Matter waves II

Mesoscopic transport of coherent bosonic matter waves

Peter Schlagheck
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Outline

- Bose-Einstein condensation
- guided atom lasers
- quasi-stationary (?) transport of interacting bosonic matter waves: mean-field description and beyond
- breakdown of Anderson localization in 1D disorder potentials
- coherent backscattering in 2D disorder potentials
- summary and FAQs

Bose-Einstein condensation

Noninteracting atoms in harmonic traps: $T > T_c$

Bose-Einstein condensation

Noninteracting atoms in harmonic traps: $T \lesssim T_c$

Bose-Einstein condensation

Noninteracting atoms in harmonic traps: $T \ll T_c$

→ ground state population: $N_0/N = (1 - T/T_c)^3$

Bose-Einstein condensation

Noninteracting atoms in harmonic traps: $T = 0$

→ ground state population: $N_0/N = (1 - T/T_c)^3$

Atom-atom interaction

→ short-range interaction potential $U(\mathbf{r}_1 - \mathbf{r}_2)$

with spatial extent $\sim \text{nm} \ll \text{mean inter-atomic distance}$

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→ replace it by a contact interaction (pseudo-)potential

$$U(\mathbf{r}_1 - \mathbf{r}_2) = U_0 \delta(\mathbf{r}_1 - \mathbf{r}_2)$$

that reproduces the asymptotic scattering properties of two atoms:

$$U_0 = \frac{4\pi\hbar^2 a_s}{m}$$

with a_s the s -wave scattering length of the atomic species

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$$(U\psi)|_{\mathbf{r}_1, \mathbf{r}_2} \equiv U_0 \delta(\mathbf{r}_1 - \mathbf{r}_2) \left. \frac{\partial}{\partial r} [r\psi] \right|_{r=|\mathbf{r}_1 - \mathbf{r}_2|}$$

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→ stable condensate for $a_s > 0$ (effective repulsion)

BEC with atom-atom interaction

BEC with atom-atom interaction

→ modified condensate wavefunction ψ and chemical potential μ

$$\left(-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial \mathbf{r}^2} + V_{\text{trap}}(\mathbf{r}) + U_0 |\psi(\mathbf{r})|^2 \right) \psi(\mathbf{r}) = \mu \psi(\mathbf{r})$$

→ Gross-Pitaevskii equation

BEC with atom-atom interaction

→ not all atoms share the same wavefunction:
finite “depletion” at zero temperature

$$\delta N \equiv N_{\text{noncondensed}} \equiv N - N_0 \sim (Na_s/a_{\text{osc}})^{6/5} \quad \text{for small } a_s$$

J. Javanainen, PRA 54, R3722 (1996)

BEC with atom-atom interaction

Mean-field limit: $N \rightarrow \infty$, $a_s \rightarrow 0$, such that $Na_s \sim \text{const}$

→ $\delta N \sim (Na_s)^{6/5} \sim \text{const} \sim \mathcal{O}(N^0)$

→ Gross-Pitaevskii description becomes perfect in that limit

E. H. Lieb and R. Seiringer and J. Yngvason, PRA 61, 043602 (2000)

BEC with atom-atom interaction

- spontaneous breaking of $U(1)$ gauge symmetry for $N \rightarrow \infty$
- atomic field operator $\hat{\psi}(\mathbf{r})$ can be replaced by a c-number
 $\psi(\mathbf{r}) \equiv \langle \hat{\psi}(\mathbf{r}) \rangle$ for the condensate

E. H. Lieb and R. Seiringer and J. Yngvason, Rep. Math. Phys. 59, 389 (2007)

BEC with interaction at finite temperature

→ thermal population of Bogoliubov quasiparticle excitations

$$\hat{H}_{\text{QP}} = \sum_{j=1}^{\infty} \hbar \omega_j \hat{\alpha}_j^{\dagger} \hat{\alpha}_j$$

