Speckle Statistics of Localized Waves

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Wave propagation in disordered media

Field
$$E(\mathbf{r}, t) = \sum_i E_i(\mathbf{r}, t)$$

Intensity
$$I(\mathbf{r}, t) = |E(\mathbf{r}, t)|^2 = \sum_{i,j} E_i(\mathbf{r}, t) E_j^*(\mathbf{r}, t)$$

Wave propagation in disordered media

Average intensity:

$$\langle I(\mathbf{r}, t) \rangle = \sum_i \langle |E_i(\mathbf{r}, t)|^2 \rangle + \sum_{i \neq j} \langle \cancel{E_i(\mathbf{r}, t) E_j^*(\mathbf{r}, t)} \rangle$$

independent paths

Weak localization and non-Gaussian statistics

Probability of return: $(A + A^*)^2 = 4A^2$, $A^2 + A^{*2} = 2A^2$
wave particle

- transport reduction
- paths correlation

- weak localization
- non-Gaussian statistics

Transmission coefficients

$$T_{ab} = |t_{ab}|^2 \quad \text{Transmitted intensity = speckle intensity}$$

$$T_a = \sum_b T_{ab} \quad \text{Total transmission} \Rightarrow \text{brightness}$$

$$T = \sum_a T_a = \sum_{ab} T_{ab} \quad \text{Transmittance = conductance}$$

Random matrix theory

$$t = U \begin{pmatrix} \sqrt{\tau_1} & & 0 \\ & \dots & \\ 0 & & \sqrt{\tau_N} \end{pmatrix} V, \quad tt^\dagger = \begin{pmatrix} \tau_1 & & 0 \\ & \dots & \\ 0 & & \tau_N \end{pmatrix}$$

$$P\{\tau_n\} \Rightarrow P(T_{ab}), P(T_a), P(T)$$

Non-Gaussian statistics of transmission

$$t_{ab} = \sum_n u_{an} \sqrt{\tau_n} v_{nb} = \sqrt{\sum_n |u_{an}|^2 \tau_n} \times t'_{ab} = \sqrt{T_a} \times t'_{ab}$$

AAC & Genack, *PRA* (2005)

$$T_{ab} = T_a \times |t'_{ab}|^2$$

$$P_a(T_{ab}) = \frac{1}{T_a} \exp(-T_{ab}/T_a) \Rightarrow P(T_{ab}) = \int_0^\infty \frac{dT_a}{T_a} P(T_a) \exp(-T_{ab}/T_a)$$

$$\langle T_{ab} T_{a'b'} \rangle = \langle T_a T_{a'} \rangle \times \langle |t'_{ab}|^2 |t'_{a'b'}|^2 \rangle$$

Kogan & Kaveh, *PRB* (1995)

Probability distribution of total transmission

$$P(T_a) = L^{-1}\{F(z)\},$$

$$F(z) = \exp[-\langle g \rangle \ln^2(\sqrt{1 + z/\langle g \rangle} + \sqrt{z/\langle g \rangle})], \quad g = \Sigma_n \tau_n$$

Nieuwenhuizen & Van Rossum, *PRL* (1995)

$$\text{var}(T_a) = 2/3 \langle g \rangle$$

AAC & Genack, *PRA* (2005)

Intensity correlation with polarization

$$\langle T_{ab} T_{a'b'} \rangle = \langle T_a T_{a'} \rangle \times \langle |t'_{ab}|^2 |t'_{a'b'}|^2 \rangle$$

$$C = \langle T_{ab} T_{a'b'} \rangle - \langle T_{ab} \rangle^2 = \cos^2 \Delta\theta + \text{var}(T_a)(1 + \cos^2 \Delta\theta)$$

Speckle statistics of localized waves

Q: How speckle pattern of localized waves looks like?

Q: How different it is from that of extended waves?

Q: What we can learn from speckle statistics of localized waves?

