Seismic Coda Waves

L. Margerin

CNRS, Toulouse, France

In collaboration with B. Van Tiggelen, M. Campillo, E. Larose, C. Sens-Schoenfelder, V. Rossetto, N. Shapiro . . .

Students: D. Anache-Ménier, T. Planès, A. Obermann

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Mesoscopic physics in complex media

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Content

- 1 Observation of seismic coda waves
- 2 Equipartition of seismic waves
- 3 Green function reconstruction
- 4 Monitoring the Earth with coda waves
- 5 Weak localization of seismic waves
- 6 Phase statistics of coda waves

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Crustal Coda Waves

Crustal Earthquakes in Japan

Space and Time Scales

- Frequency > 1 Hz
- Wavelength $\sim$ tens of meters to kilometers
- Mean free path??

Inner Core Coda Waves

Vidale and Earle, Nature (2000)

Mesosopic physics with seismic waves

- Equipartition (Hennino et al., PRL, 2001)
- Weak localization (Larose et al., PRL, 2004)
- Green function retrieval from coda waves (Campillo and Paul, Science, 2003)
- Green function retrieval from noise (Aki, Bull. Earthquake Res. Institute, 1956 ; Shapiro and Campillo, GRL, 2004)

Tomography

Monitoring

- Coda wave interferometry (Diffusing Acoustic Wave Spectroscopy) (Poupinet et al., JGR, 1984; Sens-Schoenfelder and Wegler, GJI, 2006; Brenguier et al., Nature Geoscience 2007)
- ...

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Pynion Flats Observatory Array

Data from California

Array Analysis

Wagner, JGR, 103, 1998

Stablization of Vertical to Horizontal Kinetic Energy Ratio

$$V^2 = \int_{t-T}^{t+T} \rho \omega^2 u_z^2(\omega, \tau) d\tau$$

$$H^2 = \int_{t-T}^{t+T} \rho \omega^2 (u_x^2(\omega, \tau) + u_y^2(\omega, \tau)) d\tau$$

Frequency-dependent V^2/H^2 partition ratio

Equipartition

“In a small frequency band, all the modes are excited to equal energy”
(Weaver, 1982)

"Reference" Earth = Layered Half-Space

vs1,d1,vp1,h1

vsi,di,vpi,hi

vsn,dn,vpn,hn

Half-Space

$$\mathbf{u}(t, \mathbf{r}) = \sum_n a_n \phi_n(\mathbf{r}) e^{-i\omega_n t}$$

"True" Earth = Disordered Half-Space

vs1,d1,vp1,h1

vsi,di,vpi,hi

vsn,dn,vpn,hn

Half-Space

$$\mathbf{u}(t, \mathbf{r}) = \sum_n a_n(t) \phi_n(\mathbf{r}) e^{-i\omega_n t}$$

Assumption: $\langle a_n(t) a_{n'}^*(t) \rangle = \sigma^2(t) \delta_{nn'}$

Consequence: $\langle u_i(\mathbf{x}_A, \omega) u_j(\mathbf{x}_B, \omega)^* \rangle \approx C \operatorname{Im} G_{ij}(\mathbf{x}_A, \mathbf{x}_B, \omega)$

Theory vs Observations

Velocity Model
$h_1 = 4\text{m}$ $\alpha_1 = 300\text{m/s}$ $\beta_1 = 150\text{m/s}$ $\rho_1 = 2200\text{kg/m}^3$
$h_2 = 11\text{m}$ $\alpha_2 = 900\text{m/s}$ $\beta_2 = 500\text{m/s}$ $\rho_2 = 2200\text{kg/m}^3$
$h_3 = 50\text{m}$ $\alpha_3 = 3100\text{m/s}$ $\beta_3 = 1600\text{m/s}$ $\rho_3 = 2700\text{kg/m}^3$
$\alpha_\infty = 5400\text{m/s}$ $\beta_\infty = 3000\text{m/s}$ $\rho_\infty = 2700\text{kg/m}^3$

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Experimental Set-Up

Cross-correlation of coda waves

- $C_{ij}(\mathbf{x}_a, \mathbf{x}_b, \tau) = \int u_i(\mathbf{x}_a, t - \tau/2) u_j(\mathbf{x}_b, t + \tau/2) dt$
- Source average
- Reconstruction of Rayleigh and Love waves fundamental mode
- Temporal asymmetry

Understanding the temporal asymmetry

Results from 2-D F.D. simulation

Early coda

Late coda

$$\frac{d\langle u(\mathbf{R} - \mathbf{r}/2, t - \tau/2)u(\mathbf{R} + \mathbf{r}/2, t + \tau/2) \rangle}{d\tau} \propto$$

$$E(\mathbf{R}, t) [G(\mathbf{r}, \tau) - G(\mathbf{r}, -\tau)] - 3\mathbf{J}(\mathbf{R}, t) \cdot \nabla_{\mathbf{r}} [G(\mathbf{r}, \tau) - G(\mathbf{r}, -\tau)]$$

(B.V.T., PRL, 2003)

GF reconstruction with the coda of noise based GF

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Earthquake doublets

- Same location
- Same mechanism
- Nearly identical waveforms
- Drawback: no continuous monitoring

Repeated shots recorded on Merapi Volcano

- No changes on the first arrival
- Increasing delay time in the coda
Stretching of the signal $\approx$ tiny velocity change in the medium
- Active source experiments are expensive

Monitoring with seismic noise

Example on Piton de la Fournaise

- Green function obtained from cross-correlation of seismic noise
- Measurement of stretching parameter in the coda

“Passive image interferometry” (Sens-Schoenfelder and Wegler, GJI, 2006; Brenguier et al., Nature Geoscience, 2008; Brenguier et al., Science, 2008)

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Specific issues in seismology

Detection operates at the surface

Earthquakes sources are often at depth

Problem 1: Weak localization has size of a wavelength around the source

Problem 2: The source mechanism

Enhanced backscattering in a 2-D chaotic cavity

Role of the source

Monopole

Dipole

De Rosny et al., PRE, 2001

Small-scale seismic experiment

Observation of weak localization

Rise time $\approx$ mean free time

Frequency dependence

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Phase Statistics

- Phase of the analytic signal
- Easy with seismic waves: spatial and temporal resolution
- Free from effect of absorption
- Simplifying assumptions :
Analyze the phase field of **vertical components**
Wavefield obeys **Circular Gaussian Statistics**:

Data from California

Field and Intensity

2-Point Statistics

$\Delta\phi$: Phase difference $\in (-2\pi, 2\pi]$

$\Delta\phi_u$: Unwrapped phase difference $\in (-\pi, \pi]$

Higher Phase Derivatives

Fitting Parameters:

Coefficients of Taylor series of field correlation function

Universal behavior for n^{th} derivative:

$$\text{slope} = -(1 + 2/n)$$

Phase Difference Correlation

Correlation function $\approx \frac{1}{r} e^{-r/2\ell}$

Van Tiggelen et al., *EPL*, 2006; Anache et al., *P.R.L.*, 2009