

# **Metamaterials, Transform Optics and Cloaking**

**Ross McPhedran**

**IPOS, University of Sydney, Australia**



**The University of Sydney**



**Institute for Photonics  
and Optical Science**

# Metamaterials, Transform Optics and Cloaking

Ross McPhedran



**CUDOS**

Centre for Ultrahigh bandwidth Devices for Optical Systems

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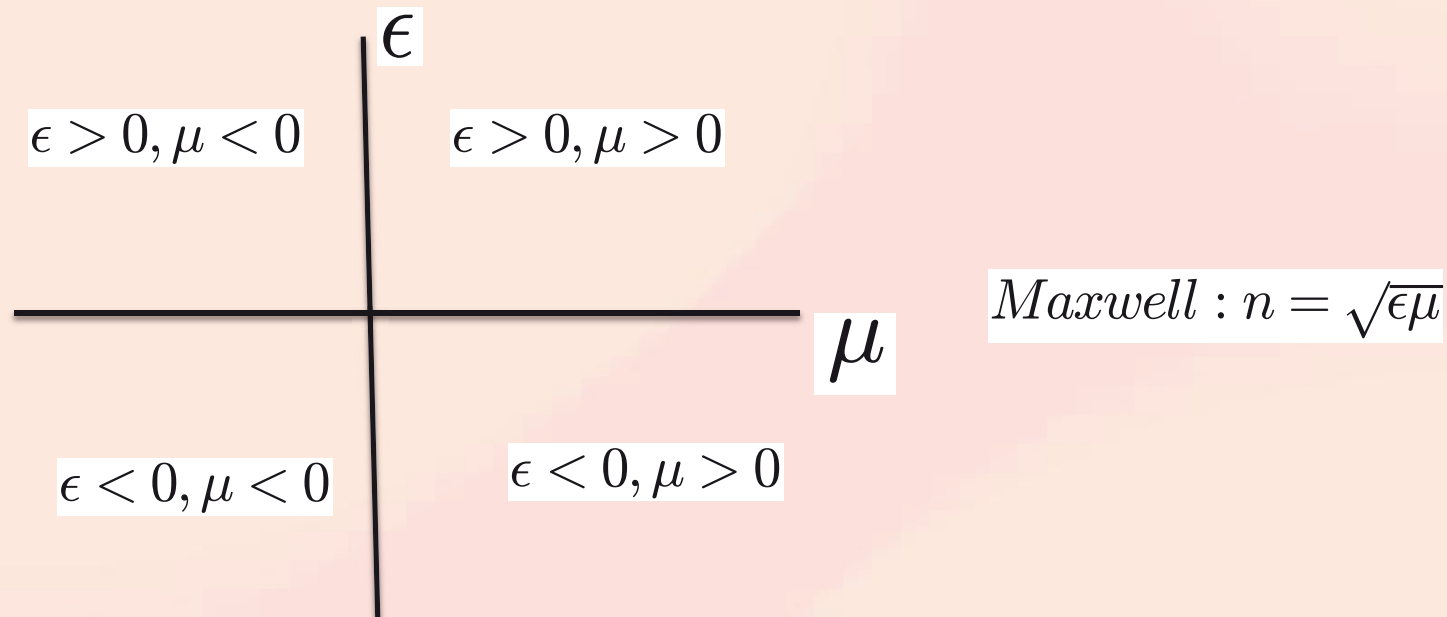
## Outline

- What are metamaterials?
- How do they differ from photonic crystals?
- What can they do?
- What is transform optics?
- Examples of some outcomes of transform optics.
- What is cloaking?
- Some different cloaking methods

## Metamaterials- Veselago

V. G. Veselago (1968 (Russian text 1967)).

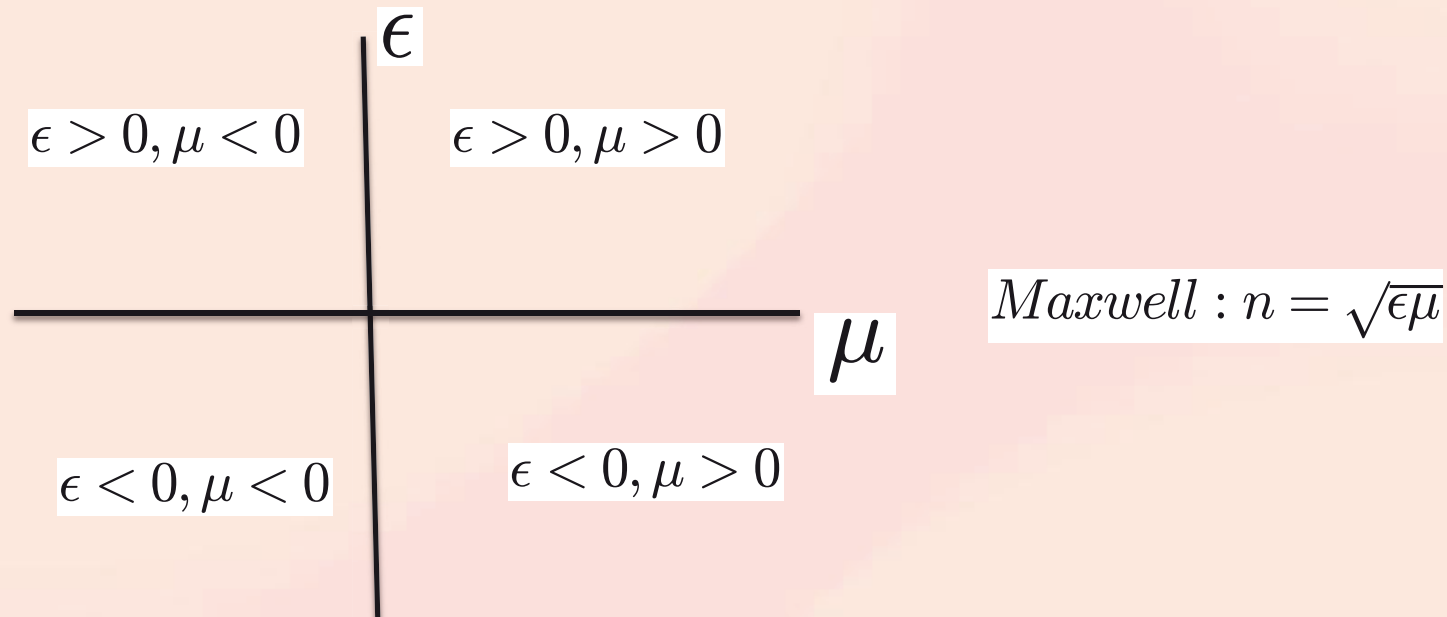
"The electrodynamics of substances with simultaneously negative values of  $\epsilon$  and  $\mu$ ". *Sov. Phys. Usp.* **10 (4): 509** □ **14.**



First, second and fourth quadrants: conventional or easily understood materials. The third quadrant gives new and striking possibilities.

