Playing with ultrasound in complex media

Arnaud Tourin

✉ bgh@h.ub[Yj]b'!`9 GD7 =DUf]gž: fUbWY

Playing with ultrasound in complex media

Arnaud Tourin

Cargèse – July, 13, 2010

The Langevin Institute

- ❑ Founded in january 2009 to gather people from
 - LOA (M. FINK) : ultrasound, sound, seismic waves, microwaves
 - LOP (C. BOCCARA) : optics, ultrasound
- ❑ New devices for manipulating waves in various kinds of media
 - Time Reversal Mirror
 - Multiwave Imaging
 - IR SNOM, thermal radiation STM
- ❑ Medical imaging & therapy,
non destructive testing,
underwater acoustics,
seismology,
telecommunications,
tactile objects,...
- ❑ Fundamental physics/applications/companies
 - Echosens, Sensitive Object, SSI, TRCOM, LLTech (~200)

The Langevin Institute

92 people

- 34 permanent researchers
- 14 for the technical & administrative support
- 29 Phd students
- 15 Post-Doc
- One Director (M. Fink) and two vice directors (R. Carminati and AT)

5 groups

- **Acoustic waves & microwaves in complex media**
- Nanophotonics & Optical waves in diffusive media
- Wave Physics for medicine & Biology
- Detection, Imaging, and characterization

Acoustic waves & microwaves in random media

Julien de Rosny
(Research scientist)

Arnaud Tourin
(Professor)

Arnaud Derode
(Professor)

Mathias Fink
(Professor)

Agnès Maurel
(Research scientist)

Geoffroy Lerosey
(Research scientist)

Marie Müller
(Associate professor)

Abdel Ourir
(Engineer)

Alice Bretagne
(PhD student)

OUTLINE

□ Wave propagation in random media

- ➡ The ballistic and coherent waves
- ➡ The incoherent wave and the diffusion approximation
- ➡ Beyond the DA : the backscattering cone

□ Using disorder for controlling wave propagation

- ➡ Disorder for guiding ultrasound
- ➡ Disorder for focusing ultrasound
- ➡ A random (or ordered) bubbly medium for filtering US

Mesoscopic physics with ultrasound

Experimental approach : measuring the S matrix

Transducer arrays

$f \sim 500 \text{ kHz} \text{ à } 5 \text{ MHz}$

$\lambda \sim \text{mm}$

$$S = \begin{pmatrix} r^{-+} & t^{--} \\ t^{++} & r^{+-} \end{pmatrix}$$

In optics, *S. Popoff et al., Phys. Rev. Lett. 104, 100601 (2010)*

Experimental approach : measuring the S matrix

- ❑ (average) time-resolved transmitted amplitude / intensity
- ❑ (average) backscattered amplitude or intensity (BS Cone)
- ❑ Perform a SVD and study the statistics of the singular values

- ❑ Focusing using time reversal, wavefront shaping, inverse filter

Experimental approach : what can be measured with ultrasound ?

Diameter : 0.8 mm

Density : 19 rods / cm²

5mm < L < 80mm

Experimental approach

Single transducer
 $\nu=3.2$ MHz, $\lambda=0.48$ mm

2D random sample

128-transducer array
Pitch : 0.42 mm

Spatio-temporal distribution of the amplitude

Water

L=7 mm

Spatio-temporal distribution of the amplitude

Spatio-temporal distribution of the amplitude

$L=70$ mm

The wave equation in a heterogeneous medium

Classical (acoustic) wave

Quantum wave

$$\Delta \Psi(\mathbf{r}, t) = \frac{1}{c(\mathbf{r})^2} \frac{\partial^2 \Psi(\mathbf{r}, t)}{\partial t^2}$$

$$k_0^2 = \frac{\omega^2}{c_0^2}$$

$$\sigma(\mathbf{r}) = k_0^2(\omega) - k^2(\omega)$$

$$\left(-\frac{\hbar^2}{2m} \Delta + V(\mathbf{r}) \right) \Psi(\mathbf{r}, t) = i\hbar \frac{\partial \Psi(\mathbf{r}, t)}{\partial t}$$

$$k_0^2 = \frac{2m\omega}{\hbar^2}$$

$$\sigma(\mathbf{r}) = \frac{2mV(\mathbf{r})}{\hbar^2}$$

$$\Delta \Psi(\omega, \mathbf{r}) + k_0^2 \Psi(\omega, \mathbf{r}) = \sigma(\mathbf{r}) \Psi(\omega, \mathbf{r})$$

$$\Psi(\omega, \mathbf{r}) = \Psi_0(\omega, \mathbf{r}) + \int_V G_0(\omega, \mathbf{r} - \mathbf{r}') \sigma(\mathbf{r}') \Psi(\mathbf{r}') d\mathbf{r}'$$

