Transport properties of coherent matter waves : superfluidity & Anderson localization

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In collaboration with:

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Light/matter interaction: mechanical effects

Fundamental process: momentum transfer between photons and atoms

Quasi-resonant interaction:

- absorption (+ spontaneous emission)
- dissipative force (e.g. $\vec{F} = -\gamma \vec{v}$)

Laser cooling
(MOT, optical molasses, ...)

Non-resonant interaction:

- dispersive interaction
- conservative force $\vec{F} = -\nabla U_{dip}$

trapping
(optical traps, optical lattices, ...)

Optical trapping

Far off resonance light induces
an electric dipole

$$\vec{p} = \alpha \vec{E}$$

The atomic induced electric
dipole then interacts with the e.m.
wave

$$U(\vec{r}) = -\vec{p} \cdot \vec{E}(\vec{r})$$

Optical trapping

Red detuning

Blue detuning

Transport of a Bose Einstein Condensate

Multipole Oscillations

(sudden offset of the trap)

« Stir »

« Atom Lasers »

Guerin, Riou, Gaebler, Josse, Bouyer, Aspect,
PRL 97 (2006)

Atoms chip

Equations of motion, 1D regime:

Adiabatic approximation:

$$a_{sc} m \omega_{\perp} / \hbar \ll n a_{sc} \ll 1 \quad a_{sc} : \text{s - wave scattering length } (> 0)$$

Order parameter: $\psi(x, t) e^{-i\mu t/\hbar}$

$$i\hbar \partial_t \psi = -\frac{\hbar^2}{2m} \partial_x^2 \psi + [U(x - Vt) + g|\psi|^2 - \mu] \psi$$

$$\Psi(\vec{r}, t) = \psi(x, t) \phi(\vec{r}_{\perp}, n) e^{-i\mu t/\hbar}$$

$$V_{\perp}(\vec{r}_{\perp}) = \frac{1}{2} \omega_{\perp}^2 r_{\perp}^2$$

$$\bullet \quad n(x, t) = \int d^2 r_{\perp} |\Psi|^2 = |\psi(x, t)|^2$$

Longitudinal density

$$\bullet \quad g = 2\hbar \omega_{\perp} a_{sc}$$

Interaction parameter

$$\bullet \quad U(x), \quad U(|x| \rightarrow \infty) = 0$$

Longitudinal potential

$$\bullet \quad \mu$$

Chemical potential

Stationary transmission modes

$$\longrightarrow \psi(x, t) = \psi(X = x - Vt)$$

$$\longrightarrow \begin{cases} \mu = gn_o & \text{Chemical potential} \\ c = \sqrt{\mu / m} & \text{Speed of sound} \\ \xi = \hbar / (mc) & \text{Healing length} \end{cases}$$

Boundary conditions

$$\psi(X \rightarrow -\infty) = \sqrt{n_o}$$

$$\psi(X) = \sqrt{n(X)} e^{iS(X)} \quad \begin{cases} A = \sqrt{n / n_o} \\ v = (\hbar / m) \partial_X S \end{cases}$$

$$\begin{cases} (v - V)n = J_\infty \\ U \frac{dA^2}{dX} = \frac{d}{dX} \left[\frac{\hbar^2}{2m} \left(\frac{dA}{dX} \right)^2 + W(A) \right] \end{cases}$$

Free modes

Transmission coefficient

$$A^2(X) = 1 + \delta\rho(X) \quad \left| \delta\rho \right| \ll \left| \frac{V^2}{c^2} - 1 \right|$$

$$T = \frac{1}{1 + m E_{cl}^+ / (2 \hbar^2 \kappa^2)}$$

$$\kappa = \frac{m}{\hbar} \sqrt{|V^2 - c^2|}$$

Superfluidity

- **Superfluidity** is a phase of matter in which unusual physical effects are observed
 - No viscosity (frictionless flow)
 - quantized vortices that persist forever
 - zero thermodynamic entropy
 - infinite thermal conductivity (no temperature gradient can exist)
 - fountain effect
- superfluidity and superconductivity are low temperature quantum mechanical effects
- The potential applications of superfluids seem more limited compared to those of superconductors