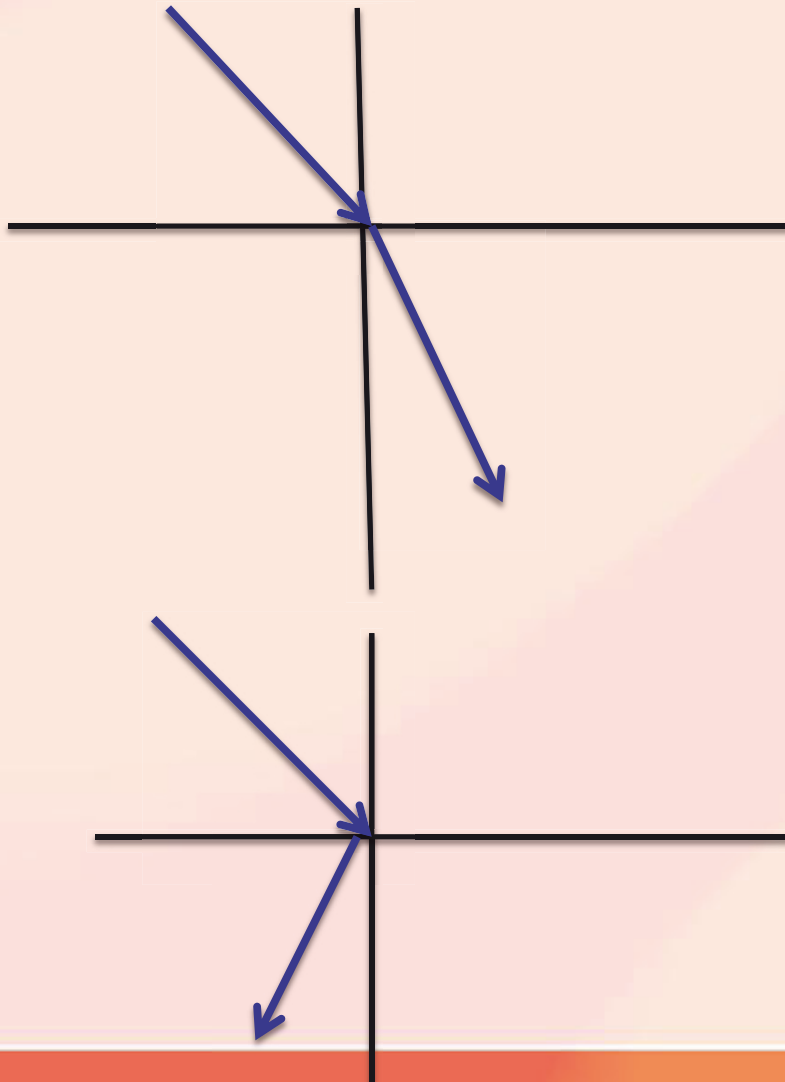
## Metamaterials- Veselago (2)

- First quadrant- conventional dielectrics
- Second quadrant-lossy magnetic
- Fourth quadrant- lossy electric



Third quadrant- free propagation, but with "left-handed" characteristics.

## Metamaterials- Veselago (3)



Normal refraction: Snell's Law

$$\text{Maxwell} : n = \sqrt{\epsilon\mu}$$

Left-handed media:  
refracted ray on same  
side of normal as  
incident ray

$$\text{Veselago} : n = -\sqrt{\epsilon\mu}$$

## Negative Refraction in a Photonic Crystal

- Veselago's idea lay dormant for 30 years: lack of materials with  $\epsilon < 0$ ,  $\mu < 0$ .
- One class of optical systems capable of displaying negative refraction is that of photonic crystals
- Animation: photonic crystal composed of 6 layers of dielectric rods (index 3.0, radius  $0.3 \times \text{period}$ ) in air; incoming beam consists of periodic set of Gaussian beams

## Negative Refraction and Structure

- Snell's law is an expression of conservation of momentum parallel to the medium interface
- Negative refraction apparently breaks momentum conservation
- Requires structured material

Homogeneous material:

$$n_i k_{i;x} = n_r k_{r;x},$$

where  $x$  runs along the interface and  $k$  denotes the wavevector (with length  $2\pi/\lambda$ ).

Structured material:

$$n_i k_{i;x} = n_r k_{r;x} + p2\pi/d_x,$$

where  $p$  is an integer and  $d_x$  is the period of the structured material along  $Ox$ .

## Sir John Pendry: Perfect Imaging

- Pendry realized that negative refraction could make possible imaging of a point by a plane dielectric slab
- Furthermore, this imaging in principle would be better than Rayleigh limit
- Rayleigh limit imposed by loss of information in evanescent orders
- Negative refraction lens can preserve information in evanescent orders

# Sir John Pendry: Perfect Imaging(2)

VOLUME 85, NUMBER 18

PHYSICAL REVIEW LETTERS

30 OCTOBER 2000

## Negative Refraction Makes a Perfect Lens

J. B. Pendry

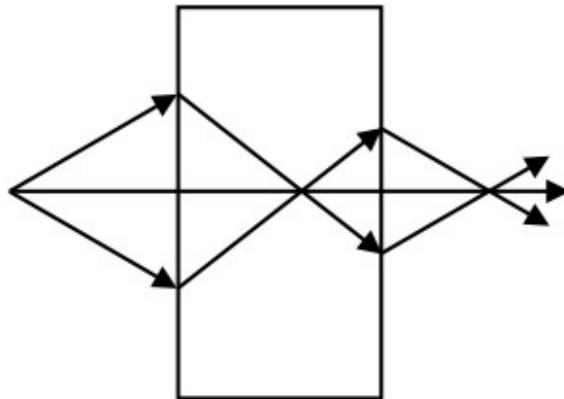


FIG. 1. A negative refractive index medium bends light to a negative angle with the surface normal. Light formerly diverging from a point source is set in reverse and converges back to a point. Released from the medium the light reaches a focus for a second time.

Key point: evanescent waves emitted from source arrive at image point with exactly the same amplitude! The negative index region compensates the positive image region: all waves have zero optical path length!

# Sir John Pendry: Building Blocks

- Negative  $\epsilon$  material: array of perfectly conducting wires

PHYSICAL REVIEW E

VOLUME 52, NUMBER 1

JULY 1995

## Photonic band gaps for arrays of perfectly conducting cylinders

N.A. Nicorovici and R.C. McPhedran

*Department of Theoretical Physics, School of Physics, University of Sydney, New South Wales 2006, Australia*

L.C. Botten

*School of Mathematical Sciences, University of Technology Sydney, New South Wales 2007, Australia*

(Received 13 March 1995)

VOLUME 76, NUMBER 25

PHYSICAL REVIEW LETTERS

17 JUNE 1996

## Extremely Low Frequency Plasmons in Metallic Mesostructures

J.B. Pendry

*The Blackett Laboratory, Imperial College London SW7 2BZ, United Kingdom*

A.J. Holden and W.J. Stewart

*GEC-Marconi Materials Technology Ltd., Caswell, Towcester, Northamptonshire NN12 8EQ, United Kingdom*

I. Youngs

## Sir John Pendry: Building Blocks (2)

- Negative  $\epsilon$  material: array of perfectly conducting wires: radius  $a$ , spacing  $d$

$$\epsilon_{\text{eff}} = 1 - \frac{\omega_p^2}{\omega^2}$$

where

$$\omega_p^2 = \frac{2\frac{1}{4}c^2}{d^2 \log(d/a)}$$

$\epsilon_{\text{eff}} < 0$  if  $\omega < \omega_p$ .

