Mixed Phase Space II and Tunneling

Roland Ketzmerick

Technische Universität Dresden, Germany

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FOR760

classical

quantum

$$H\psi_n = E_n\psi_n$$

integrable

fully chaotic

mixed phase space

coexistence of
regular and chaotic
dynamics

Mushroom Billiard

Bunimovich, Chaos 2001

Altmann, Motter, Kantz,
Chaos 2005

Dietz et al., PRE 2007

Vidmar et al., JPA 2007

Barnett, Betcke, Chaos 2007

Mushroom Billiard □ Classical Dynamics

regular

chaotic

Mushroom Billiard □ Quantum Eigenstates

regular eigenstate
lives in cap

chaotic eigenstate
lives in stem and cap

Mushroom Billiard □ Phase Space

Mushroom Billiard □ Phase Space

classical

quantum

sharply divided
regular and chaotic regions

Mushroom Billiard □ Quantum Eigenstates

regular eigenstate
lives in cap

chaotic eigenstate
lives in stem and cap

Nano wire with one-sided disorder

Garcia-Martin, Governale, Wölfle, PRB 2002

Prusty, Schanz, PRL 2006

Feist, Bäcker, R.K., Rotter, Huckestein, Burgdörfer, PRL 2006, PRB 2009

Is there a drift to the chaotic motion?

- A) No
- B) To the right
- C) To the left

Is there Anderson localization?

- A) No
- B) Yes

What determines the localization length?

kicked rotor □ standard map

rotor

change of angle

$$\varphi(t) = \varphi(0) + p t$$

periodic kicks

$$t = 1, 2, 3, \dots$$

change of momentum

$$K \sin \varphi$$

standard map

Chirikov

$$\varphi_{n+1} = \varphi_n + p_n$$

$$p_{n+1} = p_n + K \sin \varphi_{n+1}$$

integrable

mixed phase space

fully chaotic

coexistence of
regular and chaotic
dynamics

many new quantum phenomena

today: **Tunneling**

1D barrier tunneling

$$\text{WKB: } \gamma \propto \exp\left(\frac{i}{\hbar} \int_a^b p dq\right)$$

$$p = \sqrt{2m(E - V(q))}$$

Dynamical tunneling

(tunneling across dynamical barriers)

Davis, Heller 1981

Qualitative estimate: $\gamma \sim \exp\left(-C \frac{A}{\hbar}\right)$

Hanson, Ott, Antonsen 1984

Tunneling rates from regular island

- Time evolution:

$$\oint p dq = (m + \frac{1}{2})h$$

- Eigenvalues of time evolution operator:

$$U |\psi_m\rangle = e^{i(\alpha + i\gamma_m/2)} |\psi_m\rangle$$

Does not depend on details of absorbing region!

1D barrier tunneling

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$$p = \sqrt{2m(E - V(q))}$$

Dynamical tunneling

(tunneling across dynamical barriers)

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Qualitative estimate: $\gamma \sim \exp\left(-C \frac{A}{\hbar}\right)$
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Quantitative theory?

- Which properties of **regular island**?
- Which properties of **chaotic dynamics**?
- Dependence on quantum number **m**?

Direct Regular-to-Chaotic Tunneling

Hanson, Ott, Antonson, PRA 1984

Bohigas, Tomsovic, Ullmo, Phys. Rep. 1993

Podolskiy, Narimanov, PRL 2003

Sheinman, Fishman, Guarneri, Rebuzzini, PRA 2006

Approach: **Fictitious Integrable System**

$$\gamma = \left\| \underbrace{P_{\text{ch}} H}_{\text{projector on chaotic part}} |\psi_{\text{reg}}\rangle \right\|^2$$

projector
on chaotic part

$\Rightarrow \gamma$ independent of chaotic dynamics

procedure:

1. determine $H_{\text{reg}}, |\psi_{\text{reg}}\rangle$

2. determine P_{ch}

Bäcker, R.K., Löck, Schilling, PRL 2008

Designed Quantum Map

- elliptic island
- constant rotation number
- narrow hierarchical region

$$\gamma_m = c \frac{h_{\text{eff}}}{\sqrt{1-\alpha_m}} \exp \left(-2 \frac{A}{h_{\text{eff}}} \left[\sqrt{1-\alpha_m} - \alpha_m \ln \left(\frac{1+\sqrt{1-\alpha_m}}{\sqrt{\alpha_m}} \right) \right] \right)$$

m : quantum number

A : area of island

$c \approx 1$

$$\alpha_m = \left(m + \frac{1}{2} \right) \left(\frac{A}{h_{\text{eff}}} \right)^{-1}$$

Example 2

- deformed island
- narrow hierarchical region

Example 3: Standard Map

$$q' = q + p$$

$$p' = p + K \sin q'$$

$$K = 2.9$$

Mushroom Billiard - Prediction

H_{reg} = quarter circle

$$\psi_{\text{reg}}^{mn}(r, \varphi) \propto J_m(j_{mn}r) \sin m\varphi$$

ψ_{ch} random wave model
(with boundary condition)

$$\Rightarrow P_{\text{ch}}$$

$$\Rightarrow \gamma_{mn} = \frac{8}{\pi} \sum_{\substack{s=1 \\ s \neq 3, 6, \dots}}^{\infty} \frac{J_{m+2s/3}(j_{mn}a)^2}{J_{m+1}(j_{mn})^2}$$

Bäcker, R.K., Löck (Dresden)

Robnik, Vidmar (Maribor)

Höhmnn, Kuhl, Stöckmann (Marburg), PRL 2008

Mushroom Billiard $\square$ Microwave Experiment

Mushroom Billiard - Numerics

- agreement over 18 orders of magnitude
- no fit parameter!

Optical Microcavities

Bäcker, R.K., Löck, Wiersig (Magdeburg), Hentschel (MPIPKS Dresden), PRA 2009

inverse
quality factor

$$Q_{\text{dyn}}^{mn} = \frac{4n_{\text{eff}}^2}{N_{mn}^2(1 - n_{\text{eff}}^2)^2 |k_{mn}^2| \sum_{l=1}^{\infty} |I_l|^2} \quad I_l = \int_0^{R_2} \int_0^{\pi} \rho \, d\rho \, d\vartheta \, \psi_l^*(\rho, \vartheta) J_m(n_{\text{eff}} k_{mn} r) \cos(m\varphi)$$

Regular-to-Chaotic Tunneling Mechanisms

$h \lesssim A$

direct regular-to-chaotic

Bäcker, R.K., Löck, Schilling, PRL 2008

$h \ll A$

resonance assisted

Brodier, Schlagheck, Ullmo, PRL 2001, Ann. Phys. 2002

Eltchka, Schlagheck, PRL 2005

Sheinman, Fishman, Guarneri, Rebuzzini, PRA 2006

combined theory

combined theory

Löck, Bäcker, R.K., Schlagheck, PRL 2010

Standard Map $K=3.5$

Prediction: $\gamma_m = \gamma_m^d + \sum_n \left| \sum_{\{a_i\}} \prod_{a_i} \frac{V_{r_i:s_i}}{E_m - E_{m'}} \right|^2 \gamma_n^d$ with $n = m + \sum_i a_i r_i$, $a_i \in \mathbb{N}$

Nano wire with one-sided disorder

Feist, Bäcker, R.K., Rotter, Huckestein, Burgdörfer, PRL 2006, PRB 2009

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localization length

----- theory, tunneling

- two-sided disorder
- no magnetic field

number of open modes, semiclassical limit

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localization length

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