(quantization of collective modes within the condensate)

Atom lasers

Atom lasers

Level scheme of ^{87}Rb :

Atom lasers

Level scheme of ^{87}Rb :

Atom lasers

Level scheme of ^{87}Rb :

Atom lasers

Apply a radiofrequency field

Atom lasers

Apply a radiofrequency field

Atom lasers with gravity

Atom lasers with gravity

Atom lasers with gravity

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Atom Laser with a cw Output Coupler

Immanuel Bloch, Theodor W. Hänsch, and Tilman Esslinger

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(Received 3 December 1998)

Guided atom laser

Idea: add a horizontal optical waveguide (focused laser beam)

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repulsive interaction between $m_F = -1$ and $m_F = 0$ atoms

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→ study quasistationary scattering at well-defined energy

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—> experimental realization: W. Guerin *et al.*, PRL 97, 200402 (2006)

Guided atom laser

Idea: add a horizontal optical waveguide (focused laser beam)

—> experimental realization: W. Guerin *et al.*, PRL 97, 200402 (2006)

—> all-optical realization of a guided atom laser:

A. Couvert *et al.*, EPL 83, 50001 (2008)

Transport theory of Bose-Einstein condensates

Field equation for atoms in the waveguide:

$$i\hbar \frac{\partial}{\partial t} \hat{\Psi}(\mathbf{r}, t) = \left(-\frac{\hbar^2}{2m} \Delta + V_{\text{guide}}(\mathbf{r}) + U \hat{\Psi}^\dagger(\mathbf{r}, t) \hat{\Psi}(\mathbf{r}, t) \right) \hat{\Psi}(\mathbf{r}, t) \\ + \tilde{U} \hat{\Psi}_{\text{res}}^\dagger(\mathbf{r}, t) \hat{\Psi}_{\text{res}}(\mathbf{r}, t) \hat{\Psi}(\mathbf{r}, t) + K(t) \hat{\Psi}_{\text{res}}(\mathbf{r}, t)$$

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Perfect condensation in the reservoir: $\hat{\Psi}_{\text{res}}(\mathbf{r}, t) = \Psi_{\text{res}}(\mathbf{r}) e^{-i\mu t/\hbar}$

(mean-field limit: $N \rightarrow \infty$)

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→ inhomogeneous field equation with a **coherent source**

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Restriction to the transverse ground mode of the waveguide:

$$\hat{\Psi}(\mathbf{r}, t) = \chi_0(\mathbf{r}_\perp) \exp(-i\omega_\perp t) \hat{\psi}(x, t)$$

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with $g = 2\hbar\omega_\perp a_s$: effective 1D interaction strength

Transport theory of Bose-Einstein condensates

Field equation for atoms in the waveguide:

$$i\hbar \frac{\partial}{\partial t} \hat{\psi}(x, t) = \left(-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + V(x) + g \hat{\psi}^\dagger(x, t) \hat{\psi}(x, t) \right) \hat{\psi}(x, t) \\ + \underbrace{\int d^2 r_\perp \chi_0^*(\mathbf{r}_\perp) K(t) \Psi_{\text{res}}(\mathbf{r}) \exp(-i\mu t/\hbar)}_{S(x, t) : \text{coherent source}}$$

with $g = \frac{2\hbar\omega_\perp a_s}{1 - 1.46 a_s / a_\perp^{(\text{osc})}}$ M. Olshanii, PRL 81, 938 (1998)

Transport theory of Bose-Einstein condensates

Field equation for atoms in the waveguide:

$$i\hbar \frac{\partial}{\partial t} \hat{\psi}(x, t) = \left(-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + V(x) + g \hat{\psi}^\dagger(x, t) \hat{\psi}(x, t) \right) \hat{\psi}(x, t) + S(x, t) \exp(-i\mu t/\hbar)$$