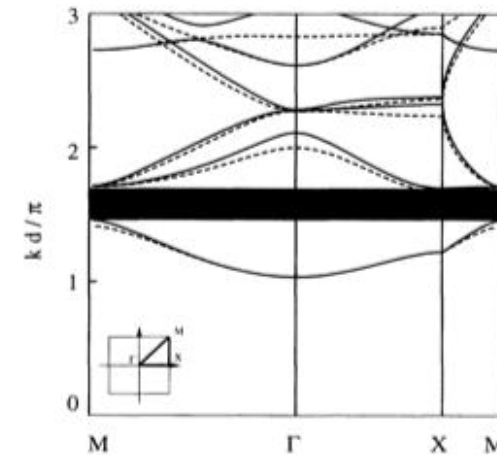
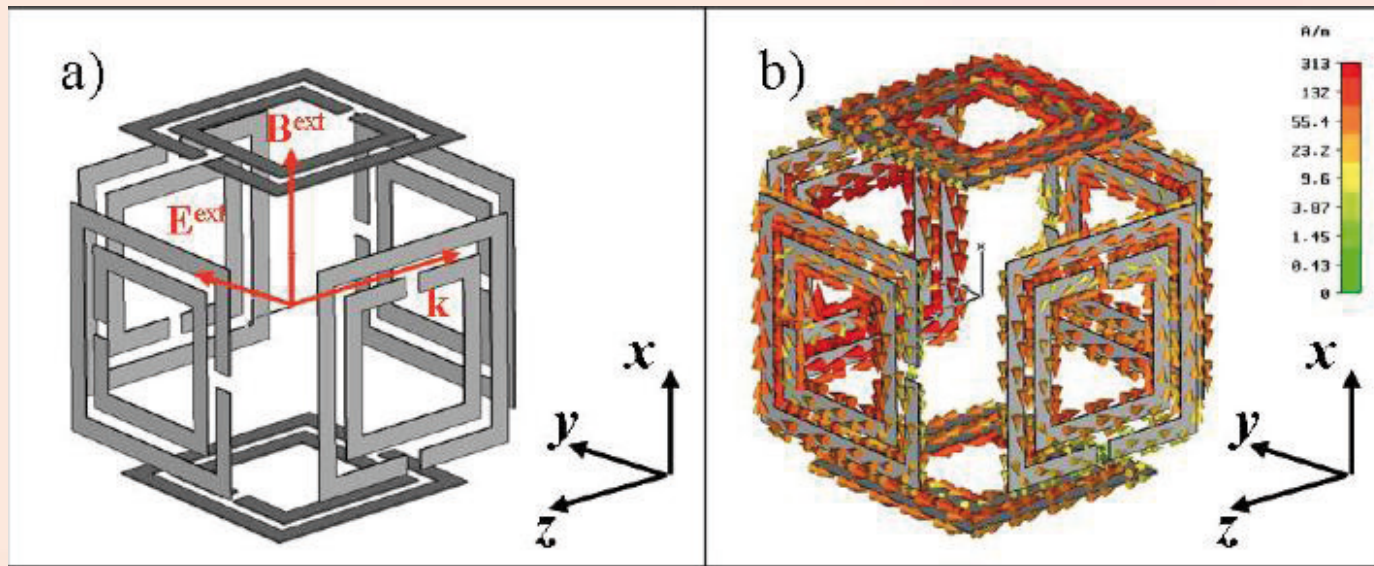


FIG. 6. Photonic band structure of a square array of perfectly conducting cylinders in air, for  $s$  polarization, with  $L = 1$  (---) and  $L = 3, 5$  (—). The ratio of the cylinder radii to the array constant is 0.187 (with the area fraction  $f = 0.11$ ). The inset shows the irreducible octant of the first Brillouin zone.

# Sir John Pendry: Building Blocks (3)

- Negative  $\mu$  material: Pendry proposed the resonant double C structure



Baena et al, Phys Rev B 2007

IEEE TRANSACTIONS ON MICROWAVE THEORY AND TECHNIQUES, VOL. 47, NO. 11, NOVEMBER 1999

Magnetism from Conductors and Enhanced Nonlinear Phenomena

J. B. Pendry, A. J. Holden, D. J. Robbins, and W. J. Stewart, *Member, IEEE*

## So What is a Metamaterial?

- Microstructured material- appears homogeneous at the wavelength of the radiation
- Differs from a photonic crystal in that both electric and magnetic fields are controlled
- Thus, typically will have resonances of both electric field and magnetic field types
- Boundary with photonic crystals not clear cut
- Concept extended to acoustic waves, phonons, elastic waves, plasmons, etc

## What Can Metamaterials Do?

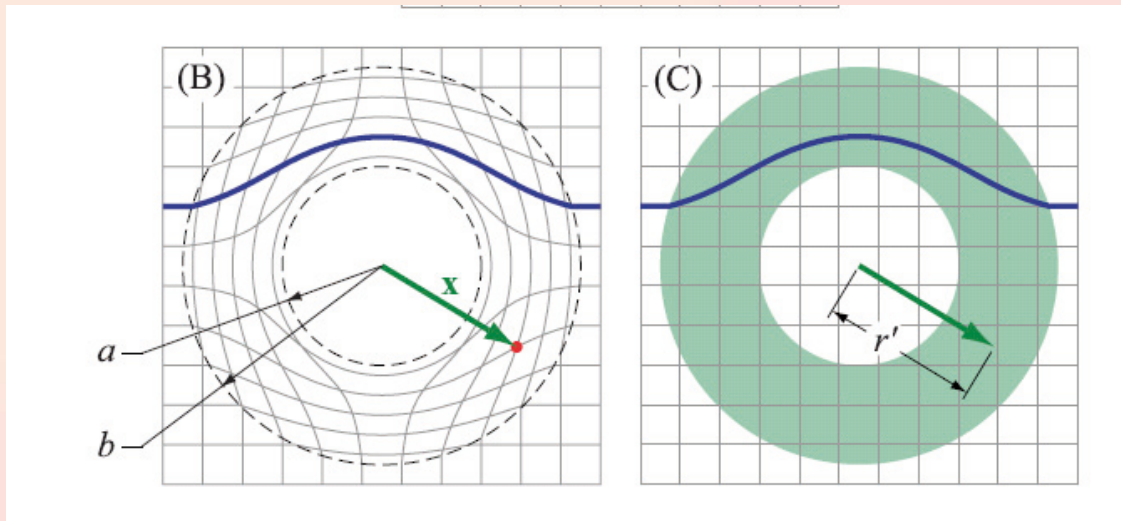
- Make waves behave in unprecedented ways
- Negative refraction, negative Cherenkov effect, negative Goos-Hanchen shift, cloaking, perfect lens, etc, etc
- But there are problems
- Bandwidth, loss, manufacture, conceptual problems, etc
- Exciting field- attracting many investigators from different areas (physics, maths, ee)

