Microwave experiments on correlated disorder

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Collaborations

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One dimensional Waveguides (1D)

Microwave set-up

Only one mode propagates in the frequency range:

$$7.5 \text{ GHz} \leq \nu \leq 15 \text{ GHz}.$$

$\Rightarrow$ wavenumber range ($k = \frac{2\pi}{c} \sqrt{\nu^2 - \nu_0^2}$): $0 \leq k/(\pi/d) \leq 1.8$

The experiment measures the transmission $\propto \frac{1}{\text{Tr}(T^{(N)})}$

One-dimensional waveguide

Only one mode propagates $\Rightarrow$ transfer matrix ansatz:

$$\begin{pmatrix} a_{n+1} \\ b_{n+1} \end{pmatrix} = T_n \begin{pmatrix} a_n \\ b_n \end{pmatrix}$$

Assumptions:

- time-reversal symmetry $\Rightarrow r_n = r_n^*$ and $t_n = t_n^*$
- no absorption $\Rightarrow |r_n|^2 + |t_n|^2 = 1$

$$T_n = \begin{pmatrix} \frac{1}{t_n} e^{ikd} & \frac{r_n}{t_n} e^{-ikd} \\ \frac{r_n^*}{t_n^*} e^{ikd} & \frac{1}{t_n^*} e^{-ikd} \end{pmatrix}$$

kd : phase shift without scatterer

Periodic structures

Every third (a) or every fourth (b) micrometer screw is introduced 3 mm in the waveguide:

Wavefunctions

Reflection measurement:
Every fifth scatterer is introduced 3 mm:

Bloch oscillations are observed.

Hofstadter butterfly

Hofstadter calculated the conductance of a twodimensional crystal with a perpendicularly applied magnetic field B (Hofstadter 1976):

The g_n obey the Harper equation:

$$g_{n+1} + g_{n-1} + 2 \cos(2\pi n\alpha - \nu) g_n = E g_n$$

with

$$\alpha = \frac{a^2 e B}{h}$$

α : number of magnetic flux quanta per unit cell

Hofstadter butterfly

α rational $\frac{p}{q} \Rightarrow$ Spectrum splits into q subbands.

α irrational $\Rightarrow$ Spectrum forms a Cantor Set

- spectra are self-similar.
- a typically 10^{-10} m $\Rightarrow B \approx 10^5$ T needed.
Alternative: superlattices (Schlösser et al. 1996)

Harper modell vs waveguide

Harper equation: $g_{n+1} + g_{n-1} + 2 \cos(2\pi n\alpha - \nu) g_n = E g_n$ too, allows a transfer matrix notation:

$$\begin{pmatrix} g_{N+1} \\ g_N \end{pmatrix} = T^{(N)} \begin{pmatrix} g_1 \\ g_0 \end{pmatrix}$$

with $T^{(N)} = \prod_{n=0}^N T_n$, where

$$T_n = \begin{pmatrix} E - 2 \cos(2\pi n\alpha - \nu) & -1 \\ 1 & 0 \end{pmatrix}$$

One-dimensional Waveguide:

$$T_n = \begin{pmatrix} \frac{1}{t_n} e^{ikd} & \frac{r_n}{t_n} e^{-ikd} \\ \frac{r_n^*}{t_n^*} e^{ikd} & \frac{1}{t_n^*} e^{-ikd} \end{pmatrix}$$

Transmission if:

$$\lim_{N \rightarrow \infty} \frac{1}{\text{Tr}(T^{(N)})} \begin{cases} \neq 0 & \text{allowed bands (Bloch band)} \\ = 0 & \text{forbidden bands (band gap)} \end{cases}$$

Spectra for different α

Micrometer screw n is introduced 3 mm in the waveguide, if $\cos(2\pi n\alpha) > 0$:

$\alpha = 1/1$ (a), $1/2$ (b), $1/4$ (c), $1/8$ (d), $1/16$ (e)

[U.K., H.-J. Stöckmann. PRL 80, 3232 (1998)]

Microwave Hofstadter butterfly

Transmission as function of k and α in greyscale printing reproduces the Hofstadter butterfly in every Bloch band:

- spectra are self-similar
- fractal depth of about 3 or 4
- low and high frequency range is not accessible due to damping