Mean-field approximation:

$$\hat{\psi}(x, t) = \langle \hat{\psi}(x, t) \rangle + \delta \hat{\psi}(x, t) \equiv \psi(x, t)$$

Transport theory of Bose-Einstein condensates

Field equation for atoms in the waveguide:

$$i\hbar \frac{\partial}{\partial t} \psi(x, t) = \left(-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + V(x) + g|\psi(x, t)|^2 \right) \psi(x, t) + S(x, t) \exp(-i\mu t/\hbar)$$

→ inhomogeneous 1D Gross-Pitaevskii equation

Scattering through double barrier potentials

Realization with focused laser beams:

Scattering through double barrier potentials

Scattering through double barrier potentials

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$$\text{Transmission} = \frac{\text{total atomic current in the quasi-stationary regime}}{\text{incident current of the beam } (\rightarrow \text{source amplitude})}$$

Transmission of the condensate

without interaction:

→ resonant transmission through quasi-bound eigenstates

Transmission of the condensate

with interaction, at fixed incident current (10^4 atoms/s)

Transmission of the condensate

with interaction, at fixed incident current (10^4 atoms/s)

→ resonant transmission suppressed due to the interaction:
nonlinear bistability phenomenon

T. Paul, K. Richter, and P.S., PRL 94, 020404 (2005)

Transmission through 2D confinement structures

Example: circular billiard

T. Hartmann *et al.*, in preparation

Transport through longer scattering potentials

Negligible interaction between the atoms:

Transport through longer scattering potentials

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Negligible interaction between the atoms:

→ stationary scattering state at the end of the propagation:

$$|\psi(x)|^2 \propto 1/\sqrt{2m[\mu - V(x)]}$$

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Finite interaction between the atoms: $g|\psi|^2 \simeq 0.1\mu$

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Finite interaction between the atoms: $g|\psi|^2 \simeq 0.1\mu$

→ permanently time-dependent scattering

B. Spivak and A. Zyuzin, PRL 84, 1970 (2000)

S. E. Skipetrov and R. Maynard, PRL 85, 736 (2000)

Transport through longer scattering potentials

Finite interaction between the atoms: $g|\psi|^2 \simeq 0.1\mu$

→ validity of the mean-field theory ??

Beyond the Gross-Pitaevskii equation

$$i\hbar \frac{\partial}{\partial t} \hat{\psi}(x, t) = \left(-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + V(x) + g \hat{\psi}^\dagger(x, t) \hat{\psi}(x, t) \right) \hat{\psi}(x, t) \\ + S(x, t) \exp(-i\mu t/\hbar)$$

Beyond the Gross-Pitaevskii equation

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Bogoliubov ansatz: $\hat{\psi}(x, t) = \langle \hat{\psi}(x, t) \rangle + \delta \hat{\psi}(x, t)$

→ derive coupled equations for the expectation values

$$\langle \hat{\psi}(x, t) \rangle, \langle \delta \hat{\psi}(x_1, t) \delta \hat{\psi}(x_2, t) \rangle, \langle \delta \hat{\psi}^\dagger(x_1, t) \delta \hat{\psi}(x_2, t) \rangle, \\ \langle \delta \hat{\psi}(x_1, t) \delta \hat{\psi}(x_2, t) \delta \hat{\psi}(x_3, t) \rangle, \dots$$

→ Hartree-Fock-Bogoliubov theory

Beyond the Gross-Pitaevskii equation

$$i\hbar \frac{\partial}{\partial t} \hat{\psi}(x, t) = \left(-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + V(x) + g \hat{\psi}^\dagger(x, t) \hat{\psi}(x, t) \right) \hat{\psi}(x, t) + S(x, t) \exp(-i\mu t/\hbar)$$

Mean-field regime:

- large density $n \propto |S|^2$ of atoms emitted by the source
- small 1D interaction strength $g \propto a_s$
- finite mean-field interaction term $\propto n a_s$