## Transform Optics (1)

- Suppose we have a problem in optics with a given spatial region in it
- We want to change the shape of that region
- We can apply a mapping which takes the old shape to the new shape
- Using that mapping in Maxwell's equations, we find we need to change the dielectric constant and the magnetic permeability tensors as a function of position
- Like general relativity- same methods of solution

## Transformation Optics (2)

On the left- transformation of space; on the right- transformation of material properties. Two equivalent viewpoints. From Schurig, Pendry and Smith- Optics Express 2006



# Transformation Optics (3)

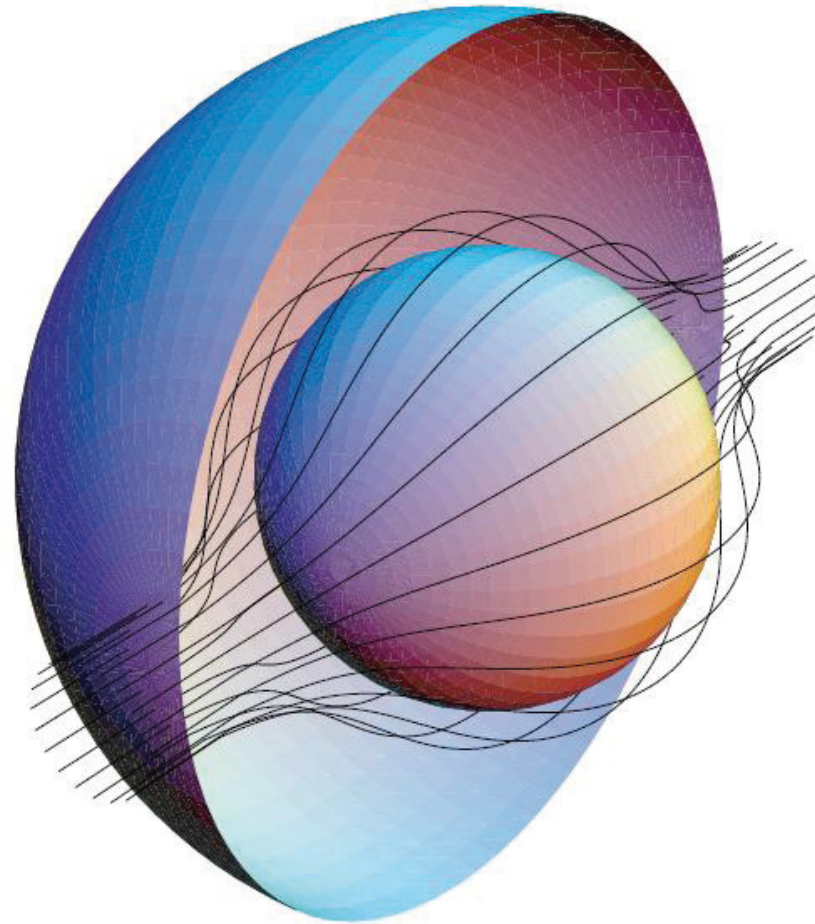
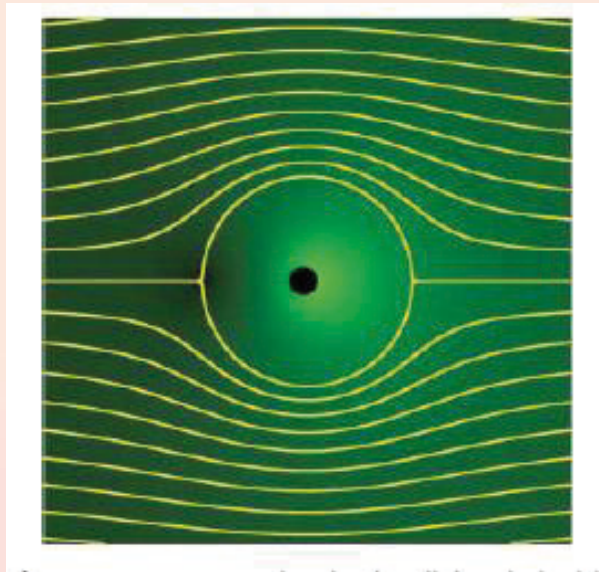


Fig. 2. Rays traversing a spherical cloak. The transformation media that comprises the cloak lies between the two spheres.

## Transformation Optics (4)

- These cloaking transformations make a "hole" in space in which the object to be hidden is placed
- Ulf Leonhardt- Science 2006-



$$n^2 = \frac{r_0}{j\omega_i \omega_1 j} - 1$$

Refractive index distribution- hole at  $w_1$ . Black circle is the cloaked region. Note range of refractive indices required.

## Transformation Optics (5)

- Experimental confirmation: theoretical transformation

$$r^0 = \left(\frac{b_i a}{b}\right)r + a, \quad \theta^0 = \theta, \quad z^0 = z$$

$$\epsilon_r = \mu_r = \frac{r_i a}{r}, \quad \epsilon_\mu = \mu_\mu = \frac{r}{r_i a}$$

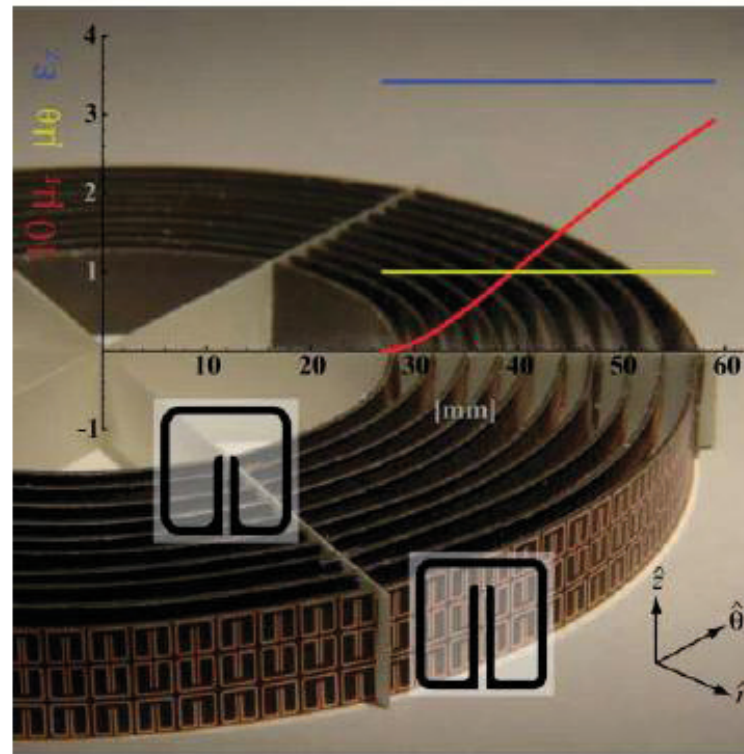
$$\epsilon_z = \mu_z = \left(\frac{b}{b_i a}\right)^2 \left(\frac{r_i a}{r}\right)$$

