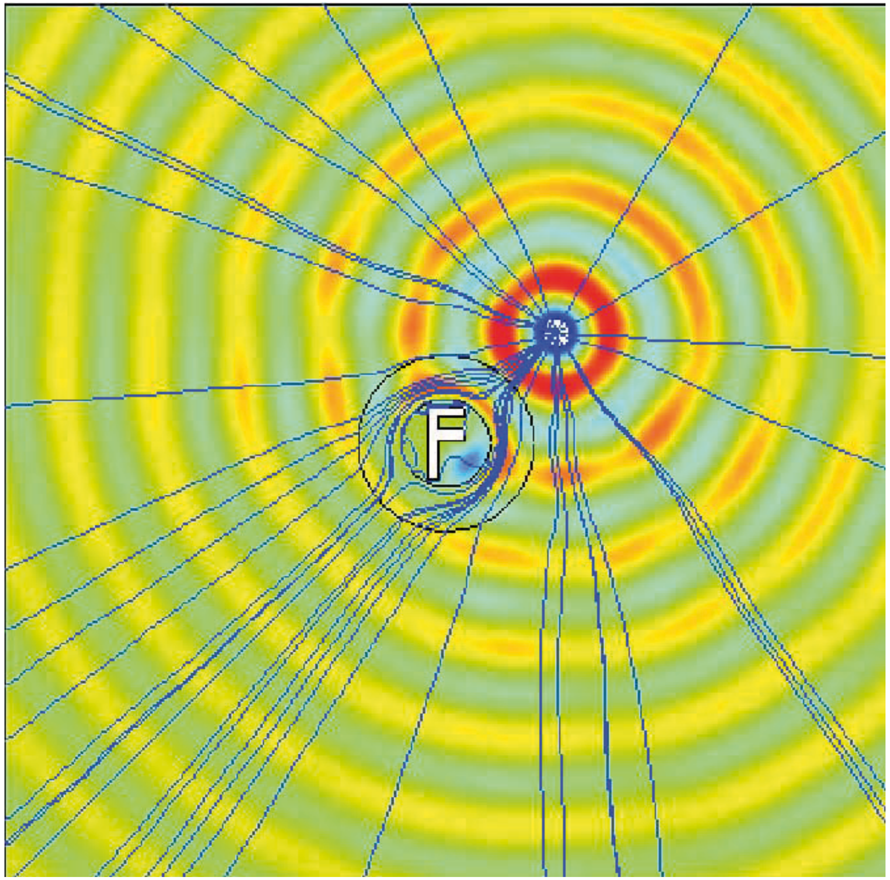
Cloaking by transformation

Remark : « Nearly » invisible in TE polarization (magnetic field perpendicular to the figure):



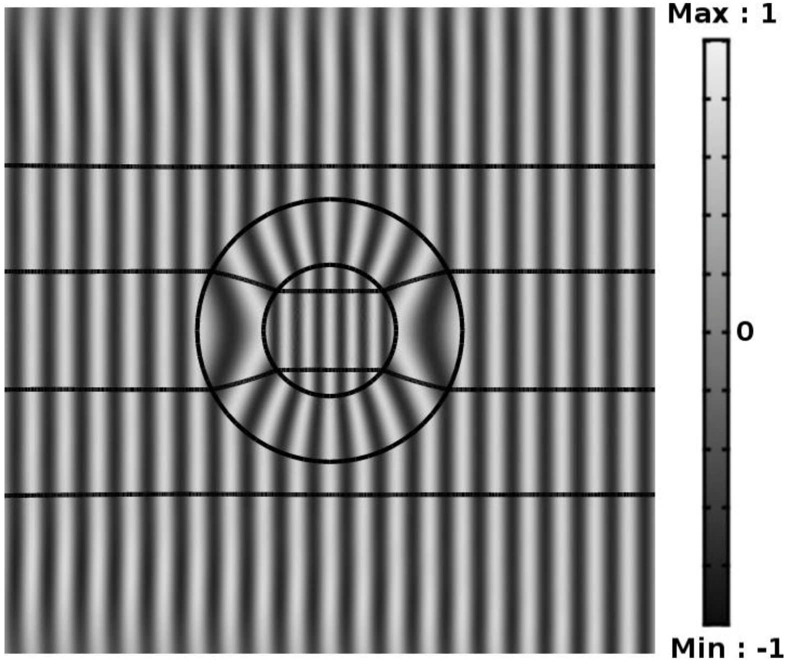
$$\varepsilon_r = \left(\frac{R_2}{R_2 - R_1}\right)^2 \left(\frac{r - R_1}{r}\right)^2$$