$\Rightarrow$ well-defined truncation scheme in the formal limit

$n \rightarrow \infty, a_s \rightarrow 0, n a_s \sim \text{const:}$

take into account terms of order $n^{1/2}, n^0, n^{-1/2},$ and n^{-1}

Beyond the Gross-Pitaevskii equation

$$\Psi(x, t) \equiv \langle \hat{\psi}(x, t) \rangle, \quad \Gamma(x, y, t) \equiv \langle \delta \hat{\psi}^\dagger(y, t) \delta \hat{\psi}(x, t) \rangle, \quad \Phi(x, y, t) \equiv \langle \delta \hat{\psi}(y, t) \delta \hat{\psi}(x, t) \rangle$$

$$i\hbar \frac{\partial}{\partial t} \Psi(x, t) = H_0^{(x)} \Psi(x, t) + g |\Psi(x, t)|^2 \Psi(x, t) + S(x, t) \exp(-i\mu t/\hbar) \quad O(n^{1/2})$$

$$+ 2g \Psi(x, t) \Gamma(x, x, t) + g \Psi^\dagger(x, t) \Phi(x, x, t) \quad O(n^{-1/2})$$

$$i\hbar \frac{\partial}{\partial t} \Phi(x, y, t) = \left[H_0^{(x)} + 2g |\Psi(x, t)|^2 + H_0^{(y)} + 2g |\Psi(y, t)|^2 \right] \Phi(x, y, t)$$

$$+ g \delta(x - y) \Psi(x, t) \Psi(y, t)$$

$$+ g \Psi^2(y, t) \Gamma(x, y, t) + g \Psi^2(x, t) \Gamma(y, x, t) \quad O(n^0)$$

$$+ g \delta(x - y) \Phi(x, y, t) + 2g [\Gamma(x, x, t) + \Gamma(y, y, t)] \Phi(x, y, t)$$

$$+ g \Gamma(y, x, t) \Phi(x, x, t) + g \Gamma(x, y, t) \Phi(y, y, t) \quad O(n^{-1})$$

$$i\hbar \frac{\partial}{\partial t} \Gamma(x, y, t) = \left[H_0^{(x)} + 2g |\Psi(x, t)|^2 - H_0^{(y)} - 2g |\Psi(y, t)|^2 \right] \Gamma(x, y, t)$$

$$+ g \Psi^2(x, t) \Phi^*(x, y, t) - g (\Psi^*(y, t))^2 \Phi(x, y, t) \quad O(n^0)$$

$$+ 2g [\Gamma(x, x, t) - \Gamma^*(y, y, t)] \Gamma(x, y, t)$$

$$+ g \Phi^*(x, y, t) \Phi(x, x, t) - g \Phi(x, y, t) \Phi^*(y, y, t) \quad O(n^{-1})$$

Depletion in extended potentials

Quasi-stationary scattering on the Gross-Pitaevskii level:

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Quasi-stationary scattering on the Gross-Pitaevskii level:

→ low depletion, coherent (mean-field-like) transport

Depletion in extended potentials

Time-dependent scattering on the Gross-Pitaevskii level:

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Time-dependent scattering on the Gross-Pitaevskii level:

→ strong depletion, breakdown of Gross-Pitaevskii description

Depletion in extended potentials

T. Ernst, T. Paul, and P.S., PRA 81, 0136431 (2010)

Depletion in extended potentials

T. Ernst, T. Paul, and P.S., PRA 81, 0136431 (2010)

Alternative: truncated Wigner approach

R. G. Scott and D. A. W. Hutchinson, PRA 78, 063614 (2008)

Transport through 1D disorder potentials

Consider spatially correlated disorder potentials with

- Gaussian correlation

- Lorentzian correlation

→ disorder on atom chips

- Bessel-type correlation

→ optical speckle fields

Transport through 1D disorder potentials

Negligible interaction between the atoms: Anderson localization

- exponential decrease of the average transmission with the length L of the disorder region
- lognormal-type probability distribution for the transmission T at fixed length L :

$$P(\ln T) = \sqrt{\frac{L_{\text{loc}}}{4\pi L}} \exp \left[-\frac{L_{\text{loc}}}{4L} \left(\frac{L}{L_{\text{loc}}} + \ln T \right)^2 \right]$$