Landau criterion

Assume we have a SF moving at velocity v . In order to create an excitation of the fluid (« heat »), the fluid must have enough energy. Suppose we create an excitation of momentum p and energy $\varepsilon(p)$ (dispersion relation). The fluid loses momentum (and energy), with

$$p = M \delta v$$

where δv the velocity variation and M the fluid mass. The energy variation of the fluid is

$$\delta E = (1/2) M 2 v \delta v = M v \delta v = v p$$

The excitations are then possible if $\delta E \geq \varepsilon(p) \Rightarrow v p \geq \varepsilon(p)$

$$v \geq v_c^L = \min \left(\frac{\varepsilon(p)}{p} \right)$$

$$\Rightarrow \text{if } \varepsilon(p) = \frac{p^2}{2m} \rightarrow v_c^L = 0$$

Interactions!

Landau's (energetical) instability of the superflow

Linear analysis of the stability of the uniform flow under a small perturbation

$$\psi(x) = \phi_0(x) + \delta\phi(x)$$

$$\Rightarrow \delta E = \int dx (\delta\phi, \delta\phi^*) \mathcal{M} \begin{pmatrix} \delta\phi \\ \delta\phi^* \end{pmatrix}$$

- Eigenvalues of $\mathcal{M} < 0$?

$$-\frac{\hbar}{2m} \partial_x^2 \psi + [U(x) + g|\psi|^2] \psi = \mu \psi$$

$$\epsilon(p) = p \sqrt{m^2 c^2 + p^2 / 4} / m$$

Critical velocity

$$v_c^L = \min(\epsilon(p)/p) = c$$

$$c = \sqrt{gn / m}$$

Speed of sound

Superfluidity

- No dissipation
- No drag force
- Perfect transmission
- Normal fluid fraction

In measurements, $v_c \leq v_c^L$

Flow past an impurity

$$U(x) = \lambda \delta(x)$$

$$V \geq c \quad \begin{cases} F \propto V^2 (1 - c^2 / V^2)^2 & 3D \quad \text{Astrakharchik-} \\ F \propto (V^2 - c^2) / V & 2D \quad \text{Pitaevskii 2004} \\ F \propto cst & 1D \quad \text{Pavloff 2002} \end{cases}$$

$$V < c \quad M_n \propto \lambda^2 / c^3$$

Experimental tests

P. Engels, C. Atherton, Phys. Rev. Lett. **99**, 160405 (2007)

Dipolar oscillation + defect

Albert et al., PRL **100**, 250405 (2008)

D. Dries, S. E. Pollack, J. M. Hitchcock and R. G. Hulet, arXiv:1004.1891

Interacting bosons in a random potential: Two contrasting phenomena

Superfluidity

- No dissipation
- No drag force
- Perfect transmission
- Normal fluid fraction

Anderson localization

- Exponential decay
- $T \propto \exp(-L/L_{loc})$
- Large L , no transmission

Interaction

Disorder

Flow through a random potential

{ A series of uncorrelated δ peaks
 Correlated Gaussian potential
 Speckle potential

$$U(x) = \frac{\hbar^2}{mb} \sum_{n=1}^{N_i} \delta(x - x_n)$$

Uncorrelated δ peaks

- $\langle U \rangle = \hbar^2 n_i / (mb)$
- $\langle U(x)U(0) \rangle - \langle U \rangle^2 = \left(\hbar^2 / m \right)^2 \sigma \delta(x)$

$$\left(\langle U \rangle \ll \mu + mV^2 / 2, \quad \sigma = n_i / b^2, \quad n_i = N_i / L \right)$$