$$\varepsilon_\theta = \left(\frac{R_2}{R_2 - R_1}\right)^2$$

$$\mu_3 = 1$$

Other transformations

Concentrator

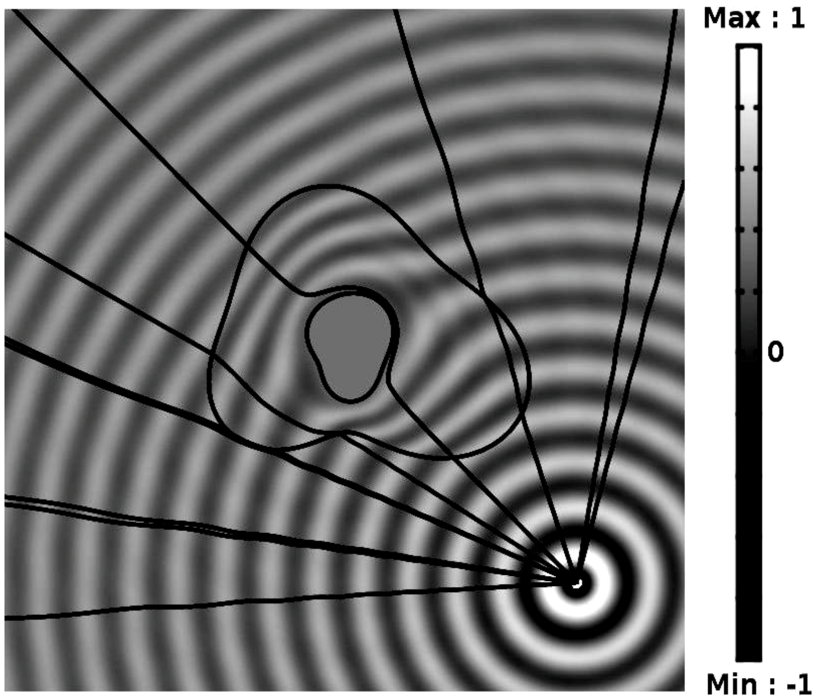


A plane wave at wavelength $\lambda = 0.2$ on a concentrator with inner radius R1 = 0.2, 'virtual' radius R2 = 0.3 and outer radius R3 = 0.4; We note that the wavelength of the field inside the disc is shrunk, which is the concentrating effect, but the device is perfectly invisible.

Other transformations

Arbitrary shaped invisibility cloak

Arbitrary shaped invisibility cloak in presence of a line source of wavelength λ = 0.3 located at (x,y) = (1,-1);



From Navier-Stokes to Helmholtz

Let u(x_1; x_2; x_3; t) satisfy the Navier-Stokes equation :

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} + \frac{\nabla p}{\rho} = g + \nu \Delta \mathbf{u}$$

Neglecting the liquid viscosity and assuming an incompressible and irrotational liquid flow and thus noting $\mathbf{u} = \nabla \phi$, we obtain :

$$\nabla^2 \Phi = 0$$

BUT : Laplace equation manifests no wave character !

From Navier-Stokes to Helmholtz

➔Waves are induced by the boundary conditions on the free surface air-liquid.
Indeed, neglecting surface tension, we obtain on the free surface :

$$\frac{\partial \Phi}{\partial t}+\frac{\left|\nabla \Phi\right|}{2}+\frac{p_0}{\rho}+g\xi=f\left(t\right),\quad \text{on}\, x_3=\xi$$

For a harmonic potential with frequency ω, we can choose the following form of Φ:

$$\Phi\left(x_1,x_2,x_3,t\right)=\Re e\left(\phi\left(x_1,x_2\right)\cosh\left(\kappa x_3\right)e^{-i\omega t}\right)$$

κ is the spectral parameter and ϕ the reduced potential, solution of the 2D Helmholtz equation on a free surface.

From Navier-Stokes to Helmholtz

2D Helmholtz equation on a free surface

$$\nabla^2 \phi + \kappa^2 \phi = 0$$

Furthermore, κ is linked to the wave-frequency ω via the relation :

$$\omega^2 = g\kappa\left(1 + d_c^2 \kappa^2\right)\tanh\left(\kappa h\right)$$

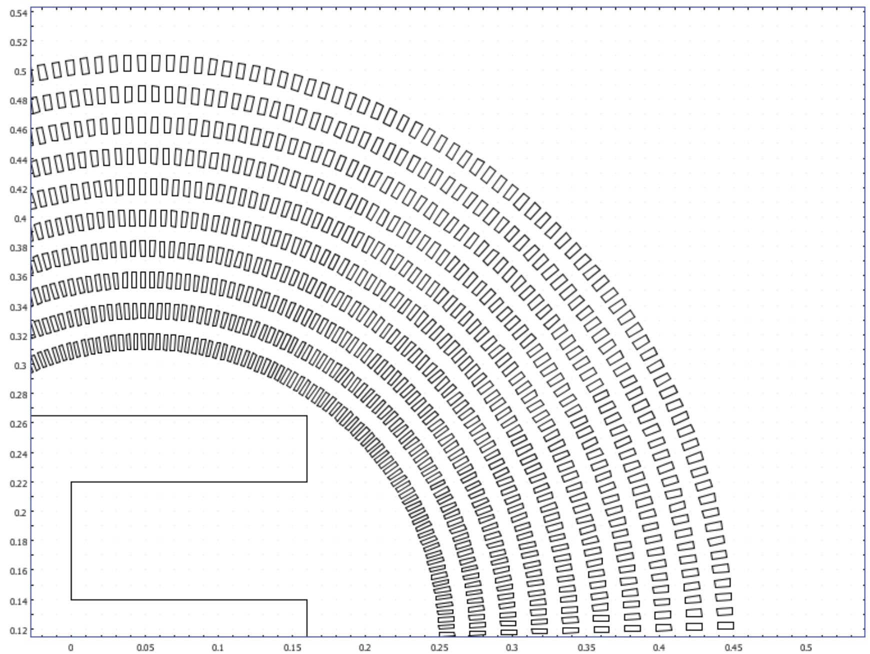
involving the depth of liquid h, its capillarity d_c and the gravity g.