The wave equation in a heterogeneous medium

The Born expansion

Ψ

$= \Psi_0$ unscattered wave

$+ \int G_0(\omega, \mathbf{r} - \mathbf{r}_1) \sigma(\mathbf{r}_1) \Psi_0(\mathbf{r}_1) d\mathbf{r}_1$ single scattered wave

$+ \iint G_0(\omega, \mathbf{r} - \mathbf{r}_1) \sigma(\mathbf{r}_1) G_0(\mathbf{r}_1 - \mathbf{r}_2) \sigma(\mathbf{r}_2) \Psi_0(\mathbf{r}_2) d\mathbf{r}_1 d\mathbf{r}_2$ double scattered wave

$+ \dots$

The Dyson equation

$$\Delta G(\omega, \mathbf{r}, \mathbf{r}') + k_0^2 G(\omega, \mathbf{r}, \mathbf{r}') = \sigma(\mathbf{r}) G(\omega, \mathbf{r}, \mathbf{r}') + \delta(\mathbf{r} - \mathbf{r}')$$

$$\langle G(\omega, \mathbf{r}, \mathbf{r}') \rangle = G_0(\omega, \mathbf{r} - \mathbf{r}') + \int G_0(\omega, \mathbf{r} - \mathbf{r}_1) \langle \sigma(\mathbf{r}_1) G(\omega, \mathbf{r}_1, \mathbf{r}') \rangle d\mathbf{r}_1$$

$$\langle G(\omega, \mathbf{r} - \mathbf{r}') \rangle = G_0(\omega, \mathbf{r} - \mathbf{r}') + \int \int G_0(\omega, \mathbf{r} - \mathbf{r}_1) \Sigma(\omega, \mathbf{r}_1 - \mathbf{r}_2) \langle G(\omega, \mathbf{r}_2 - \mathbf{r}') \rangle d\mathbf{r}_1 d\mathbf{r}_2$$

$$I_{\text{coh}} = \left| \langle G_{\omega}(\mathbf{r}, \mathbf{r}_0) \rangle \right|^2 \propto \frac{\exp i k_e |\mathbf{r} - \mathbf{r}_0|}{4\pi |\mathbf{r} - \mathbf{r}_0|} \exp - \frac{|\mathbf{r} - \mathbf{r}_0|}{\ell_s}$$

Beer-Lambert

$$l_s = 1/n\sigma$$

Scattering mean free path

How to build an estimator of the ensemble average ?

Source

*Receiving
array*

The coherent wave

$L=30$ mm

average on 80
Configurations

The coherent wave

$$\ell = 4 \text{ mm}$$

$$1/\ell = 1/\ell_s + 1/\ell_a$$

- Ballistic amplitude
- Ballistic amplitude averaged over the 127 transducers
- △ Ballistic amplitude averaged over 80 random sample positions

The coherent wave

The coherent wave

Scattering from a single inclusion

$$\Psi(\vec{r}) \xrightarrow{r \rightarrow \infty} \exp(i\vec{k} \cdot \vec{r}) + f(\vec{k}, \vec{k}') \frac{\exp(ikr)}{r}$$

Scattering cross section

$$\sigma = \frac{P_S}{dP_{inc} / dS} = 2\pi \int_0^\pi \sin \theta |f(\theta, k)|^2 d\theta$$

Optical theorem

$$\sigma = \frac{4\pi}{k_0} \text{Im} f(k_0, 0)$$

Scattering from a single inclusion

Scattering from a single inclusion

$$p = p_{rigid} + p_{elastic}$$

$$\rightarrow \sigma = \sigma_{rigid} + \sigma_{elastic} + interference$$

The incoherent wave

$L=70$ mm

The Bethe Salpeter equation

The incoherent intensity

$$I_{\text{incoh}} = \left\langle |G(\mathbf{r}, \mathbf{r}_0; t)|^2 \right\rangle$$