- Electric field along axis of cylinder- only three components relevant

$$\epsilon_z = \left(\frac{b}{b_i a}\right)^2 \mu_r = \left(\frac{r_i a}{r}\right) \mu_\mu = 1$$

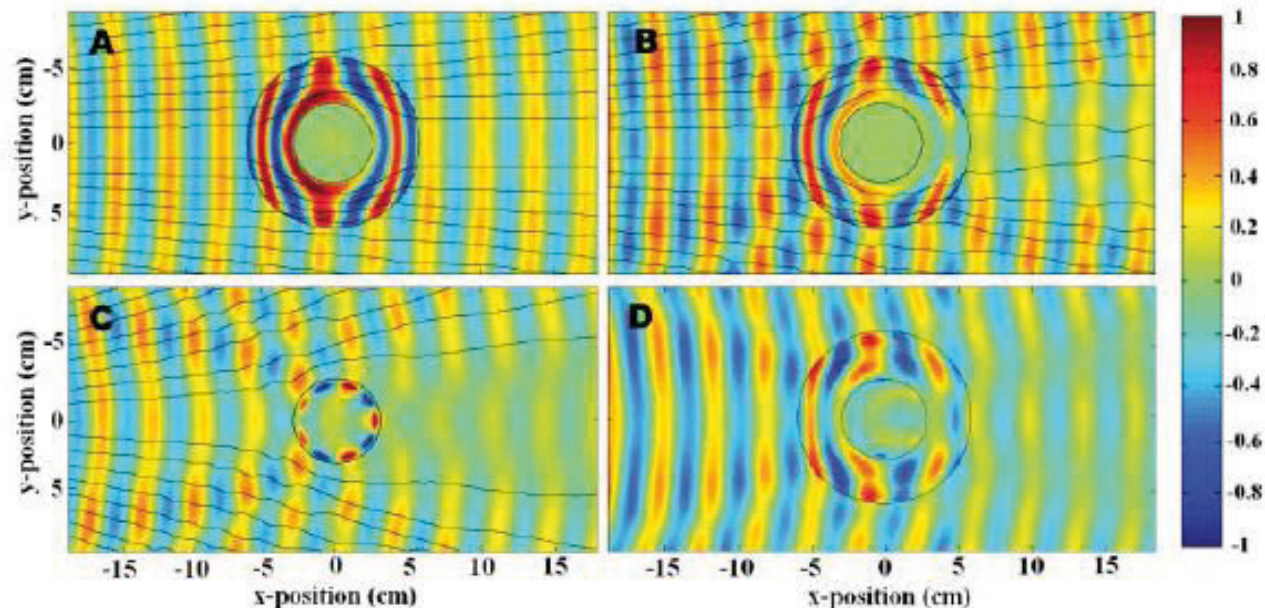
# Transformation Optics (6)

**Fig. 1.** 2D microwave cloaking structure (background image) with a plot of the material parameters that are implemented.  $\mu_r$  (red line) is multiplied by a factor of 10 for clarity.  $\mu_\theta$  (green line) has the constant value 1.  $\epsilon_z$  (blue line) has the constant value 3.423. The SRRs of cylinder 1 (inner) and cylinder 10 (outer) are shown in expanded schematic form (transparent square insets).



Schurig et al, Science, 2006

# Transformation Optics (7)



**Fig. 4.** Snapshots of time-dependent, steady-state electric field patterns, with stream lines [black lines in (A to C)] indicating the direction of power flow (i.e., the Poynting vector). The cloak lies in the annular region between the black circles and surrounds a conducting Cu cylinder at the inner radius. The fields shown are (A) the simulation of the cloak with the exact material properties, (B) the simulation of the cloak with the reduced material properties, (C) the experimental measurement of the bare conducting cylinder, and (D) the experimental measurement of the cloaked conducting cylinder. Animations of the simulations and the measurements (movies S1 to S5) show details of the field propagation characteristics within the cloak that cannot be inferred from these static frames. The right-hand scale indicates the instantaneous value of the field.

# Transformation Optics (8)

- Fancier things:

Rahm et al, Photonics and Nanostructures, 2008

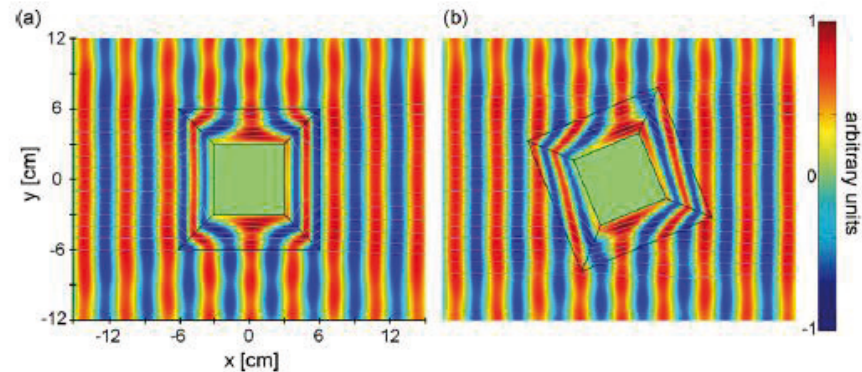
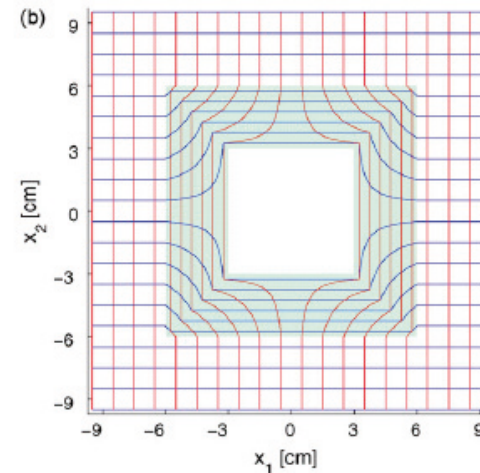


Fig. 6. Electric field distribution in the interior and exterior region of the square cloak. The direction of the power flow is in the positive  $x$ -direction. The wave is smoothly bent around the corners of the square cloak. (a) Phase fronts parallel to one side of the cloak and (b) cloak rotated by  $\pi/8$  with respect to the phase fronts of the incoming wave.