Experimental setup

Alumina sample

alumina sphere:

$d=0.9 \text{ cm}$

$n=3.14$

$f=0.068$

copper tube:

$D=7.3 \text{ cm}$

Correlations with polarization

$$C(\Delta\theta_S, \Delta\theta_D) = \cos(\Delta\theta_S) \cos(\Delta\theta_D) + A_2 (\cos(\Delta\theta_S) + \cos(\Delta\theta_D)) + A_3 (1 + \cos(\Delta\theta_S) + \cos(\Delta\theta_D) + \cos(\Delta\theta_S) \cos(\Delta\theta_D))$$

$$C(90,90) = A_3, \quad C(0,90) = C(90,0) = A_2 + 2A_3$$

Chabanov et al, *PRL* **92**, 173901 (2004)

Intensity correlation of localized waves

Alumina localized sample, $L = 4\xi$

From fit: $A_2 = 1.00 \pm 0.02$

$A_3 = 4.38 \pm 0.04$

Intensity correlation of localized waves

$$\frac{\langle T_{ab} T_{a'b'} \rangle}{\langle T_{ab} \rangle^2} = C(\Delta\theta_S, \Delta\theta_D) + 1 = (1 + A_3)(1 + \cos(\Delta\theta_S))(1 + \cos(\Delta\theta_D))$$

$$\langle I^2 \rangle = 4(1 + A_3) \quad \Delta\theta_D = 0, \Delta\theta_S = 0$$

$$\langle I_1 I_2 \rangle = 1 + A_3 \quad \Delta\theta_D = 90, \Delta\theta_S = 90$$

$$\frac{\langle I^2 \rangle}{\langle I_1 I_2 \rangle} = 4$$

Intensity correlation of localized waves

$$L = 2\xi$$

$$L = 0.4\xi$$

$$\langle I^2 \rangle = 4(1 + A_3)$$

$$\langle I_1 I_2 \rangle = 1 + A_3$$

$$\frac{\langle I^2 \rangle}{\langle I_1 I_2 \rangle} = 4$$

Statistics of transmission quantities in localized regime

$$T_{ab} = |u_{a1}|^2 \tau_1 |v_{1b}|^2 ,$$

$$T_a = |u_{a1}|^2 \tau_1 ,$$

$$T = \tau_1 .$$

We now switch to the normalized quantities, $s_{ab} = T_{ab}/\langle T_{ab} \rangle$, $s_a = T_a/\langle T_a \rangle$, and $s = T/\langle T \rangle$.

$$\langle s_{ab}^n \rangle = (n!)^2 \langle s^n \rangle ,$$

$$\langle s_a^n \rangle = n! \langle s^n \rangle .$$

These lead to the relations between the statistics of the transmission quantities,

$$P(s_a) = \int_0^\infty \frac{ds}{s} P(s) \exp(-s_a/s) ,$$

$$P(s_{ab}) = \int_0^\infty \frac{ds_a}{s_a} P(s_a) \exp(-s_{ab}/s_a) = 2 \int_0^\infty \frac{ds}{s} P(s) K_0(2\sqrt{s_{ab}/s}) ,$$

Correlations: $\Gamma_{s_{ab}}(\Delta\theta_{in}, \Delta\theta_{out}) = \langle s^2 \rangle (1 + \cos^2(\Delta\theta_{in})) (1 + \cos^2(\Delta\theta_{out}))$

Statistics of transmission quantities in localized regime

$$\frac{\langle I^{r+q} \rangle}{\langle I_1^r I_2^q \rangle} = \left(\frac{(r+q)!}{r!q!} \right)^2$$

Statistics of conductance in localized regime

$$\ln(s_{ab}) = \ln|u_{a1}|^2 + \ln(s) + \ln|v_{1b}|^2$$

$$F_{\ln(s_{ab})}(\omega) = F_{\ln(s)}(\omega)\Gamma(1+i\omega)^2$$

$$P[\ln(s)] = \int_{-\infty}^{+\infty} dz \exp[iz \ln(s)] F_{\ln(s)}(z)$$

Statistics of conductance in localized regime

Summary

- In a given quasi-1D random configuration, the statistics of transmitted field is Gaussian for both diffusive and localized waves; non-Gaussian mesoscopic field statistics arise in ensemble of configurations due to mesoscopic fluctuations of transmission.
- For localized waves, transmission statistics is rather simple: only statistics of conductance matters, whereas statistics of total transmission and speckles follow. The distribution of conductance can be easily determined from transmission or speckle intensity distributions
- The factorization of the speckle statistics can be used as indication of the localization threshold, because only one transmission contributes.