$$\begin{aligned} \langle G(\mathbf{r}, \mathbf{r}_0; \omega - \Omega/2) G^*(\mathbf{r}', \mathbf{r}'_0; \omega + \Omega/2) \rangle &= \langle G(\mathbf{r}, \mathbf{r}_0; \omega - \Omega/2) \rangle \langle G^*(\mathbf{r}', \mathbf{r}'_0; \omega + \Omega/2) \rangle + \\ &\int \langle G(\mathbf{r}, \mathbf{r}_1; \omega - \Omega/2) \rangle \langle G^*(\mathbf{r}', \mathbf{r}_2; \omega + \Omega/2) \rangle U(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3, \mathbf{r}_4) \langle G(\mathbf{r}_3, \mathbf{r}_0; \omega - \Omega/2) G^*(\mathbf{r}_4, \mathbf{r}'_0; \omega + \Omega/2) \rangle d\mathbf{r}_1 d\mathbf{r}_2 d\mathbf{r}_3 d\mathbf{r}_4 \end{aligned}$$

- Boltzmann approximation $\Rightarrow$ Transfer radiative equation for the specific intensity

- Diffusion approximation $\Rightarrow$ Diffusion equation $\frac{\partial I_{\text{incoh}}}{\partial t} = D \Delta I_{\text{incoh}} \quad D = \frac{1}{3} V_E \ell^*$

The incoherent wave

□ Time-of-flight distribution

$$I_{\text{incoh}} = \langle |G|^2 \rangle$$

$$D = 3.2 \text{mm}^2 / \mu\text{s}$$

$$\ell^* = 4.8 \text{mm}$$

$$\ell_a = 350 \text{mm} \quad (\tau_a = 250 \mu\text{s})$$

□ Total transmission varies as $\sim \ell / L$ (ohm's law)

Beyond the diffusion approximation

The backscattering cone

M.P. van Albada and A. Lagendijk, Phys. Rev. Lett. **55**, 2692 (1985)

E. Wolf and G. Maret, Phys. Rev. Lett. **55**, 2696 (1985)

A. Tourin, B. A. Van Tiggelen, A. Derode, P. Roux, M. Fink, Phys. Rev. Lett **79**, 3637 (1997)

The backscattering cone

$$t=11\mu s, \Delta T=5\mu s$$

Speckle

One realisation

Averaging over
80 realisations

The backscattering cone

$$I = |A|^2 = \sum_{i,j} A_i A_j^*$$

Speckle

Incoherent contribution

Coherent contribution

The backscattering cone

The backscattering cone

Dynamic CBE

Stationary CBE

Weak localisation

cf. John Page's lecture

**Using disorder
for controlling wave propagation**

1. A random fiber

H. De Raedt, Ad Lagendijk, Pedro de Vries

« *Transverse localization of light* »

Phys. Rev. Lett. **62**, 47 (1989)

$$2ikn_0 \frac{\partial \psi}{\partial z} = H\psi$$

$$\begin{cases} H \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + V(x, y) \\ V(x, y) \equiv k^2 (n^2(x, y) - n_0^2) \end{cases}$$

Lee, P. A. et al., Rev. Mod. Phys. **57**, 287-337 (1985)

$$\xi_{loc} = l^* \exp\left(\frac{\pi}{2} k_F l^*\right)$$

$$k_F \equiv k_T \propto \omega_0^{-1}$$

1. A random fiber

Schwartz et al., Nature **446**, 52-55 (2007)

**Intensity distribution
after $L=10$ mm propagation**

hexagonal lattice 15% positional disorder
(average over 100
realizations)

1. A random fiber

Alice Bretagne, AT

Ordered sample
(triangular lattice, $a = 2.4 \text{ mm}$)

Disordered sample

Ordered (blue)
Disordered (red)

2. A random lens

Transducer arrays

$f \sim 500 \text{ kHz à } 5 \text{ MHz}$

$\lambda \sim \text{mm}$

$$F < \ell_s$$

2. A random lens

2. A random lens

Waveform transmitted in water and received at transducer # 64

Waveform transmitted through the rods and received at transducer #64

Time reversed wave recorded at the source location

2. A random lens

Spatial Focusing : the hyperfocusing effect

Beamwidth at -12 dB : 1 mm (scattering medium) / 35 mm (free space)

Spatial resolution does not depend on the array aperture !