On a rigid obstacle, the boundary condition is a **Neuman condition.**

Cloaking Linear Water Waves

The viscosity matrix is such that the liquid will flow faster in the azimutal direction and will be bent around the central region of the cloak.

➔ Goal : homogenization of the structured cloak to obtain an effective anisotropic fluid.



Cloaking Linear Water Waves

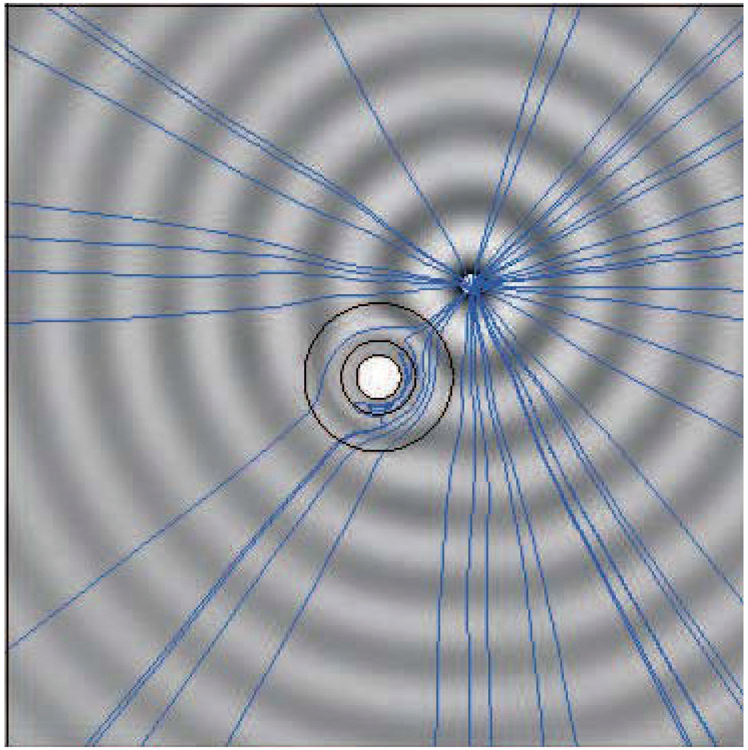
Taking the two-scale limit in the fast-oscillating potential field, we obtain the following homogenized problem when $\eta \rightarrow 0$:

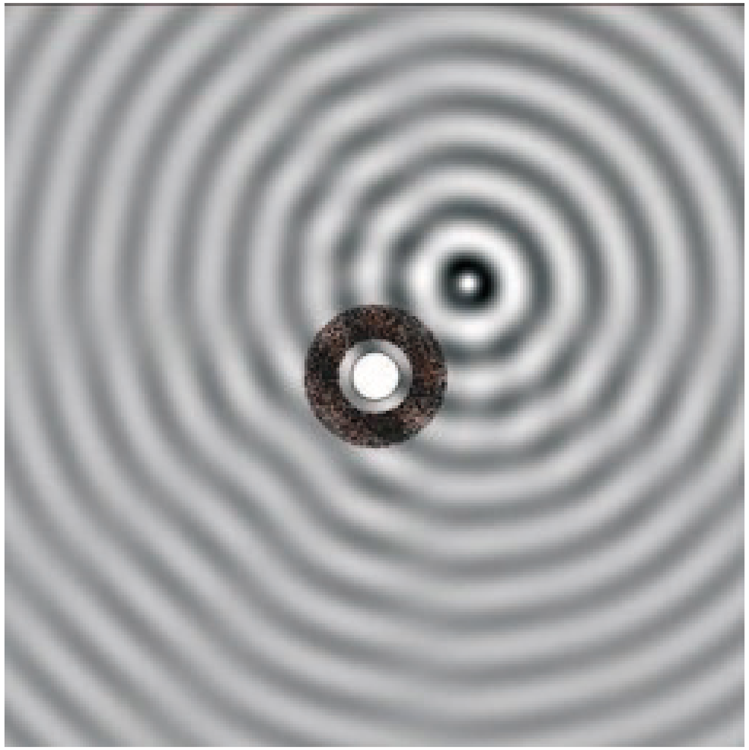
$$\nabla\left(\left[\mu_{\mathrm{hom}}\right]\nabla\phi_{\mathrm{hom}}\left(\mathbf{x}\right)\right)=\kappa^{2}\phi_{\mathrm{hom}}\left(\mathbf{x}\right)$$