[U.K., H.-J. Stöckmann. PRL 80, 3232 (1998)]

Butterfly for different d

Using only every second micrometer screw doubles the number of Bloch bands in the accessible frequency range:

- butterflies in second and third band complete
- usage of half of the scatterers $\Rightarrow$ smaller fractal depth

Experiment versus scattering theory

Calculating the transmission for 100 scatterers in a rectangular scattering arrangement by multiplying

$$T_n = \begin{pmatrix} \frac{1}{t_n} e^{ikd} & \frac{r_n}{t_n} e^{-ikd} \\ \frac{r_n^*}{t_n^*} e^{ikd} & \frac{1}{t_n^*} e^{-ikd} \end{pmatrix}$$

Correlated disorder

Band edges

Correlated disorder (tight binding)

The tight-binding model

$$\psi_{n+1} + \psi_{n-1} = (E + \epsilon_n)\psi_n,$$

with ϵ_n : site potential,

The inverse localization length within the band ($|E| < 2$) is given by:

$$l^{-1} = \frac{\langle \epsilon_n^2 \rangle \varphi(\mu)}{8 \sin^2 \mu}$$
$$\varphi(\mu) = 1 + 2 \sum_{k=1}^{\infty} \xi(k) \cos(2\mu k), \quad \text{where } E = 2 \cos \mu$$

$\xi(k)$: two-point correlators of the site potential ϵ_n

$$\xi(k) = \frac{2}{\pi} \int_0^{\pi/2} \varphi(\mu) \cos(2k\mu) d\mu$$

Arbitrary band structures can be generated e.g. via:

$$\epsilon_n = A \sum_{k=-\infty}^{\infty} \xi(k) Z_{n+k}$$

Correlated disorder (Kronig-Penney)

Discrete Kronig-Penney model

$$\psi_{n+1} + \psi_{n-1} = [2 \cos(kd) + U_n kd \sin(kd)] \psi_n$$

Split potential into mean ϵ and fluctuating ϵ_n parts:

$$U_n = \epsilon + \epsilon_n.$$

Equivalent to tight-binding model with

$$E \rightarrow 2 \cos(kd) + k\epsilon \sin(kd)$$

$$\epsilon_n \rightarrow k\epsilon_n \sin(kd) : \text{random potential, } d \text{ scatterer distance}$$

Localization length

$$l_{\infty}^{-1} = \frac{\epsilon_0^2}{8k^2} \frac{\sin(kd)}{\sin^2(\gamma)}, \quad \epsilon_0 = \langle \epsilon_n^2 \rangle$$

with

$$2 \cos(\gamma) = 2 \cos(kd) + \frac{\epsilon_0}{k} \sin(kd), \quad E = k^2$$

Band structures

Transmission through generated correlated disorder arrangement

Numerics:

$$N = 10^4$$

$$\langle \epsilon_n \rangle = -0.1$$

$$\sqrt{\langle \epsilon_n^2 \rangle} = 0.1$$

Measurement:

Single (N=100) and average of 5

Multiplication of 5 measurements (N=500)

[U.K., F. M. Izrailev, A. A. Krokhin, H.-J. Stöckmann. APL 2000]

Inverse band structures

Transmission through generated correlated disorder arrangement

Numerics:

$$N = 10^4$$

$$\langle \epsilon_n \rangle = -0.1$$

$$\sqrt{\langle \epsilon_n^2 \rangle} = 0.1$$

Measurement:

Single (N=100) and average of 5

Multiplication of 5 measurements (N=500)

[U.K., F. M. Izrailev, A. A. Krokhin, H.-J. Stöckmann. APL 2000]

Random structures

Transmission through a random scattering arrangement:

Micrometer screws are introduced randomly between 0 and 3mm.
The transmission is roughly **a percent of the transmission for the correlated disorder arrangements!**

Correlated disorder

Enhanced localization

Transmission wave functions (random)

Transmission wave functions (correlated)

Reflection wave functions (random)

Reflection wave functions (correlated)

Strongly localized wave functions

- transmission shows band-gap structure
- seen in reflection as well
- localization within the gaps
- delocalization elsewhere
- reduced localization length by a factor of 15 (to ≈ 10)

[U.K., F. M. Izrailev, A. A. Krokhin, H.-J. Stöckmann. APL 2000]

[U.K., F. M. Izrailev, A. A. Krokhin, PRL 2008]

Disorder

Fabry-Perot bands

Index variation (periodic lattice)

Periodic lattice of teflon ($n \approx 1.4$) pieces:

Single mode propagation within
 $7.5 \text{ GHz} \leq \nu \leq 15 \text{ GHz}$.