## Transformation Optics (9)

- Wormholes!

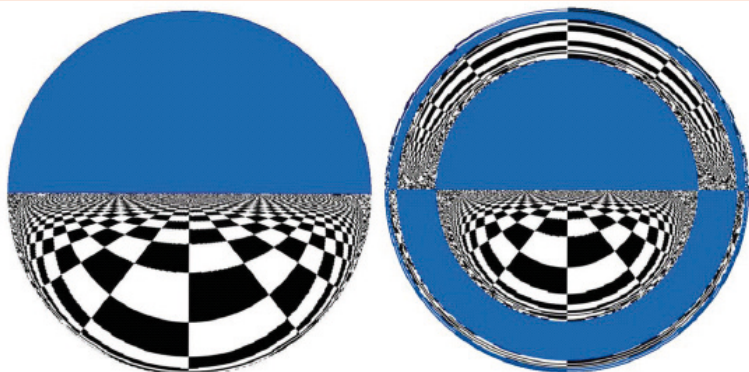


FIG. 2 (color). Ray-tracing simulations of views through the bores of two wormholes. The distant ends are above an infinite chess board under a blue sky. On left,  $L \ll 1$ ; on right,  $L \approx 1$ . Note that the blue is used for clarity; the wormhole construction should be considered essentially monochromatic.

Greenleaf et al, PRL, 2007

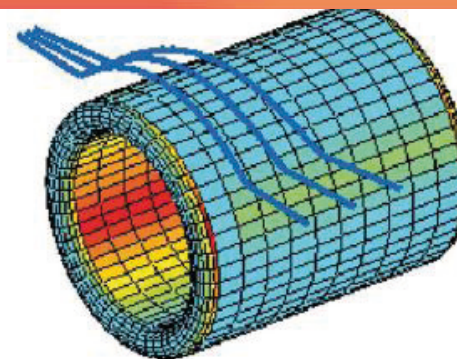


FIG. 3 (color). Obstacle  $K$  and light rays going around the wormhole. The exterior  $N$  of  $K$  consists of two components; the bore  $N_2$  of the wormhole device, and the outside  $N_1$ .

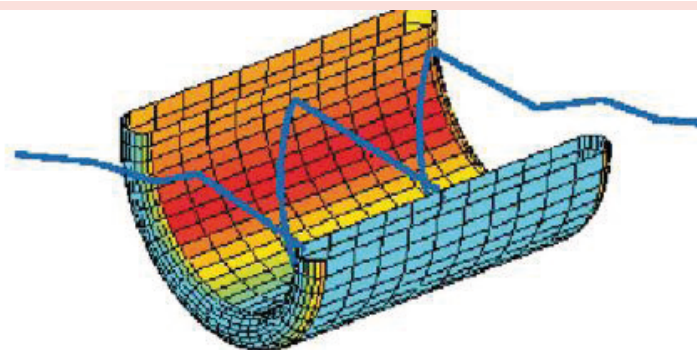


FIG. 4 (color). A typical light ray traversing the bore of the wormhole when  $L \gg 1$ . The upper part of  $K$  is not shown.

# Early Example of Transformation Optics

Appl. Phys. 18, 39—52 (1979)

Applied  
Physics

© by Springer-Verlag 1979

## Crossed Gratings: A Theory and its Applications

G. H. Derrick\*, R. C. McPhedran\*, D. Maystre, and M. Nevière

Faculté des Sciences et Techniques, Laboratoire d'Optique Electromagnétique, Equipe de Recherche Associée au C.N.R.S. No. 597, Centre de St-Jérôme, F-13397 Marseille Cédex 4, France

Received 5 June 1978/Accepted 2 September 1978

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G. H. Derrick et al.

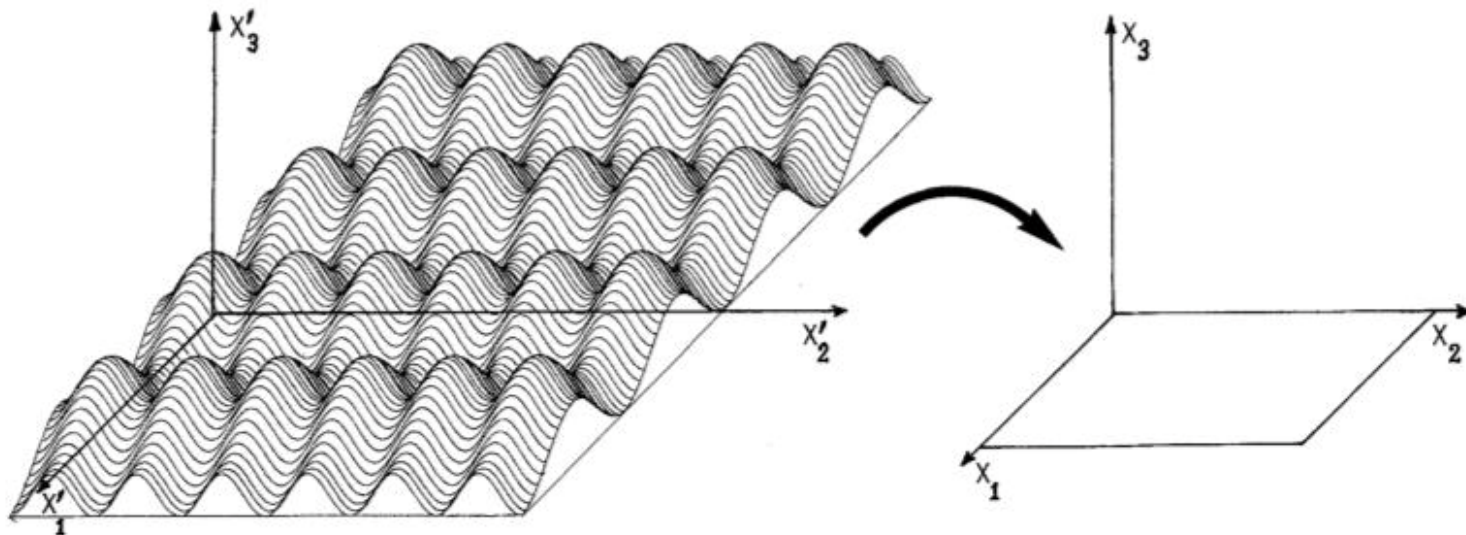


Fig. 1. Schematic representation of the diffraction problem for a crossed grating before and after the coordinate transformation

# Early Example of Transformation Optics (2)

Cartesian system: Maxwell's equations

$$\text{curl}E^0 = jk\eta_0 H^0, \text{curl}H^0 = -\frac{jk n^2}{0} E^0$$

The transformation of coordinates is represented by a metric tensor  $g_{lm}$ .