The fluid flowing within the homogenized cloak is characterized by the shear viscosity matrix :

$$\left[\mu_{\mathrm{hom}}\right]=\begin{bmatrix}1.7 & 0\\ 0 & 8.2\end{bmatrix}$$

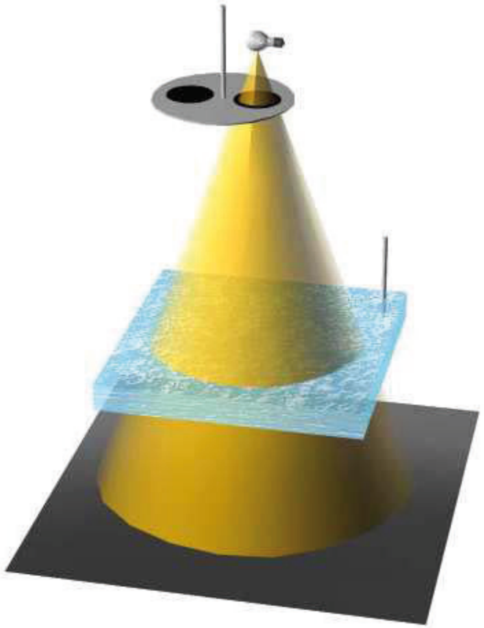
Cloaking Linear Water Waves

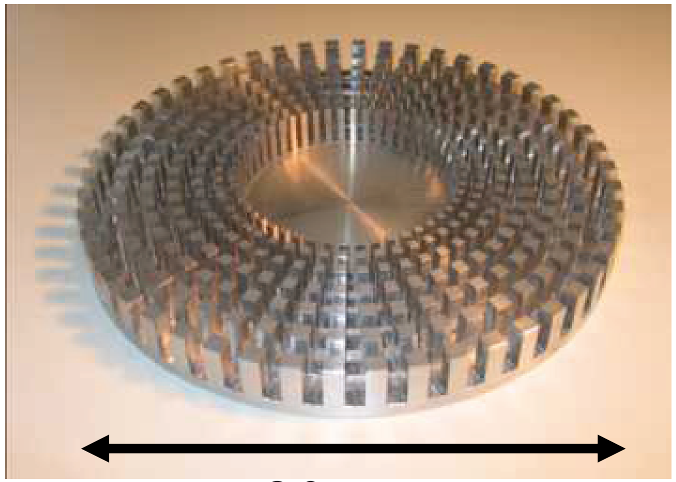




Cloaking Linear Water Waves

A classical experimental setup used in secondary schools:



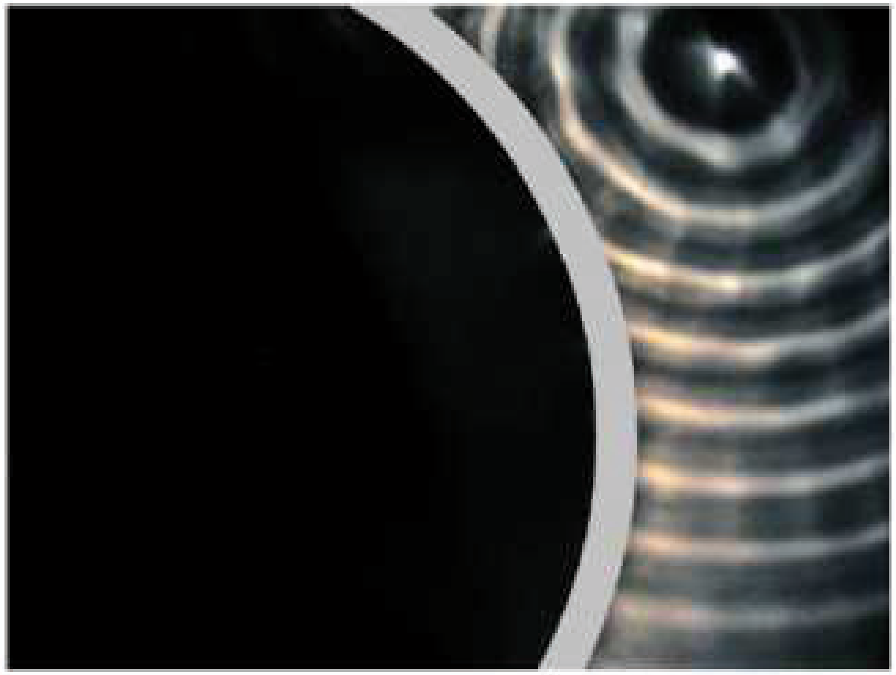


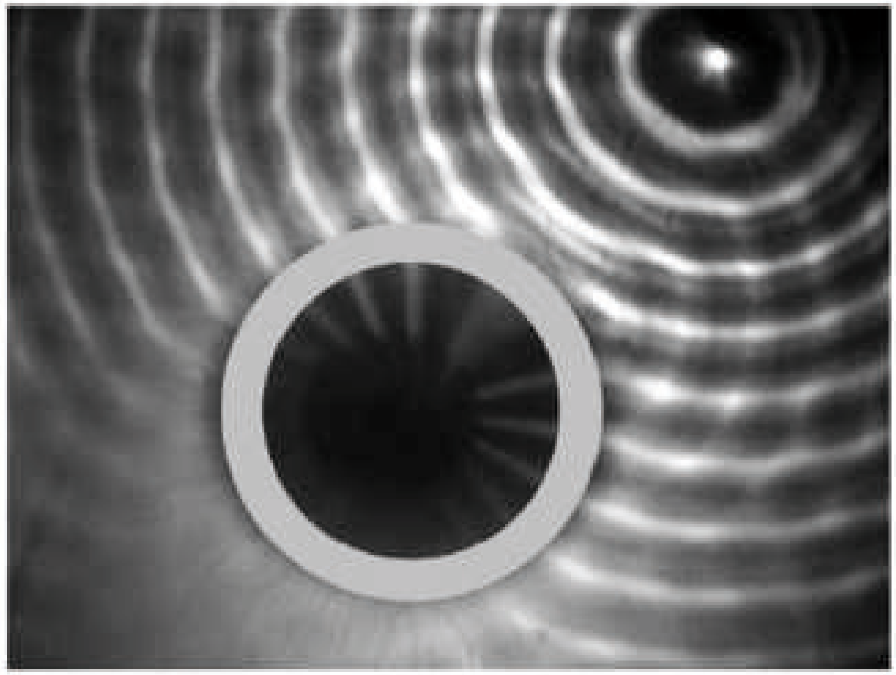
20 cm

M. Farhat, S. Enoch, S. Guenneau, and A. B. Movchan, "Broadband cylindrical acoustic cloak for linear surface waves in a fluid", *Physical Review Letters* 101,.134501 (2008).

Cloaking Linear Water Waves

Methoxynonafluorobutane:
viscosity 0,61 mm^2/s
density 1,529 g/mL.

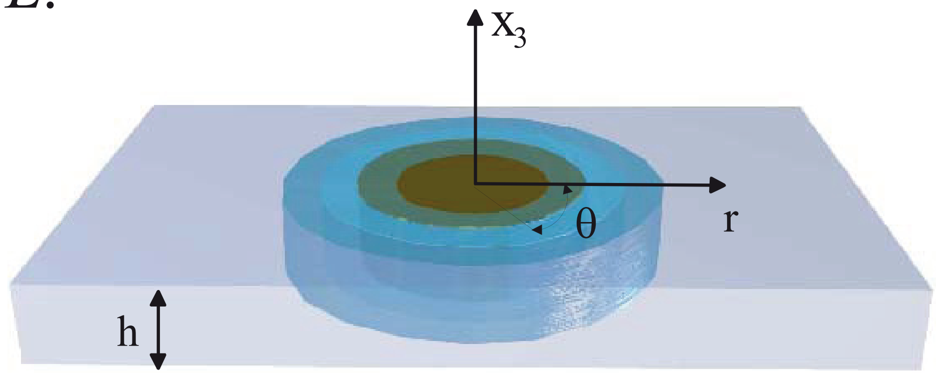




Thin Elastic Plates

Transformation Electrodynamics is still challenging but …

The wavelength λ is supposed to be large enough compared to the thickness of the plate h and small compared to its in-plane dimension L, *i.e.* $h << \lambda << L$.