Summary of Results

SF Regime: Subsonic motion

$$n(X) = n_0 + \delta n(X)$$

$$\delta n(X) \cong -\frac{mn_0}{\hbar^2 \kappa} \int_{-\infty}^{\infty} dy U(y) \exp[-2\kappa|X - y|]$$

$$\boxed{\kappa = \frac{m}{\hbar} \sqrt{|c^2 - V^2|}} \quad \text{Effective wave vector}$$

$$M_n / M = \frac{\lambda^2 n_i \xi}{2} (1 - V/c)^{-3/2}, \quad (\kappa \gg n_i)$$

Density locally perturbed in the region of the potential,
 → perfect transmission
 → no dissipation nor drag exerted on the potential

Non-perturbative supersonic flow

$$L_{loc}(\kappa) < L$$

$$\lambda_j = m E_{cl}^{(j)} / (2 \hbar^2 \kappa^2)$$

$$P(\lambda, j) d\lambda$$

$$t = j / (\kappa^2 b^2)$$

Diffusion process:

$$\frac{\partial P}{\partial t} = \frac{\partial}{\partial \lambda} \left[\lambda (1 + \lambda) \frac{\partial P}{\partial \lambda} \right]$$

(DMPK Equation)
(Dorokov-Mello-Pereyra-Kumar)

$$\langle \ln T \rangle = - \frac{L}{L_{loc}(\kappa)}$$

$$L_{loc}(\kappa) = \frac{\kappa^2 / \sigma}{\hat{C}(2\kappa)}$$

$$\kappa = \frac{m}{\hbar} \sqrt{|V^2 - c^2|}$$

$$\kappa = \frac{m}{\hbar} \sqrt{|V^2 - c^2|} \rightarrow k$$

$$L_{loc}(\kappa) \rightarrow L_{loc}(k)$$

Antsygina-Pastur-Slyusarev
formula

Uncorrelated delta peaks

$$L / L_{loc}(\kappa) = 0.1, 0.5, 1, 2$$

$$P(T) = \frac{L_{loc}}{L} \exp\left[-\frac{L_{loc}}{L}(1-T)\right]$$

$$P(\ln T) = \sqrt{\frac{L_{loc}}{4\pi L}} \exp\left[-\frac{L_{loc}}{4L}\left(\frac{L}{L_{loc}} + \ln T\right)^2\right]$$

Speckle potential

$$L / L_{loc}(\kappa) = 0.31, 0.52, 0.68$$

$$L / L_{loc}(\kappa) \ll 1$$

$$L / L_{loc}(\kappa) \gg 1$$

Crossover to the time-dependent regime

$$\frac{\partial P}{\partial t} = \frac{\partial}{\partial \lambda} \left[\lambda(1 + \lambda) \frac{\partial P}{\partial \lambda} \right]$$

$$P(\lambda_{\max}, t) = 0$$

$$P_s(t^*) = 1/2$$

$$L^*(\kappa) \approx L_{loc}(\kappa) \left[\ln \left(\frac{V^2}{16c^2} \right) - 1 \right]$$

Crossover from superfluid to time-dependent regime: A problem of extreme value statistics

Slowly varying disordered potential : $\xi / l_c \ll 1$

Albert, Paul, Pavloff, Leboeuf *PRA(R)* 2010

Experiments with Dipole Oscillations

Disordered potential

Disorder:

Theory: M. Albert, T. Paul, N. Pavloff, P. Leboeuf, Phys. Rev. Lett. 100, 250405 (2008)

Experiment: D. Dries, S. E. Pollack, J. M. Hitchcock and R. G. Hulet, arXiv:1004.1891.