2. A random lens

One-channel time reversal mirror

Directivity patterns of the time-reversed waves

2. A random lens

An alternative approach : wave front shaping

- *I.M. Vellekoop and A.P. Mosk*
Optics Lett. 32, 2309 (2007)
- *E. Herbert, M. Pernot, M. Tanter, G. Montaldo, M. Fink*
IEEE transactions on ultrasonics, ferroelectrics, and frequency control **56** (11), 2388 (2009)

Two problematic issues

- low sensitivity (one by one)
- a lot of iterations are needed

2. A random lens

In the case of two transducers, the intensity at the intended focus is :

$$I = \frac{P_0^2}{\rho c^2} [A_1^2 + A_2^2 + 2A_1A_2 \cos(\Phi_1 - \Phi_2)]$$

i.e., $I = B + C \cos(\Phi)$

When a phase x is added to the 2nd transducer, the intensity at focus is :

$$I(x) = B + C \cos(\Phi + x)$$

At least 3 measurements must be performed

Finally, A_1 and $A_2 e^{-i\Phi}$ are transmitted from the two transducers

2. A random lens

Extension to a N-transducer array using spatial coded excitations

Hadamard matrix

$$H = \frac{1}{2} \begin{matrix} & \begin{matrix} V_0 & V_1 & V_2 & & \end{matrix} \\ \begin{pmatrix} 1 & 1 & 1 & 1 & \dots \\ 1 & -1 & 1 & -1 & \\ 1 & 1 & -1 & -1 & \\ 1 & -1 & -1 & 1 & \\ \dots & & & & \dots \end{pmatrix} \end{matrix}$$

Transmission of
 $V_0(x, \omega) + e^{jx} \cdot V_i(x, \omega)$
 $(1 < i < N)$

Recording of $I = I(x, \omega)$
at focus

Repeat the transmission
for four different values
 $(x=0, \pi, \pi/2, -\pi/2)$

2. A random lens

$$\tan(\Psi_i) = \frac{I(x = \pi / 2) - I(x = -\pi / 2)}{I(x = \pi) - I(x = 0)}$$

$$B_i = \frac{\sqrt{I(0) + I(\pi) - I(\pi / 2) + I(-\pi / 2)}}{2}$$

$$\begin{pmatrix} A_1 e^{j\Phi_1} \\ A_2 e^{j\Phi_2} \\ \dots \\ A_i e^{j\Phi_i} \\ A_N e^{j\Phi_N} \end{pmatrix} = H \begin{pmatrix} B_1 e^{j\Psi_1} \\ B_2 e^{j\Psi_2} \\ \dots \\ B_i e^{j\Psi_i} \\ B_N e^{j\Psi_N} \end{pmatrix}$$

2. A random lens

Definition of
spatio-temporal
vectors

$V_1(t)$

$V_{31}(t)$

$V_{41}(t)$

2. A random lens

A. Bretagne, J. Aulbach, M. Tanter, M. Fink, A. Tourin

Typical waveform received at the focal point
after a Hadamard emission

2. A random lens

Monochromatic case

2. A random lens

Broadband case

Time Reversal

Wavefront shaping

2. A random lens

Broadband case

3. A random (or periodic) bubbly medium

Minnaert Resonance

$$\omega_M = \sqrt{\frac{k}{m}} = \frac{1}{R_0} \sqrt{\frac{3\beta_{\text{air}}}{\rho_{\text{eau}}}}$$

$$R = 10 \mu\text{m} \quad \longrightarrow \quad f_M = 300 \text{ kHz}$$

$$\sigma_s \gg \sigma_{\text{geo}}$$

M. Kafesaki et al.

Air Bubbles in water : a strongly Multiple Scattering Medium for Acoustic Waves

Phys. Rev. Lett. **84**, 6050 (2000)

10 % volume fraction

3. A random (or periodic) bubbly medium

V. Leroy, A. Bretagne, M. Fink, H. Willaime, P. Tabeling, A. Tourin
« Design and characterization of bubble phononic crystals »
App. Phys. Lett. 95, 171904 (2009)

$h=78\text{ }\mu\text{m}$, $a_x=200\text{ }\mu\text{m}$, $a_z=360\text{ }\mu\text{m}$

NATURE|Vol 462|24/31 December 2009

APPLIED PHYSICS

Bubbly but quiet

Edwin L. Thomas

An array of air bubbles in a rubber-like material can be made to block the transmission of sound. This finding might help in the design of soundproof walls for music rooms and urban apartments.

Thank you for your attention !