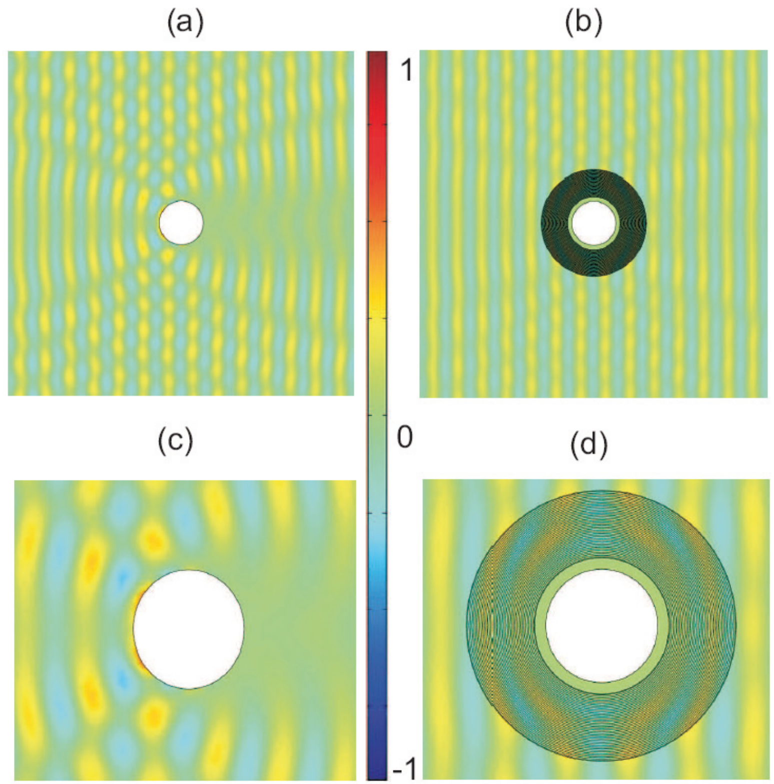
The out-of-plane displacement u= $(0;\ 0; U(r;\ \theta))$ in the x_3-direction (along the vertical axis) is solution of:

$$\langle\lambda\rangle\nabla\cdot\left(\underline{\underline{\zeta}}^{-1}\nabla\left(\langle\lambda\rangle\nabla\cdot\left(\underline{\underline{\zeta}}^{-1}\nabla U\right)\right)\right)-\beta_0^4 U=0$$

Thin Elastic Plates

Left: Real part of the displacement scattered by a rigid clamped obstacle of radius 0:25m for an incoming plane wave of frequency 22:5Hz

Right: Real part of the displacement scattered by a rigid clamped obstacle surrounded by a multilayered isotropic cloak (inner radius a = 0.3m, outer radius b = 0.6m and 20 layers, for an incoming plane wave of frequency 22.5Hz



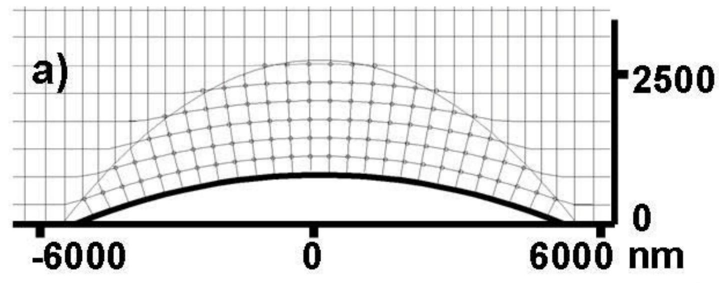
M. Farhat, S. Guenneau, S. Enoch, A. B. Movchan, "Cloaking bending waves propagating in thin elastic plates," *Physical Review B* 79, 033102 (2009).

M. Farhat, S. Guenneau, and S. Enoch, "Ultrabroadband Elastic Cloaking in Thin Plates", Physical Review Letters 103, 024301 (2009)

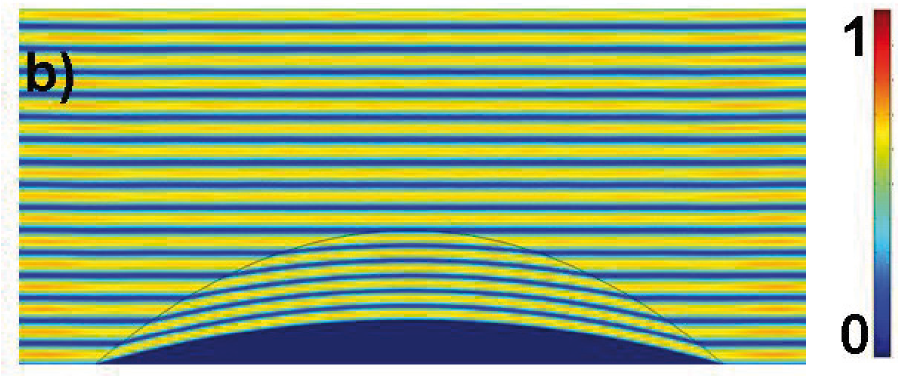
Plasmonic Carpet

We consider the following transformation

$$\begin{cases} x' = \dfrac{x_2(y) - x_1(y)}{x_2(y)} x + x_1(y)\,, 0 < x < x_2(y)\,, \\ y' = a < y < b\,, \\ z' = z\,, 0 < z < +\infty\,, \end{cases}$$