Recent experiments on localization with cold atoms

- Orsay : Billy *et al*, *Nature* 2008 : Weakly interacting BEC in a **speckle disorder**

- Firenze : Roati *et al*, *Nature* 2008 : Non interacting cold atoms in a **quasi-periodic potential**

$$\hat{H}_{exp} = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + V_1 \cos^2(k_1 x) + V_2 \cos^2(k_2 x)$$

« Half » way between a periodic and a random system

Anderson localization and nonlinearity in one dimensional disordered photonic lattices

Yoav Lahini¹, Assaf Avidan¹, Francesca Pozzi², Marc Sorel², Roberto Morandotti³, Demetrios N. Christodoulides⁴ and Yaron Silberberg¹

$$i \frac{\partial A_k}{\partial z} = -C (A_{k+1} + A_{k-1}) + \gamma |A_k|^2 A_k + \epsilon_k A_k$$

- C : tunneling rate between adjacent waveguides
- ϵ_k : on-site energy
- γ : strength of the nonlinearity of the medium (>0 , self defocusing medium)
- Paraxial approximation

Superfluid Motion of Light

$$i \frac{\partial A_k}{\partial z} = -C (A_{k+1} + A_{k-1}) + \gamma |A_k|^2 A_k + \epsilon_k A_k$$

Non-linear equation, similar to the mean-field description of cold atoms => does superfluid motion of light exists?

$$v_c^L \propto \sqrt{\gamma C |A|^2} \approx 2 \times 10^{-2}$$

Critical speed

$$\epsilon_k = \epsilon_0 + \frac{1}{2} \omega^2 k^2 + U_0 \delta_{k,0}$$

(or altern. twisted array of waveguides)

$$\epsilon_k = \epsilon_0 + \frac{1}{2}\omega^2 k^2 + U_0 \delta_{k,0}$$

P. Leboeuf, S. Moulieras
to appear

Concluding remarks

- *We provide analytical and numerical evidence of AL in the presence of interaction for # types of disorder. Generalization of DMPK diffusion equation to include interactions. No localization as L goes to infinity.*
- *Renormalization of the localization length due to interactions*
- *The flow is SF at small velocities either for an obstacle or a disordered potential*
- *Existence of a time-dependent regime, at intermediate velocities or large sample lengths*
- *Estimates of crossovers to an unstable flow, either from the sub or supersonic regimes*
- *Dipole oscillations. Experiments on BEC show SF as well as the presence of unstable flow in presence of disorder*
- *Extension of the concept of superfluidity to motion motion of light in NL media*

Some Related References

- # [Bose-Einstein beams: coherent propagation through a guide](#)
P. Leboeuf, N. Pavloff
Phys. Rev. A 64 (2001) 033602
- # [Breakdown of superfluidity of an atom laser past an obstacle](#)
N. Pavloff
Phys. Rev. A 66 (2002) 013610
- # [Solitonic transmission of Bose-Einstein matter waves](#)
P. Leboeuf, N. Pavloff, S. Sinha
Phys. Rev. A 68 (2003) 063608
- # [Nonlinear transport of Bose-Einstein condensates through waveguides with disorder](#)
T. Paul, P. Leboeuf, N. Pavloff, K. Richter, P. Schlagheck
Phys. Rev. A 72, 063621 (2005)
- # [Superfluidity versus Anderson localization in a dilute Bose gas](#)
T. Paul, P. Schlagheck, P. Leboeuf, N. Pavloff
Phys. Rev. Lett. 98, 210602 (2007)
- # [Dipole oscillations of a BEC in the presence of defects and disorder](#)
M. Albert, T. Paul, N. Pavloff, P. Leboeuf
Phys. Rev. Lett. 100, 250405 (2008)
- # [Anderson localization of a weakly interacting one-dimensional Bose gas](#)
T. Paul, M. Albert, P. Schlagheck, P. Leboeuf, N. Pavloff
Phys. Rev. A 80, 033615 (2009)
- # [Localization by bichromatic potentials versus Anderson localization](#)
M. Albert, P. Leboeuf
Phys. Rev. A **81**, 013614 (2010)
- # [Breakdown of superfluidity of a matter wave in a random environment](#)
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- # [Superfluid motion of light](#)
P. Leboeuf, S. Moulieras
to appear