The transformed Maxwell's equations are

$$\frac{z^{ilm}}{\sqrt{|jg|}} \frac{\partial E_m}{\partial x^l} = jk\eta_0 g^{il} H_l, \frac{z^{ilm}}{\sqrt{|jg|}} \frac{\partial H_m}{\partial x^l} = -\frac{jk n^2}{0} g^{il} E_l.$$

- **Gain:** boundary conditions easier on flat surface.
- **Loss:** medium above surface replaced by inhomogeneous, anisotropic media, which is given by the metric tensor of the coordinate transformation.

## Cloaking and stealth technology

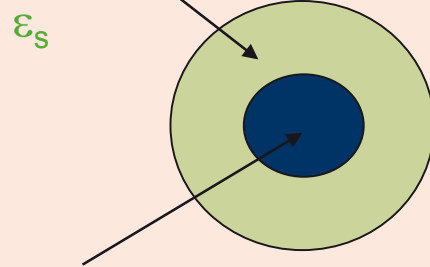
- Cloaking is related to older fields: optical composite materials, artificial dielectric and stealth technology
- Purpose of latter two is narrower: centred around avoidance of radar detection or sonar detection
- Wavelength range much narrower than for cloaking
- Two techniques- shaping to avoid reflection, coatings to absorb probe radiation

## Cloaking and stealth technology (2)

- Many results are classified
- Some of the Russian stealth technology seems flavoured by Veselago; anticipates aspects of metamaterials for cloaking
- Cloaking work aims for broader wavelength range- includes visible and near infrared
- Short wavelengths require much better lithography
- Also material losses in metals more of a problem

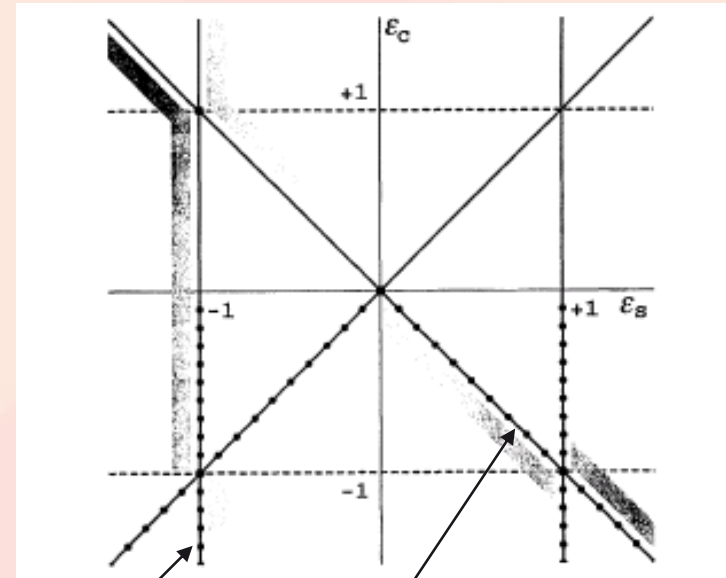
# Cloaking by plasmonic resonance (1)

Shell: radius  $r_s$ ,  
dielectric constant  $\epsilon_s$



Core:  
radius  $r_c$ ,  
dielectric constant  $\epsilon_c$

Matrix:  
dielectric constant  $\epsilon_m$



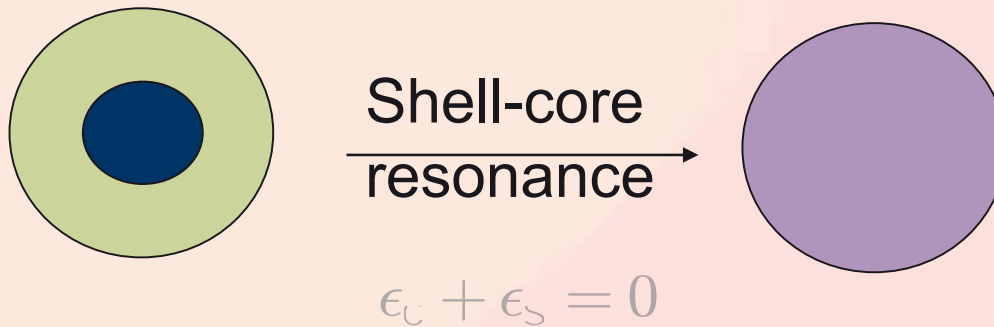
Line of shell-matrix  
resonances:  $\epsilon_s + \epsilon_m = 0$

Line of core-shell  
resonances:  $\epsilon_c + \epsilon_s = 0$

Nicorovici et al, Proc Roy Soc. A, 1993

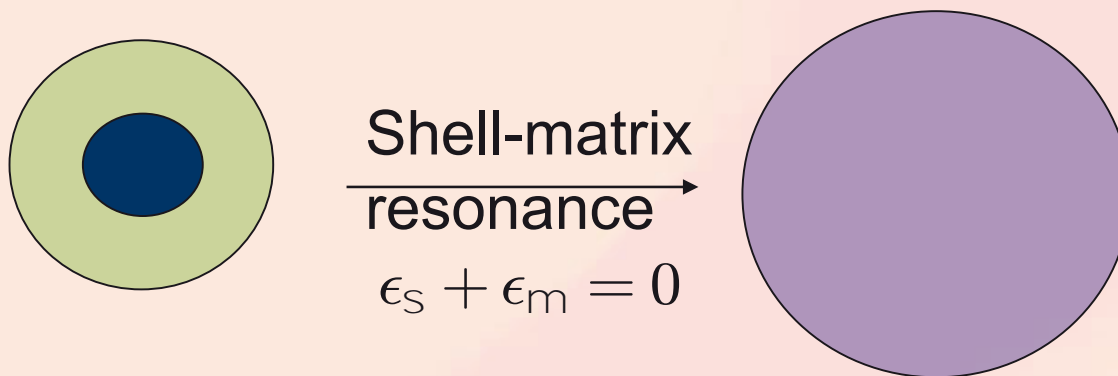
## Shell-Core Resonance

- The shell hides itself (cloaks itself) by making the core behave as if it extended out to  $r_s$ , not  $r_c$