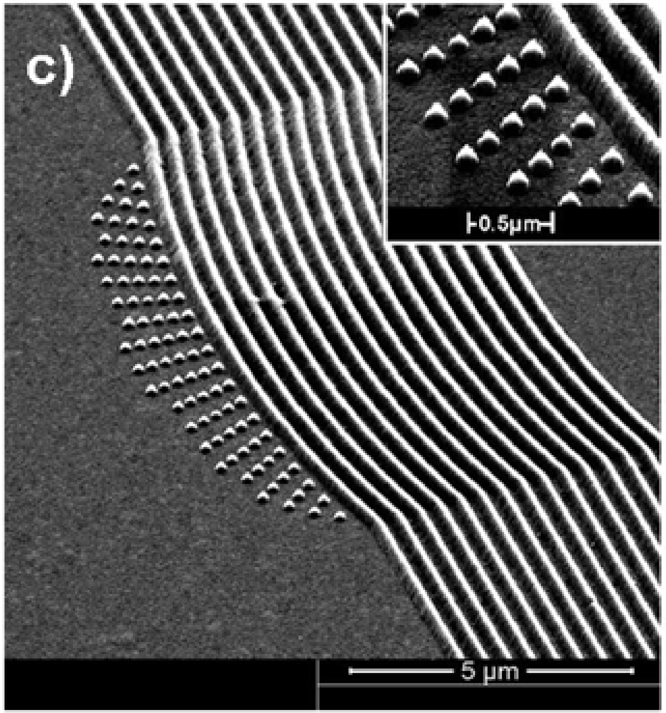


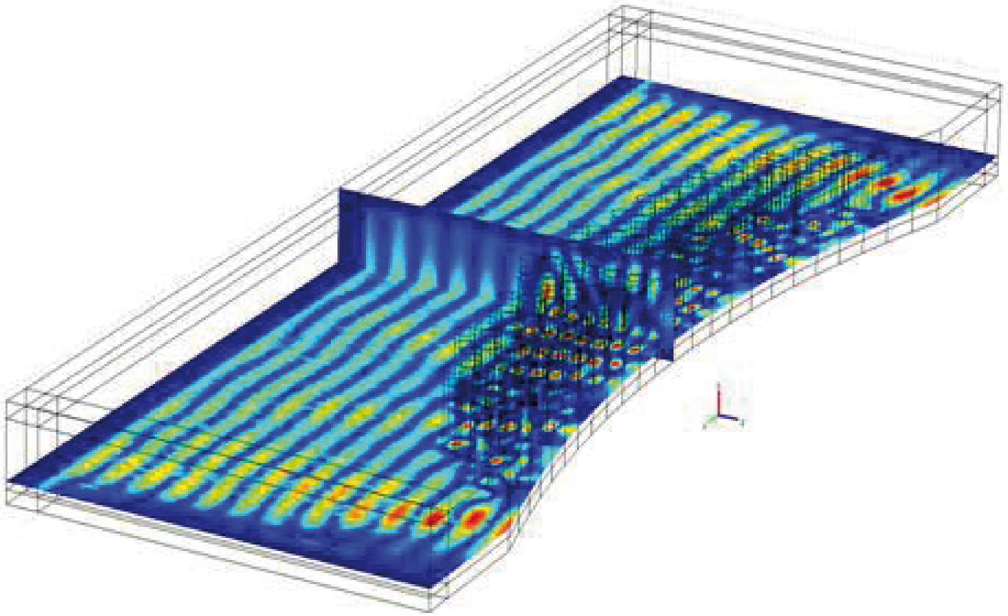
That gives us the permittivity and permeability tensors.



J. Li, and J. B. Pendry, "Hiding under the carpet: a new strategy for cloaking," Phys. Rev. Lett. **101**(20), 203901(2008).

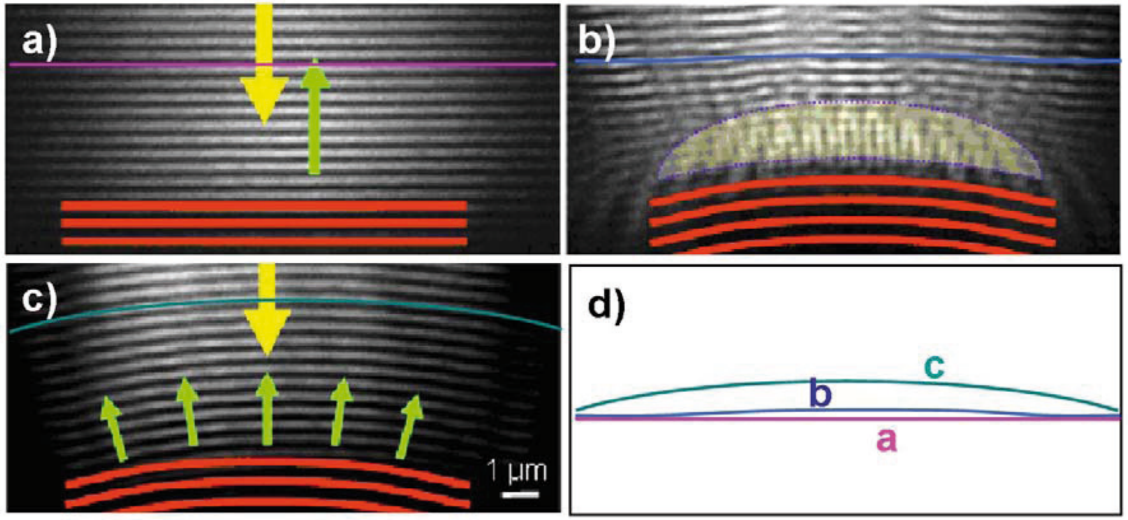
Plasmonic Carpet





We chose a configuration in which a gold surface is structured with TiO_2 nanostructures.
The shape of the obtained TiO2 particles is conical (h = 200 nm, r = 210 nm) as a consequence of the etching anisotropy.