- periodic structures $\Rightarrow$ bands and gaps
- “photonic crystal”

Impurity

- impurities generate bound states in the gap
- impurity position $\Leftrightarrow$ band oscillations
- good agreement with transfer matrix simulations
- derived exact theoretical description

[G. Luna-Acosta, H. Schanze, U.K., H.-J. Stöckmann, New J. of Physics 10, 043005 (2008)]

Positional disorder with Fabry-Perot

- No disorder in length of Teflon pieces
⇒ Perfect transmission at resonance frequency

26-cells arrays:

Disorder:

(a) weak $\epsilon = 3.0 \cdot 10^{-2}$

(b) medium $\epsilon = 12.3 \cdot 10^{-2}$

(c) strong $\epsilon = 49.0 \cdot 10^{-2}$

black: numerics

red: experiments

Dashed: Analytics

blue: teflon resonances

- two different band types
- behaviour on disorder strength different
- theoretical description for weak disorder

[G. A. Luna-Acosta, F. M. Izrailev, N. M. Makarov, U.K., and H.-J. Stöckmann, Phys. Rev. B 80, 115112 (2009)]

Quasi one dimensional Waveguides (Q1D)

Experimental set-up

- 2 moveable antennas
- scatterers here:
brass bars
- absorbers at both ends
mimic an infinite system

Experimental set-up

- 2 moveable antennas
- scatterers here:
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- measurement in position space
- Fourier transform $\Rightarrow$ scattering matrix in mode representation
- wavenumber for each mode n :

$$k_{x,n} = \sqrt{k^2 - \left(\frac{n\pi}{d}\right)^2}$$

Transmission (position representation)

- fixed antenna positions ($y_1=y_2=55$ mm)
- vertical lines indicate mode openings
- input data for Fouriertransform

Mode extraction

- modulus square of scattering matrix elements $|S_{nm}|$
- $\nu = 11$ GHz where 7 open modes exist

Transmission for 4th mode

- mode propagation for the 4th mode through of the empty waveguide (dark)
- $S_{nn} \sim G_n, \quad T_n \sim |G_n|^2$
 G_n : Green function for rectangular waveguide

Periodic structures

Band structure example for 3rd (blue) and 4th mode (yellow)

- clear band structure which is the same for all modes
- vertical lines indicate band edges
- description in terms of an effective bilayer model
- use band edges to determine the free parameters $(n_{a/b}, \alpha)$ from the dispersion relation:
$$\cos(\gamma) = \cos(k_x n_a a) \cos(k_x n_b b) - \alpha \cdot \sin(k_x n_a a) \sin(k_x n_b b)$$

Correlated disorder

Band structure of 3rd mode:

- transmission for white noise disorder in yellow
- transmission for correlated disorder in blue
- blue dotted lines indicate the expected correlation gaps:
 $\gamma/\pi = 0.3$ and $\gamma/\pi = 0.7$

Fabry-Perot band

- width and height of metallic inset always the same
⇒ Fabry-Perot band
- induced gap close to Fabry-Perot resonance disappeared
- other is shifted

[O. Dietz, U.K., H.-J. Stöckmann, F.M. Izrailev, N.M. Makarov, in preparation]

Summary

- One dimensional waveguide
 - realization of Hofstadter butterfly
 - induced band by correlated disorder
 - enhanced localization by correlated disorder
 - Fabry-Perot bands
- Quasi 1D systems
 - separation of modes
 - transport suppression in each modes by correlated disorder
 - interplay between Fabry-Perot band and correlations

Publications that the presentations strongly depended on

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- O. Dietz, U. K., H.-J. Stöckmann, F. M. Izrailev, N. M. Makarov, in preparation (October 2010)