## Shell-Matrix Resonance

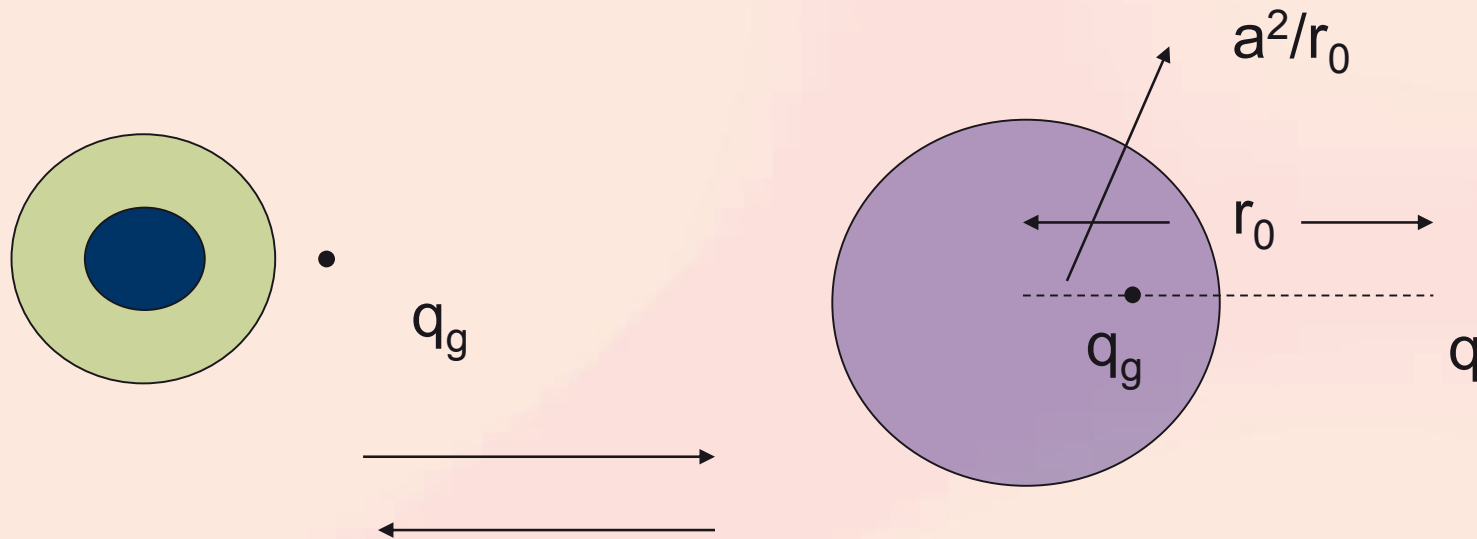
- The shell magnifies the core by making it behave as if it extended out to  $r_s^2/r_c$ , not  $r_c$



Homeopathic behaviour: the smaller the core, the bigger its effect after magnification by the shell! Limited of course by the requirement that  $r_s^2/r_c$  not correspond to intersecting cylinders.

## Ghost Sources

- Interaction of a charge at  $r_0$  with the coated and enlarged cylinders



The ghost source is in the matrix medium if

$$r_0 > a, \frac{a^2}{r_0} > r_s \quad \text{or} \quad a < r_0 < \frac{a^2}{r_s}$$

Pendry perfect image!

## Cloaking by plasmonic resonance (2)

- This resonance mechanism can be used for long-wavelength (quasistatic) cloaking
- The core and matrix dielectric constants must be the same (if they are not, the cloaking system may be visible even if it cloaks: the ostrich effect)
- The coated cylinder can then cancel out the local field acting on an external polarizable particle (if it is in the cloaking region)
- Result: the particle is hidden from external electric fields

# Cloaking by plasmonic resonance (3)

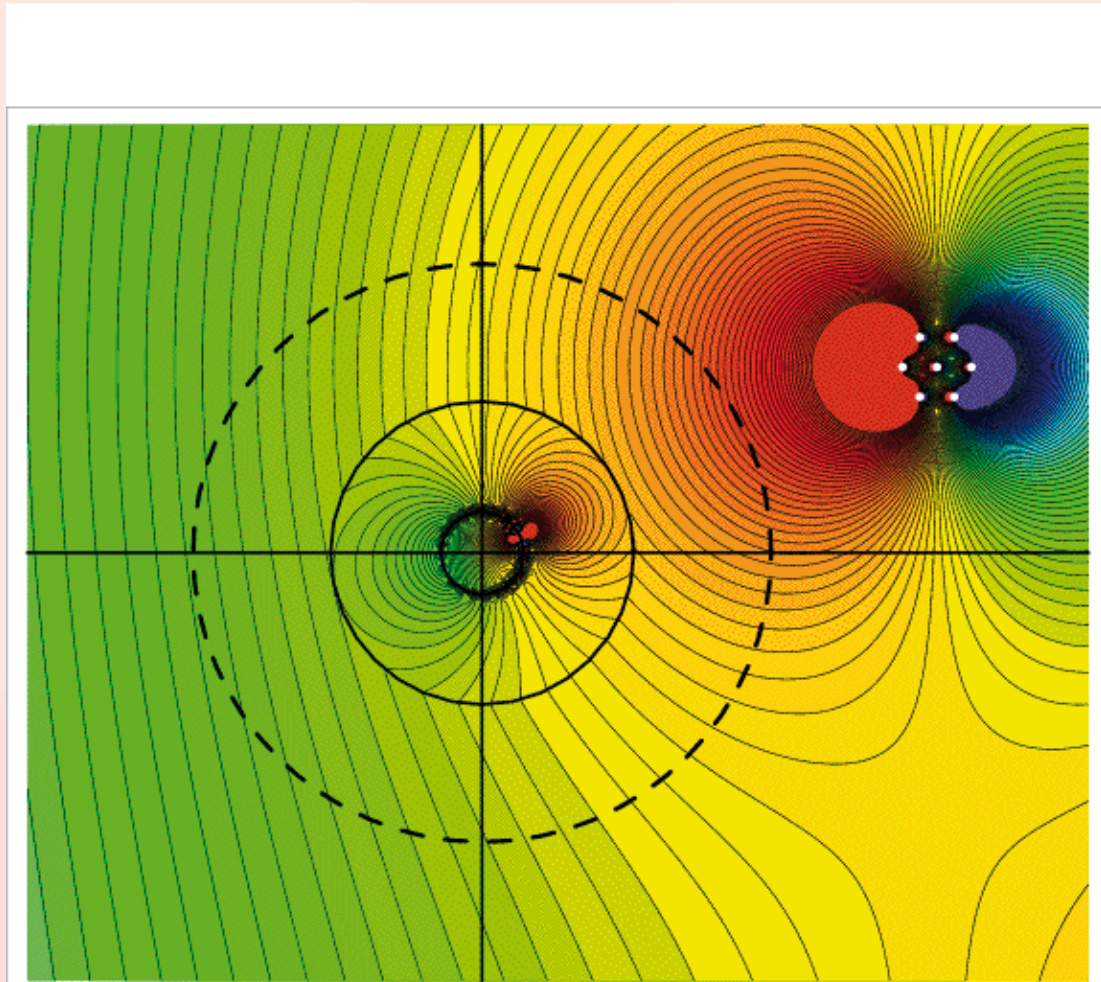
Optics

Express: 15,  
3614 (2007)

$$\epsilon_s = -1 + i \cdot 10^{-12}$$

Cloaking  
radius:

$$R_{\#} = \sqrt{\frac{r_c}{\epsilon_s}}$$



## Cloaking by plasmonic resonance (4)

- This sort of resonant cloaking has been studied up till now in the long wavelength limit, in 2D
- Currently extending it to shorter wavelengths
- Also studying interaction effects among systems of cloaking particles
- Has to be extended to spheres
- Problem is of course getting  $\epsilon_s = -1$