Plasmonic Carpet



Comparing the areas under the numerically averaged curves b (curved mirror with carpet) and c (curved mirror without carpet) leads to reduction by a factor 3.7.

J. Renger, M. Kadic, G. Dupont, S. S. Aćimović, S. Guenneau, R. Quidant , S. Enoch, "hidden progress: broadband plasmonic invisibility" *optics express* **18**, 15757-15768 (2010).

P. A. Huidobro, M. L. Nesterov, L. Martin-Moreno, and F. J. García-Vidal, “Transformation Optics for Plasmonics,” http://arxiv.org/abs/1003.1154.

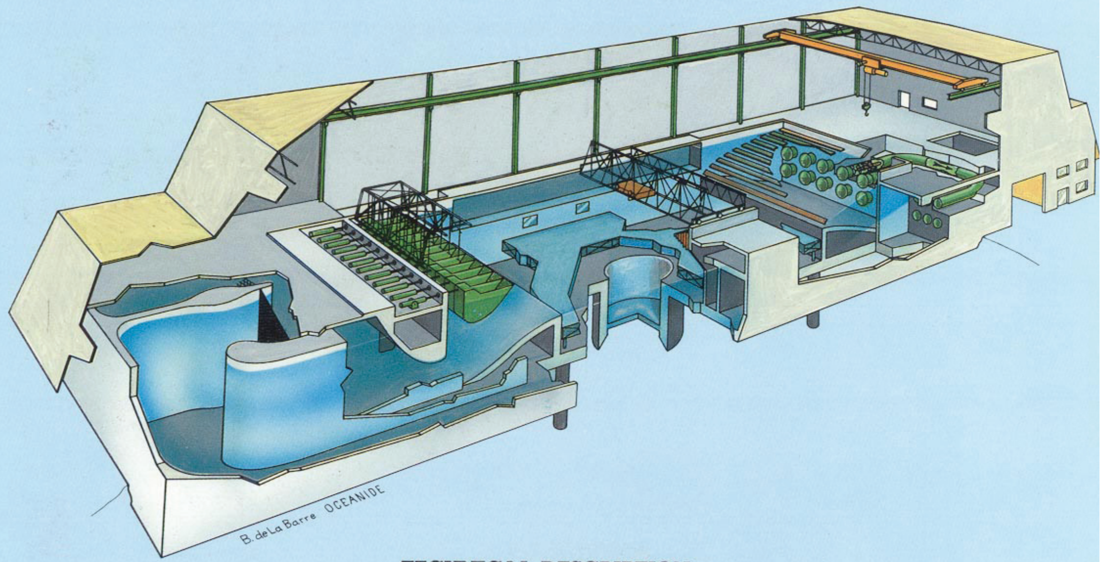
Y. Liu, T. Zentgraf, G. Bartal, and X. Zhang, “Transformational Plasmon Optics,” http://arxiv.org/abs/1003.1326.

Conclusion

- Larger scale
Nonlinearity
Collaboration with B. Molin (IRPHE, Marseille)

- Applications : anti vibration systems, fish farming…

- Apply metamaterials concepts to other type of waves.



TECHNICAL DESCRIPTION:

DIMENSIONS:

Overall dimensions	Length	:	65 m
	Width	:	26 m
Dimensions of the test channel	Length	:	24 m
	Width	:	16 m
	Depth	:	6 m
Central pit	Diametre	:	5 m
	Depth of bottom	:	- 10 m

WAVE CHARACTERISTICS :

Regular and complex aves, unidirectional in the initial phase.
Wave generation using 12 horizontal cylindrical floats moving back and forth horizontally.

CURRENT CHARACTERISTICS :

Water layer concerned	3 m
Velocity in direction of wave propagation	V ≤ 0.4 m/s
Velocity in the opposite direction	V ≤ 0.25 m/s

Current generation using 12 Helical pumps

MOVEABLE BOTTOM:

Mobile bottom - 5 m to 0.
Inclinable ± 7°.

MODEL TOWING:

X-Y platform covers the entire area of the basin.

Longitudinal velocity	V ≤ 1.5 m/s
Transversal velocity	V ≤ 0.8 m/s

BGO FIRST, La Seyne sur Mer