Dorokhov (1982); Mello, Pereyra, Kumar (1988); Pichard (1991)

Transport through 1D disorder potentials

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- same expression $L_{\text{loc}} \equiv L_{\text{loc}}(\kappa)$ as for noninteracting atoms, with a “renormalized” wavenumber $\kappa = \sqrt{k^2 - mg|\psi|^2/\hbar^2}$

T. Paul *et al.*, PRL 98, 210602 (2007); PRA 80, 033615 (2009)

Lognormal distribution for speckle disorder

Lognormal distribution for speckle disorder

Transport through 1D disorder potentials

Finite interaction between the atoms:

- permanently time-dependent scattering, except for very short disorder samples
- compute time-averaged transmission $\bar{T} = \frac{1}{\Delta t} \int_{t_0}^{t_0 + \Delta t} T(t) dt$
- algebraic (Ohm-like) decrease of the average transmission:

$$\bar{T} \simeq \frac{1}{1 + L/L_0}$$

→ signature for incoherent transport

T. Paul, P. Leboeuf, N. Pavloff, K. Richter, and P.S., PRA 72, 063621 (2005)

Two-dimensional disorder

→ measure angle-resolved flux of backscattered atoms

Weak localization in two-dimensional disorder

- Constructive interference between reflected paths and their time-reversed counterparts

- enhanced coherent backscattering of laser light from disordered media

M. P. Van Albada and A. Lagendijk, PRL 55, 2692 (1985)

P.-E. Wolf and G. Maret, PRL 55, 2696 (1985)

Weak localization in two-dimensional disorder

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Transport of condensates through 2D disorder

Stationary scattering state of the condensate

Coherent backscattering of the condensate

Coherent backscattering of the condensate

Coherent backscattering of the condensate

→ inverted cone in presence of finite interaction:
crossover from constructive to destructive interference

M. Hartung, T. Wellens, C. A. Müller, K. Richter, P.S., PRL 101, 020603 (2008)

Summary

- guided atom lasers for studying quasi-stationary transport of coherent bosonic matter waves
- extended scattering potentials and interaction may lead to a loss of coherence
 - breakdown of Anderson localization
- interaction-induced inversion of coherent backscattering

Healing length

Q: The healing length is much larger than any other relevant length scale. Doesn't this invalidate the applicability of the Gross-Pitaevskii equation?

Healing length

Q: The healing length is much larger than any other relevant length scale. Doesn't this invalidate the applicability of the Gross-Pitaevskii equation?

A: In a closed 1D system yes. But we have here a scattering process in an open system with a finite extent of the interaction region. The large healing length should therefore not be a relevant length scale.

“Roton”-like excitations in 1D

Q: What about excitations along the second Lieb-Liniger branch?

$$k_F = \pi|\psi|^2, \epsilon_F = \hbar^2 k_F^2 / (2m)$$

“Roton”-like excitations in 1D

Q: What about excitations along the second Lieb-Liniger branch?

A: Such excitations cannot be properly accounted for by HFB studies. In closed systems, however, they do not seem to play an important role in the structure factor.

A. Y. Cherny and J. Brand, PRA 79, 043607 (2009)

J.-S. Caux and P. Calabrese, PRA 74, 031605 (2006)

Infinitely long disorder regions

Q: What happens for infinitely long 1D disorder regions? Is it possible to have stationary transport (with 100% reflection) for finite interaction strengths $g > 0$?

Infinitely long disorder regions

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A: We did not attempt to explore this issue with our means. More careful studies [M. Johansson *et al.*, EPL 86 10009](#) indicate that the answer is yes.